A method introducing deteriorating bond at the interfaces and declining load transfer at the joints in the existing design methods for concrete pavements

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Abstract

Most of the concrete highways of the Belgian road system have a structure composed of a CRCP surface layer, a thin intermediate asphalt bond layer and a lean concrete base layer on one or more granular sub bases**.**

They are designed as an equivalent composite rigid layer on an elastic granular sub grade. The equivalent Young's modulus of the composite layer is assumed to be equal to the modulus of the concrete surface layer, the thickness is assumed to be equal to the sum of the thickness of the three layers; the equivalent moment of inertia and the equivalent stiffness depend on the values of the thickness and the moduli of each layer and the bond conditions between the layers.

Several investigations regularly performed on pavements currently thirty years old have shown a systemic deterioration of the bond between the surface layers and of the load transfer at the cracks of the CRCP or at the joints of concrete slabs.

In this paper we present a mathematical model that permits the computation of the influence of this systemic deterioration on the lifetime of the pavements.

1. Introduction

For many years engineers have known that the mechanical strength of a structure diminishes over time as a result of fatigue.
One of the consequences is that the lifetime of a road can be estimated from the laws governing the fatigue performance of the materials from which it is made as determined in the laboratory.
This is the basis from which the design methods used nowadays have been developed.

This for example is the case for the design software of the Walloon Ministry of Equipment and Transportation (MET, Belgium), presented at the 9th International Symposium on Concrete Roads (Leonard et al, 2004).

Nonetheless road managers have observed that the lifetimes seen in reality are shorter, sometime markedly so, than those indicated by calculation. For example,

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most of our highways built since forty years with a three layered pavement structure, actually show an accelerating deterioration with the appearance of longitudinal cracks. Bond between the layers has completely disappeared. Computations performed in the hypothesis of full bond during the whole life of the pavement lead to a probability of failure after 40 years of 0.3 %; this structure is over designed. However computations performed with the varying bond values utilized in the example of section 2.4 lead to a probability of failure after 40 years of 12.2 %, value generally accepted for important highways.

Numerous reasons have been forward as possible explanations of this phenomenon: inadequacy of the computation model, the argument favored by those in the field; poor execution, lack of maintenance, overloaded vehicles, which are the arguments favored by the theoreticians.

Both camps are right. Mechanical strength is undermined not only by fatigue, but also because features of the structure as such deteriorate, such as changes in the mechanical characteristics of the layers of the structure, changes in the bond at the interfaces, and changes in load transfer at joints.

In this contribution we describe two refinements that have been recently made to the model underlying the MET software in the hope that in this way the physical reality will be better reflected and that this will be of service to both theoretical and field engineers.

First we consider deteriorating of bond between the pavement layers and secondly declining of load transfer at the joints.

We define bond, varying from 1 (full bond) to 0 (full slip), as the mean value measured by the Iowa's shear device on a number of cores taken out of a sufficiently large pavement surface at a given moment of pavement's life (M.J. Knutson ,1981).

We define load transfer as the ratio of the actual shear force transfer at the joint to the shear force in a slab without joints or cracks (CRCP). This ratio also varies from 1 (full transfer) to 0 (no transfer). It can be determined by means of deflection measurements at both sides of the joint. When the deflections are equal, the load transfer ratio is equal to 1. When the deflection at the surface of the unloaded side of the joint is 0, the load transfer ratio is equal to 0. Many relations for the in between values have been proposed. We use a relation developed by Stet (Van Cauwelaert, Stet, 1998).

We develop our model in four steps.

First we propose a model for calculating the lifetime of a concrete pavement that takes the decline in bond between the pavement and an asphalt base course into account.

We further present a second model for calculating the lifetime of an asphalt layer that considers the decline in bond between the asphalt layer and an underlying lean concrete layer and extends the model by taking into account the moment at which the lean concrete fails.

We then develop the general model of a concrete pavement on an intermediate asphalt layer on a lean concrete layer taking into account the moment at which the lean concrete fails.

We have some field information about an approximate linear decline of bond between the pavement layers measured on cores taken after 10, 20 and 30 years.
We generally observed total loss of bond after 30 or 40 years between the concrete pavement and the under laying asphalt layer. Hence we have made the assumption that the bond declines linearly over time (from 1 to 0 in thirty years) but, in order to simplify the computations, we assume that this diminution progresses in steps over periods of ten years.

