RISC - A MECHANISTIC METHOD OF RIGID PAVEMENT DESIGN

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This paper deals with the development and verification of a stress analysis model (RISC) and the use of this model in a program for the design of rigid pavements. RISC is the result of coupling finite element plate theory with multilayer elastic layer theory and consists of a two-layer rigid slab resting on a semi-infinite three-layér elastic solid foundation. The features of RISC include variable thickness of slabs, the effects of curling, warping, voids and partial contact, the effect of load transfer across joints and cracks by specifying either dowel bars, aggregate interlock, or load or shear transfer efficiency, the ability to analyze up to nine slabs in any configuration, and the ability to determine the stresses in both rigid layers as well as anywhere in the subbase and subgrade layers. Model verification includes comparison with known solutions whenever possible; however, since this model is an extension beyond the scope of existing theories, qualitative comparisons are also included. The pavement design program uses RISC for stress analysis; however the input requirements have been drastically simplified for the convenience of the user. The output includes the expected design life.

The design of rigid pavements requires the consideration of traffic (volume, weight, placement), environment, foundation support, and geometrical and component analysis such as type of pavement (i.e., (CRC or jointed) transverse joint construction, type of shoulders and longitudinal joint construction, etc.

Ideally, a pavement design method should be able to predict the formation of all the distresses, such as blowups, corner breaks, cracking, "D" cracking, faulting, frozen joints, pumping, punchouts, spalling and sealant failure, as well as to determine the effect of each of these distresses on pavement performance as measured by present serviceability, riding quality, or some other rating method that reflects the motoring public's preception of pavement serviceability. Unfortunately, the state of the art in pavement design is not yet to this point, although research currently under way is making significant progress in this direction.

The observed distresses can be divided into two catagories: (i) those that need predictive models, such as fatigue cracking, shrinkage cracking, pumping, faulting, punchouts (ii) those that can be controlled through design specifications, material selection, construction practices, drainage, etc., such as D-cracking, spalling, frozen joints, blowups, sealant failures.

The review of existing distress models indicates prediction methods are available for predicting faulting and load-induced cracking (including corner breaks for undoweled jointed pavements), but prediction models for the other distresses have as yet not been developed. Therefore, the design method presented in this paper is limited to thickness design based on limiting faulting and load-induced cracking to acceptable values.

This paper describes the development and verification of a mechanistic rigid pavement design method based on coupling a finite element slab theory with multilayer elastic solid subgrade. The finite element formulation is used to describe the slab, joint, dowel bar, tie bar system and multilayer elastic theory is used to characterize subgrade support. The model has been subjected to extensive verification, both theortically and experimentally - the highlights of this verification will also be presented.

Literature Review

Fatigue Cracking Models

Cracking occurs in concrete pavement slab when the tensile stress caused by traffic loading and environmental conditions exceeds the tensile strength of the concrete. A number of laboratory studies (1, 2) have shown that cracking of concrete beams can also occur as a result of repeated application of stresses smaller than the tensile strength of the concrete. This type of cracking is referred to as concrete fatigue cracking. The results of the fatigue tests on concrete beams show that the number of repeated loads that concrete can sustain in flexure before fracture depends on the ratio of the applied stress to the ultimate static strength (modulus of rupture, MR) of the concrete. The results of concrete fatigue studies are usually plotted in terms of the stress ratio versus log of the number of load applications to failure. Since concrete fatigue life is directly proportional to its modulus of rupture, many of the factors that affect modulus of rupture, such as water-cement ratio, type and amount of cement, curling, etc., affect the fatigue life of the concrete.

Although some researchers have suggested and used fatigue limits for concrete in the design procedures (3) laboratory studies (1) have shown that no fatigue limit exists for concrete for up to 10-20 million load applications.