The bond values currently used in the MET software are presented in table 1:

Table 1. Bond between pavement layers vs time

Periods	0 – 10 yrs	10 – 20 yrs	20 – 30 yrs	30 –
Concrete on Asphalt	1	2/3	1/3	0
Asphalt on Concrete	½	1/3	1/6	0
Concrete on Concrete	½	0	0	0

Finally we develop a concrete pavement model that takes account of the reduction of load transfer at cracks or joints.

The transfer values currently used in the MET software depend on the type of joint as follows

Transfer at cracks in CRCP: 80 %.
Transfer at the joints of dowelled slabs: 50 %.
Transfer at the joints of non-dowelled slabs: 20 %.

The practical calculations are mainly based on Westergaard's model. For the case of an asphalt layer on a lean concrete layer, the calculations are based on Burmister's model. The two models are described in detail in Van Cauwelaert (2004).

The models used for the calculation of lifetimes are based on the usual fatigue laws, which are expressed here in simple form for the clarity of the calculations.

For layers with a hydraulic binder

$$\log N(i) = c\left(1 - \frac{\sigma_{fl}(i)}{\sigma_{br}(i)}\right) \tag{1}$$

where

$\sigma_{fl}(i)$ is the bending stress at the bottom of the layer corresponding to period i.

$\sigma_{br}(i)$ is the failure stress of the layer corresponding to period i.
c is a constant determined by fatigue tests on the material concerned.

The value of c used in the MET software is $c = c1 = 15$ for pavement concrete and $c = c2 = 14$ for lean concrete.

For bituminous binders

$$N(i) = \left(\frac{\varepsilon_{br}(i)}{\varepsilon_{fl}(i)} \right)^c \tag{2}$$

where

$\varepsilon_{fl}(i)$ is the strain at the bottom of the layer corresponding to period i.
$\varepsilon_{br}(i)$ is the failure strain of the layer corresponding to period i.
c is a constant determined by fatigue tests on the material concerned.

The value of c used in the MET software is $c = 4.76$ for all bituminous mixes.

2. Influence of changes in bond conditions on the residual lifetime of a concrete pavement on a bituminous base

Consider a highway consisting of a concrete pavement on a bituminous base.

The characteristics of the structure deteriorate in a regular fashion with time as the result of the decline in bond with the bituminous layer. This deterioration takes place unnoticeably, by infinitesimal increments, as time goes by. Even if it should prove to be possible, which we doubt, it does not seem to be reasonable to formulate a mathematical model capable of taking a phenomenon of this kind into account.
Consequently we propose varying the characteristics in a discrete manner by periods of a number of years. The observations available to us have led us to choose for ten year periods.

The following is the first proposed model. In it we have limited ourselves to the determination of the lifetime of the concrete surface layer, ignoring the fatigue sustained by the bituminous base.

The computations of the stresses in the concrete surface layer are based on Westergaard's multilayer model (Stet, Van Cauwelaert, 2004). This model allows considering bond variation between the concrete surface layer, the bituminous intermediate layer and the lean concrete base.

Consider a period i.

Let N(i) be the allowed number of axle loads for the period considered as determined by the fatigue law, where in each period of its lifetime the highway is considered as new.

Let Q(i) be the number of axle loads anticipated by the designer for the same period.
Let NC(i) be the allowed number of axle loads for this period after correction for the cumulative fatigue of the preceding periods.

Consider the successive periods.

2.1. Period 1

The fatigue law for the new structure writes (1):

$$\log N(1) = c\left(1 - \frac{\sigma_{fl}(1)}{\sigma_{br}(1)}\right)$$

Let us calculate the value of $\sigma_{br}(2)$ after the passage of Q(1) axle loads.

$$\log[N(1) - Q(1)] = c\left(1 - \frac{\sigma_{fl}(1)}{\sigma_{br}(2)}\right)$$

Hence

$$\sigma_{br}(2) = \frac{c - \log N(1)}{c - \log[N(1) - Q(1)]}\sigma_{br}(1)$$

2.2. Period 2

The fatigue law for the new structure is written (1):

$$\log N(2) = c\left(1 - \frac{\sigma_{fl}(2)}{\sigma_{br}(1)}\right)$$

The corrected fatigue law for a structure that has undergone the passage of Q(1) axle loads is written:

$$\log NC(2) = c\left(1 - \frac{\sigma_{fl}(2)}{\sigma_{br}(2)}\right) \quad (3)$$

This results in

$$log\ NC(2) = \left\{c - \frac{(c - log\ N(2))}{(c - log\ N(1))}(c - log[N(1) - Q(1)])\right\} \quad (4)$$

Let us calculate the value of $\sigma_{br}(3)$ after the passage of Q(2) axle loads.

$$\log\left[NC(2)-Q(2)\right]=c\left(1-\frac{\sigma_{fl}(2)}{\sigma_{br}(3)}\right)$$

Hence

$$\sigma_{br}(3)=\frac{\left(c-\log NC(2)\right)\left(c-\log N(1)\right)}{\left(c-\log\left[NC(2)-Q(2)\right]\right)\left(c-\log\left[N(1)-Q(1)\right]\right)}\sigma_{br}(1) \qquad (5)$$

2.3. Period n

Relations (4) and (5) can be easily generalized for period n

$$logNC(n)=\left\{c-\frac{\left(c-logN(n)\right)\ldots\left(c-log\left[NC(2)-Q(2)\right]\right)}{\left(c-logN(n-1)\right)\ldots\left(c-logNC(2)\right)\left(c-logN(1)\right)}\left(c-log\left[N(1)-Q(1)\right]\right)\right\}$$

(6)

$$\sigma_{br}(n)=\frac{\left(c-\log NC(n-1)\right)\cdots\left(c-\log NC(2)\right)\left(c-\log N(1)\right)}{\left(c-\log\left[NC(n-1)-Q(n-2)\right]\right)\cdots\left(c-\log\left[NC(2)-Q(2)\right]\right)\left(c-\log\left[N(1)-Q(1)\right]\right)}\sigma_{br}(1)$$

2.4. Example

Consider a two-layered structure consisting of dowelled concrete slabs laid on a bituminous base on a sub grade with following characteristics:

Concrete: *E1 = 30000 N /mm², h1 = 200 mm,* $\mu 1 = 0.2$*, c1 = 15,* $\sigma_{br} = 6$ *N/mm².*
Asphalt: *E2 = 8000 N/mm², h2 = 50 mm,* $\mu 2 = 0.3$*.*
Sub grade: *k = 0.05 N/mm³.*
Load: *square with a side of 200 mm, p = 1.625 N/mm² (axle load of 130 kN).*
Bond*: years 0-10 : 1; years 10-20 : 2/3; years 20-30 : 1/3; subsequent years: 0.*
Load transfer at the joint: *0.5*

The values of the stresses at the bottom of the concrete slabs are at the joint:

σ_{fl} *l(1) =2.55 ;* σ_{fl} *(2/3) = 2.88 ;* σ_{fl} *(1/3) = 3.21 ;* σ_{fl} *(0) = 3.51 N/mm²*

The allowed number of axle loads is determined by $\log N(i) = 15\ (\ 1 - \sigma fl(i)/6)$

N(1) = 4.22 E 08 ; N(2) = 6.31 E 07 ; N(3) = 9.45 E 06 ; N(4) = 1.68 E 06

We assume that the anticipated number of axle loads is a constant 500,000 per year.
The results are presented in table 2

Table 2. Allowed axle loads on a concrete pavement laid on a bituminous base

Periods (yrs)	N(i)	Q(i)	NC(i)
Period 0 – 10	422,000,000	5,000,000	422,000,000
Period 10 – 20	63,100,000	5,000,000	62,256,237
Period 20 – 30	9,450,000	5,000,000	8,479,921
Period 30 -	1,680,000	-	563,499

Failure occurs after the passage of
5,000,000 + 5,000,000 + 5,000,000 + 563,499 = 15,563,499 axle loads of 130 kN, which is after 31.125 years.
Had there been no deterioration of bond over time, the number of allowed axles would have been 422,000,000 and hence incredibly high lifetime! Even if we admit the bond conditions of period 2 for the whole life of the road, the number of allowed axles would still have been 63,100,000 and the lifetime of the road 126,2 years, i.e. 4 times as much as the final result.

This figures fully justify the fact to take into account other factors than fatigue alone when designing a new road.

3. Influence of changes in the bond conditions on the residual lifetime of an asphalt layer on a lean concrete sub-layer when the lifetime of the lean concrete is taken into account

Consider part of a road structure consisting of an asphalt layer on a lean concrete sub-layer.

3.1. Influence of changes in bond conditions on the residual lifetime of an asphalt layer on a lean concrete sub-layer

This is the second proposed model. Here we have limited ourselves to the determination of the lifetime of the asphalt layer, and ignore the fatigue sustained by the lean concrete.

The variables are the same as in paragraph 2.

Consider the successive periods.