Laboratory studies have shown also that concrete fatigue life is affected by the rate of loading (2), sequence of loads, rest periods during loading. As the rate of loading, duration of rest periods, moisture condition and age of the concrete increases, the fatigue strength increases also. In addition, fatigue life is increased if stresses below 50 percent of the modulus of rupture are initially applied to the specimen (1).

The results of the laboratory fatigue studies on concrete beams have been employed by various investigators for development of design procedures for PCC pavements (3-5). Graphical representation of these models and comparisons with the laboratory test data was made by Darter (5) and is shown in Figure 1.

PCA researchers (3) developed a fatigue prediction model assuming an endurance limit of 0.5 for concrete. Darter (17) developed a probablistic fatigue model for plain jointed concrete pavement based on laboratory fatigue test results. ARE researchers (6) developed a fatigue cracking model based on AASHO Road Test data and employing elastic layer theory to compute midslab maximum stresses. ARE researchers used the AASHO equivalency factors to account for mixed traffic. U.S. Air Force (7) developed a fatigue distress model based on performance observation of airfield pavements and include a wide range of pavement thicknesses, foundation types and strengths, and load magnitudes and locations. The U.S. Air Force model has been also used by the Army Corps of Engineers. Vesic and Saxena (8) developed a fatigue distress model by analyzing the AASHO Road Test data but using Westergaard plate theory to determine the slab stresses. RII researchers developed fatigue distress model (RISC) based on a considerably more sophisticated analysis of the AASHO Road Test data (9). RISC distress function is based on plate theory resting on multilayered elastic solid foundation, and incorporates such effects as actual load placement, slab geometry, effects of load transfer devices, variation of material properties by loop, lane, location, etc.

Faulting Models

Faulting of joints or cracks in PCC pavements is a difference in elevation of two adjacent slabs at the joints or cracks. This is a serious type of distress which can lead to very rough pavement and accelerate failure. Faulting is not likely to be noticed visually during its early stages, nor can it be felt by the motorist. However, a fault of more than 1/16 mm (1.6 inch) usually causes sufficient noise from tire impact to be audible (10, 11). Various researchers (10-14) have suggested different values for intolerable faulting, but it is generally agreed that an average fault of greater than 1/8 mm (83.2 inch) results in a very rought ride and is objectionable to the motorist. Following is a summary of intolerable faulting values suggested by various investigators:

FAULTING VALUES

Investigation	Intolerable Faulting
NCHRP Synthesis 56 (12)	Avg. Fault > 3.2 mm (0.125 inch)
Spellman, et. al. (13)	Avg. Fault > 3.0-6.2 mm (0.12-0.25 in.)
Gulden (14)	Fault Index > 25 Avg. Fault > 3.8 mm (0.15 in)
Brokaw (10)	SVF >12-14 Avg. Fault > 4.8 mm (0.19 in)
Packard (11)	Avg. Fault > 3.8-4.8 mm (0.15-0.19 in)

Although faulting is a type of distress produced by many applications of heavy axle loads, there are a number of other factors which have a significant effect on faulting, such as: dowel bars at joints, slab thickness, joint spacing, drainage, existence of free water, and erodable materials under the concrete slab. Field studies (10) have shown that faulting can be reduced greatly by: use of dowel bars at joints, increase of slab thickness, reduction in joint spacing, use of better drainage and non-erodable base and shoulder materials.

A number of fault prediction models are available in literature (10, 11, 14); however, most of them have been developed either for a particular design (10, 11), a particular geographical region (14), or based on limited field data (15). Therefore, they cannot be used as general fault models in their existing forms.

Structural Analysis

The first requirement of a mechanistic rigid pavement design is the ability to determine the stresses and displacements in the concrete slab and in the supporting layers below the slab as well as the effect of the various pavement features on critical stresses and displacements included in the analysis model.

Several theories have been developed over the years for the determination of stresses and deflection in concrete slabs; these could be divided into four major groups:

1. Models of continuously supported slabs.
2. Layered systems.
3. Finite-element and discrete-element models.
4. Coupled models.