3.1.1. Period 1

The fatigue law for the new asphalt layer is written (2):

$$\log N(1) = c \log \frac{\varepsilon_{br}(1)}{\varepsilon_{fl}(1)} = c\left[\log \varepsilon_{br}(1) - \log \varepsilon_{fl}(1)\right]$$

Let us calculate the value of $\varepsilon_{br}(2)$ after the passage of Q(1) axle loads.

$$\log\left(N(1) - Q(1)\right) = c\left[\log \varepsilon_{br}(2) - \log \varepsilon_{br}(1)\right]$$

Hence

$$\log \varepsilon_{br}(2) = \log \varepsilon_{br}(1) - \frac{1}{c} \log \frac{N(1)}{\left[N(1) - Q(1)\right]}$$

3.1.2. Period 2

The fatigue law for the new asphalt layer is written (2):

$$\log N(2) = c\left[\log \varepsilon_{br}(1) - \log \varepsilon_{fl}(2)\right]$$

The corrected fatigue law for a layer that has undergone the passage of Q(1) axle loads is written

$$\log NC(2) = c\left[\log \varepsilon_{br}(2) - \log \varepsilon_{fl}(2)\right]$$

This results in

$$log\ NC(2) = log\ N(2) - log\ \frac{N(1)}{\left[N(1) - Q(1)\right]}$$

$$NC(2) = \frac{N(2)\left[N(1) - Q(1)\right]}{N(1)} \qquad (7)$$

Let us calculate the value of $\varepsilon_{br}(3)$ after the passage of Q(2) axle loads.

$$\log\left[NC(2) - Q(2)\right] = c\left[\log \varepsilon_{br}(3) - \log \varepsilon_{fl}(2)\right]$$

Hence

$$\log \varepsilon_{br}(3) = \log \varepsilon_{br}(1) - \frac{1}{c}\log \frac{NC(2)N(1)}{\left[NC(2) - Q(2)\right]\left[N(1) - Q(1)\right]} \qquad (8)$$

3.1.3. Period n

Relations (7) and (8) can be easily generalized for period n

$$NC(n) = \frac{N(n)\left[NC(n-1) - Q(n-1)\right]\ldots\left[NC(2) - Q(2)\right]\left[N(1) - Q(1)\right]}{NC(n-1)\ldots NC(2)N(1)} \qquad (9)$$

$$log\ \varepsilon br(n) = log\ \varepsilon br(1) - \frac{1}{c}\, log \frac{NC(n-1)\ldots NC(2)N(1)}{\left[NC(n-1) - Q(n-1)\right]\ldots\left[NC(2) - Q(2)\right]\left[N(1) - Q(1)\right]}$$

3.2. Calculation of the lifetime of the lean concrete layer

We calculate the lifetime of the lean concrete up to the time of cracking. This is the third proposed model. The cracking will take place within a specific period of time. Consequently one of these periods must be split into two sub-periods and the number of periods, initially equal to n, will be equal to n+1.

Let QBM be the total allowed number of axle loads prior to the failure of the lean concrete.

Consider a period i.

Let N(i) be the allowed number of axle loads for this period when the structure (with sound or cracked lean concrete) is regarded as new. Cracked lean concrete will be regarded as equivalent to a granular layer.
Let NA(i) be the allowed number of axle loads for this period on the pavement when both asphalt layer and lean concrete layer are considered as new.
Let NBM(i) be the allowed number of axle loads for this period on the lean concrete when this layer is considered as new.
Let NBF(i) be the allowed number of axle loads for this period on the pavement when the asphalt layer is regarded as new but the lean concrete is cracked.
Let Q(i) be the anticipated number of axle loads on the highway during this period.
Let NC(i) be the corrected allowed number of axle loads for this period after taking account of the cumulative fatigue during the preceding periods.

NA(i) is calculated using the fatigue law (2) for bituminous mixes where $c = 4.76$.
NBM(i) is calculated using fatigue law (1) for lean concrete where $c = c2 = 14$.
When NA(1) ≤ NBM(1) we see immediately that the total number of allowed axle loads is equal to NA(1).
We then consider this improbable conclusion as implicit and do not reintroduce it at each stage of the calculation.

For the first period NC(1) = NBM(1).
If NC(1) <= Q(1), the lean concrete fails during the first period.
Hence QBM = NC(1), N(1) = NA(1), N(2) = NBF(1) … N(n) = NBF(n-1), N(n+1) = NBF(n) and Q(n) = Q(n-1), … Q(3) = Q(2), Q(2) = Q(1) – QBM, Q(1) = QBM.

If NC(1) > Q(1), we calculate NC(2) using (3) with $c = c2 = 14$.

If NC(2) <= Q(2), the lean concrete fails during the second period.
Hence QBM = Q(1) + NC(2), N(1) = NA(1), N2 = NA(2), N3 = NBF(2)…
N(n) = NBF(n-1), N(n+1) = NBF(n) and Q(n) = Q(n-1), … Q(3) = Q(1) + Q(2) - QBM,
Q(2) = QBM - Q(1), Q(1) = Q(1).

When NC(2) > Q(2), we calculate NC(3) using (6) with $c = c2 = 14$.

By extension, we obtain:

If NC(n-1) <= Q(n-1), the lean concrete fails during period (n-1).
Hence QBM = Q(1) + Q(2) … + NC(n-1), N(1) = NA(1), N2 = NA(2), …N(n) = NBF(n-1), N(n+1) = NBF(n) and Q(n) = Q(1) + Q(2) … + Q(n-1) - QBM,
Q(n-1) = QBM - Q(1) - Q(2) … - Q(n-2), …Q(2) = Q(2) , Q(1) = Q(1).

•If NC(n-1) > Q(n-1), we calculate NC(n) by using (6) with $c = c2 = 14$.

The lean concrete will fail during the n-th and last period.
Hence QBM = Q(1) + Q(2) + … Q(n-1) + NC(n), N(1) = NB(1), N2 = NB(2), …
N(n+1) = NBF(n) and Q(n) = QBM - Q(1) - Q(2) … - Q(n-1), … Q(2) = Q(2), Q(1) = Q(1).

As the changes in bond between the asphalt layer and the cracked lean concrete, assimilated to a granular layer, are not taken into account, NBF(i) corresponds to an allowed number of axle loads independent of the period and is thus constant.

3.3. Calculation of the lifetime of the structure

We now determine the residual lifetime of the asphalt layer when the lean concrete has cracked.

The calculation is carried out on the basis of relationship (9) by considering, in this case, (n+1) periods and a corrected number of axle loads given by

$$NC(n+1) = \frac{N(n+1)[NC(n)-Q(n)]\cdots[NC(3)-Q(3)][NC(2)-Q(2)][N(1)-Q(1)]}{NC(n)\cdots NC(3)NC(2)N(1)}$$

3.4. Example

Let us consider a two-layered structure consisting in an asphalt surface layer laid on a base of lean concrete on a granular sub-base and a sub grade with following characteristics:

Asphalt concrete: *E1 = 10000 N /mm², h1 = 100 mm,* $\mu 1 = 0.3$*, c = 4.76,* $\varepsilon br = 0.0016$.
Lean concrete: *E2 = 15000 N/mm², h2 = 200 mm,* $\mu 2 = 0.2$*, c2 = 14,* $\sigma_{br} = 1.5$ *N/mm².*
Cracked lean concrete: *E2 = 3000 N/mm², h2 = 200 mm,* $\mu 2 = 0.5$.
Granular sub-base: *E3 = 600 N/mm², h3 = 300 mm,* $\mu 3 = 0.5$.
Sub grade : *CBR = 5%, E4 = 50 N/mm²,* $\mu 4 = 0.5$.
Load : *circular with a radius of 126 mm, p = 1.0 N/mm² (axle load of 100 kN).*
Bond: *years 0-10 : 1/2; years 10-20 : 1/3; years 20-30 : 1/6;subsequent years: 0.*
Bond after failure of the lean concrete: *0.5.*

The values of the strains at the bottom of the asphalt layer on a sound lean concrete are:

$\varepsilon(1/2) = 4.63$ E-06; $\varepsilon(1/3) = 1.20$ E-05; $\varepsilon(1/6) = 2.42$ E-05; $\varepsilon(0) = 7.31$ E-05

The value of the strain at the bottom of the asphalt layer on a cracked lean concrete is:

$\varepsilon(1/2) = 7.53$ E-05;

The allowed number of axle loads is determined by $N(i) = (0{,}0016/\varepsilon(i))^{4,76}$

N(1) = 1.21 E 12; N(2) = 1.30 E 10; N(3) = 4.62 E 08; N(4) = 2.40 E 06
NBF(i) = 2.08 E 06.

The values of the stresses at the bottom of the sound lean concrete are:

σ_{fl} (1/2) = 0.74; σ_{fl} (1/3) = 0.77; σ_{fl} (1/6) = 0,825; σ_{fl} (0) = 1.049 N/mm²

The allowed number of axle loads is determined by *log N(i) = 14 (1 – $\sigma fl(i)/1,5$)*

NBM(1) = 1.24 E 07; NBM(2) = 6.51 E 06 ; NBM(3) = 2.00 E 06; NBM(4) = 1.62 E 04

Assume an anticipated number of axle loads of 500,000 a year.

The results are presented in table 3.

Table 3. Allowed axle loads on an asphalt pavement laid on a lean concrete base

Period	NA(i)	NBF(I)	NBM(i)	Period	Q(i)	NC(i)	Lean concrete
0 – 10	1.21 E12	2.08 E06	1.24 E07	1	5.00 E06	1.21 E12	sound
10 – 20	1.30 E10	2.08 E06	6.51 E06	2.1	3.80 E06	1.30 E10	sound
				2.2	1.20 E06	2.08 E06	cracked
20 – 30	4.62 E08	2.08 E06	2.00 E06	3	-	8.83 E05	cracked
30 -	2.40 E06	2.08 E06	1.62 E04	4	-	0	

Failure occurs after the passage of
5,000,000 + 3,800,000 + 1,200,000 + 883,000 = 10,883,000 axle loads of 100 kN, which is after 21.80 years.

4. Influence of changes in bond conditions on the residual lifetime of a concrete pavement on a lean concrete base with or without an intermediate asphalt layer taking the lifetime of the lean concrete into account

Let us now consider a highway consisting of a concrete pavement on a possible intermediate asphalt layer on a lean concrete base.

We determine the lifetime of the structure by first considering the lifetime of the lean concrete and then, after cracking of the lean concrete, the residual lifetime of the concrete pavement. No account is taken of the lifetime of the possible intermediate layer. This is the fourth proposed model.

4.1. Calculation of the lifetime of the lean concrete

First we calculate the lifetime of the lean concrete up to the time of cracking. This cracking will take place within a specific period of time. Consequently one of these periods must be split into two sub-periods and the number of periods, initially equal to n, will be equal to n+1.

Let QBM be the total allowed number of axle loads prior to the failure of the lean concrete.

Consider a period i.

Let NB(i) be the allowed number of axle loads for this period on the pavement when both concrete layer and lean concrete layer are considered as new.
Let NBF(i) be the allowed number of axle loads for this period when the concrete layer is regarded as new but the lean concrete is cracked.

The other variables are the same as in paragraph 3.2.

NB(i) is calculated using the fatigue law (1) for pavement concrete with $c = c1 = 15$.
NBM(i) is calculated using the fatigue law (1) for lean concrete with $c = c2 = 14$.

The computations are the same as in paragraph 3.2., provided replacement of NA(i) by NB(i).

4.2. Calculation of the lifetime of the structure

The calculation of the lifetime of the structure is carried out on the basis of relation (6) by considering, in this case, (n+1) periods and a corrected number of axles given by

$$\log NC(n+1) = \left\{ c - \frac{(c - \log N(n+1))\ldots(c - \log[NC(2) - Q(2)])}{(c - \log N(n1))\ldots(c - \log NC(2))(c - \log N(1))}(c - \log[N(1) - Q(1)]) \right\}$$

where $c = c1 = 15$.

4.3. Example

Let us consider a three-layered structure consisting of a cement concrete pavement with dowelled slabs, an intermediate asphalt layer and a lean concrete base on a sub grade with following characteristics:

Cement concrete : $E1 = 30000\ N/mm^2$, $h1 = 200\ mm$, $\mu 1 = 0.2$, $c1 = 15$, $\sigma_{br} = 6\ N/mm^2$.
Asphalt concrete : $E2 = 8000\ N/mm^2$, $h2 = 100\ mm$, $\mu 2 = 0.3$.
Lean concrete: $E3 = 15000\ N/mm^2$, $h3 = 200\ mm$, $\mu 3 = 0.2$, $c2 = 14$, $\sigma_{br} = 1.5\ N/mm^2$.
Cracked lean concrete : $E3 = 3000\ N/mm^2$, $h3 = 200\ mm$, $\mu 3 = 0.5$.
Sub grade: $k = 0.05\ N/mm^3$.
Sub grade + cracked lean concrete: $k_{eq} = 0.12\ N/mm^3$.
Load: *square with a side of 200 mm,* $p = 1.625\ N/mm^2$ *(130 kN axle).*
Bond cement concrete – asphalt concrete: *years 0-10 : 1; years 10-20 : 2/3; years 20-30 : 1/3; subsequent years : 0.*
Bond asphalt concrete – lean concrete: *years 0-10 : 1/2 ; years 10-20 : 1/3 ; years 20-30 : 1/6; subsequent years: 0.*

Load transfer: *0.5.*

The values of the stresses at the bottom of the slabs on a sound lean concrete are at a joint:

σ_{fl} *(1,1/2) = 1.10;* σ_{fl} *(2/3,1/3) = 1.75;* σ_{fl} *(1/3,1/6) = 2.18;* σ_{fl} *(0,0) = 2.46 N/mm²*

The allowed number of axle loads is determined by $\log N(i) = 15\ (1 - \sigma fl(i)/6)$

NB(1) = 1.78 E 12 ; NB(2) = 4.22 E 10 ; NB(3) = 3.55 E 09 ; NB(4) = 7.08 E 08

The values of the stresses at the bottom of the lean concrete are at a joint:

σ_{fl} *(1,1/2) = 0.93 ;* σ_{fl} *(2/3,1/3) = 1.08 ;* σ_{fl} *(1/3,1/6) = 1.17 ;* σ_{fl} *(0,0) = 1.23 N/mm²*

The allowed number of axle loads is determined by $\log N(i) = 14\ (1 - \sigma fl(i)/1.5)$

NBM(1) = 2.09 E 05; NBM(2) = 8.32 E 03; NBM(3) = 1.20 E 03 ; NBM(4) = 3.31 E 02

The values of the stresses at the bottom of the slabs on a cracked lean concrete are at a joint:

σ_{fl} *(1,1/2) = 2.30 ;* σ_{fl} *(2/3,1/3) = 2.59 ;* σ_{fl} *(1/3,1/6) = 2.88 ;* σ_{fl} *(0,0) = 3.16 N/mm²*

The allowed number of axle loads is determined by $\log N(i) = 15\ (1 - \sigma fl(i)/6)$

NBF(1) = 1.78 E 09 ; NBF(2) = 3.35 E 08 ; NBF(3) = 6.31 E 07 ; NBF(4) = 1.25 E 07

Assume an anticipated number of axle loads of 500,000 a year.

The results are presented in table 4.

Table 4. Allowed axle loads on a concrete pavement on a lean concrete base

Period	NB(i)	NBF(i)	NBM(i)	Period	Q(i)	NC(i)	Lean concrete
0 – 10	1.78 E12	1.78 E09	2.09 E05	1.1	209,000	1.78 E12	sound
				1.2	4,791,000	1.79 E09	cracked
10 – 20	4.22 E10	3.35 E08	8.32 E03	2	5.00 E 06	3.34 E08	cracked
20 – 30	3.55 E 09	6.31 E07	1.20 E03	3	5.00 E06	6.18 E07	cracked
30 -	7.08 E08	1.25 E07	3.31 E02	4		1.11 E07	cracked

Failure occurs after the passage of 209,000 + 4,791,000 + 5,000,000 + 5,000,000 + 11,100,000 = 26,100,000 axle loads of 130 kN, which is after 52.20 years.

5. Influence of changes in load transfer conditions at the joint on the lifetime of a concrete pavement on a granular base

5.1. Influence of load transfer at the joints

A reduction over time of the quality of load transfer at the joint appears to be probable .

The influence of such a reduction can be assessed on the basis of the methods developed above, by taking account of the cumulative fatigue of the concrete.

It is therefore not necessary to formulate a completely new model. The models are already available, it is enough to apply them. A simple example serves to demonstrate this.

Let us return to the example discussed under paragraph 2.4.

Consider a two-layered structure consisting of dowelled concrete slabs laid on a sub grade with following characteristics:

Concrete: $E1 = 30000\ N/mm^2$, $h1 = 200\ mm$, $\mu 1 = 0.2$, $c1 = 15$, $\sigma_{br} = 6\ N/mm^2$.
Sub grade: $k = 0.532\ N/mm^3$.
Load: *square with a side of 200 mm,* $p = 1.625\ N/mm^2$ *(axle load of 130 kN).*

In this example we have dropped the bituminous layer in order to eliminate those problems relating to bond at the interfaces. Nonetheless, so that we can compare the results with those indicated above, we assume that the bearing capacity of the subgrade is 0.532 N/mm³, so that for a load transfer of 0.50, the stresses at the joint are the same in both cases.

We assume that the degree of transfer between dowelled slabs declines in stages until it is the same as that between non-dowelled slabs after a period of about thirty years.
We use following values:

Transfer: *years 0-10: 0.5; years 10-20 : 0.4; years 20-30 : 0.3; subsequent years: 0.2.*

The values of the stresses at the bottom of the concrete are at a joint:

σ_{fl} *(0.5) = 2.55 ;* σ_{fl} *(0.4) = 2.72 ;* σ_{fl} *(0.3) = 2.89 ;* σ_{fl} *(0.2) = 3.06 N/mm²*
The allowed number of axle loads is determined by $\log N(i) = 15\ (1 - \sigma fl(i)/6)$

N(1) = 4.22 E 08 ; N(2) = 1.58 E 08 ; N(3) = 5.96 E 07 ; N(4) = 2.24 E 07

Assume a constant number of anticipated axles of 500,000 a year.

The results are presented in table 5.

Table 5. Allowed axle loads on a concrete pavement with declining load transfer

Periods	N(i)	Q(i)	NC(i)
Period 0 – 10	422,000,000	5,000,000	422,000,000
Period 10 – 20	158,000,000	5,000,000	156,003,473
Period 20 - 30	59,600,000	5,000,000	56,800,434
Period 30 -	22,400,000	-	19,308,663

Dowels lose their effectiveness after the passage of 5,000,000 + 5,000,000 + 5,000,000 + 19,308,663 = 34,308,663 axle loads of 130 kN, which is after 68.6 years.

5.2. Influence of the simultaneous decline of load transfer at the joint and bond between layers

The simultaneous influence of the decline of load transfer at the joints and the loss of bond between the layers is also easy to illustrate by the foregoing examples.

Let us return to the example in paragraph 2.4.

Consider a two-layered structure consisting of dowelled concrete slabs laid on a asphalt base and a sub grade with the following characteristics:

Cement concrete: $E1 = 30000$ *N /mm²*, $h1 = 200$ *mm*, $\mu 1 = 0.2$, $c1 = 15$, $\sigma_{br} = 6$ *N/mm²*.
Asphalt concrete: $E2 = 8000$ *N/mm²*, $h2 = 50$ *mm*, $\mu 2 = 0.3$.
Sub grade: $k = 0.05$ *N/mm³*.
Load: *square with a side of 200 mm, p = 1.625 N/mm² (axle load of 130 kN).*
Bond*: years 0-10 : 1; years 10-20 : 2/3; years 20-30 : 1/3; later years: 0.*
Load transfer: *years 0-10: 0.5; years 10-20 : 0.4; years 20-30 : 0.3; subsequent years: 0.2*

The values of the stresses at the base of the concrete are at a joint:

σ_{fl} *(1,0.5) = 2.55 ;* σ_{fl} *(2/3,0.4) = 3.07 ;* σ_{fl} *(1/3,0.3) = 3.64 ;* σ_{fl} *(0,0.2) = 4.22 N/mm²*

The allowed number of axle loads is determined by $\log N(i) = 15\ (1 - \sigma fl(i)/6)$

N(1) = 4.22 E 08 ; N(2) = 2.11 E 07 ; N(3) = 7.94 E 05 ; N(4) = 2.82 E 04

Assume a constant number of anticipated axles of 500,000 a year.

The results are presented in table 6.

Table 6. Allowed axle loads on a concrete pavement with declining load transfer and declining bond at the interface with a bituminous layer

Periods	N(i)	Q(i)	NC(i)
Period 0 – 10	422,000,000	5,000,000	422,000,000
Period 10 – 20	21,134,890	5,000,000	20,833,767
Period 20 - 30	794328	-	564,027
Period 30 -	28184	-	0

Failure occurs after the passage of 5,000,000 + 5,000,000 + 564,027 = 10,564,027 axle loads of 130 kN, which is after 21.5 years,

6. Conclusion

In conclusion we believe that we can say that an accurate estimate of the service life of a cement concrete road requires to take into account the deterioration of certain of its elements, which had been ignored so far. In this first analysis, we have confined ourselves to two elements which deterioration can be assessed relatively precisely by on site measurements.

Therefore, we hope having demonstrated that with a little analysis and imagination it becomes possible to develop simple algorithms for a better approximation, in fact better mathematical models, of the physical reality that we are confronted with. The main difficulty lies in selecting the most appropriate physical characteristics.

One particular advantage of the model that is worth mentioning, is that it is not restricted to analytical applications. It can easily be integrated into FEM applications.

7. References

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