Each of these models has some advantages and disadvantages for pavement analysis but, a model that incorporates the effect of all variables and features associated with rigid pavements has not yet been developed.

Models of Continuously Supported Slabs. The first use of slab theory for concrete pavement analysis was made by Westergaard in 1926. (16). In this analyses classical bending theory of a medium-thick plate (no deformation within the slab thickness, and no membrane action) was used to represent the slab and the subgrade was assumed to act as a set of uniformly distributed springs supporting the concrete slab. The stiffness of these springs was referred to as a modulus of subgrade reaction (k), and was determined from the results of plate-load tests performed on the subgrade. Assuming the concrete pavement slab to be infinitely large and having a constant thickness, Westergaard developed closed-form solutions for critical stresses and deflections under a single load for three cases of loadings, namely: interior, edge and corner.

Pickett and Ray (17) extended the work of Westergaard to include the effect of any loading configuration in form of influence charts. The computerized version of the Pickett and Ray's influence charts were prepared by the Portland Cement Association (PCA) for concrete pavement slab analysis.

The above slab models are based upon the assumption that the slabs extend a considerable distance from the loaded area. However, concrete slabs are relatively narrow, and have many discontinuities in forms of joints and cracks, with various degrees of load transfer across these discontinuities. These effects cannot be evaluated with the above model, nor can the effects of voids and partial contact be included.

Models of Layered Systems. Layer theory assumes that each pavement layer is represented by its modulus of elasticity, thickness, and Poisson's ratio. All layers are assumed to extend horizontally to infinity with the bottom layer also extending vertically to infinity. The basic problem of stress analysis of a single layer was first formulated and solved by Boussinesq in 1885 (18). In this study applied load was assumed to be concentrated and acting normal to a semi-infinite elastic and homogenous solid system.

The first solution for a generalized elastic multiple-layered system was presented by Burmister (19). The work of Burmister was later extended by other investigatores to include more generalized loading conditions and for viscoelastic multiple-layered systems.

With the development of high speed, high capacity computers, it was possible to analyze multiple-layered systems consisting of any number of layers and subjected to multiple loads. The first computer program for analysis of five-layer elastic systems subjected to a single load was developed by Chevron Research Company (20) in 1963. Since then a number of other computer programs have become available for analyzing multiple-layered systems the most popular of these are ELSYM5 (21) (for multiple loads) and the VESYS System utilizing viscoelastic layer properties.

Despite the capabilities of multiple-layered system models for evaluation of pavements with multiple layers and pavements with elastic and viscoelastic material properties, the effects of discontinuities (finite slab size, joints, load location, etc) cannot be analyzed.

Finite-Element and Discrete Element Models. Use of discrete-element method for structural analysis of plates was pioneered by Newmark (23). In developing a discrete-element model, Newmark considered the slab as an assemblage of deformable hinges, rigid links, and coil springs. The first use of discrete-element model for concrete pavement slab analysis was made by Hudson and Matlock (24). In this analysis subgrade was idealized as Winkler foundation (similar to the Westergaard's analysis). The effects of joints and shrinkage cracks in this model was taken into consideration by reduction of original bending stiffness of the slab at those locations where a joint or crack exist.

The model developed by Hudson and Matlock was later modified and improved by other investigators to include elements of different sizes (25), anisotropic skew slabs (26), and idealization of subgrade as a semi-infinite elastic solid (27).

The major disadvantages of discrete-element formulations are that elements of varying sizes are not easily incorporated into the analysis, and that special treatment is needed at the free edge where stresses cannot be determined uniquely.

A number of finite-element models have been developed for analysis of the concrete pavement system. These may be divided into one of the following major classes:

1. Slab Models
2. Plane Strain Models
3. Prismatic Models
4. Axisymmetric Models
5. Three-Dimensional Models

The slab models idealize concrete pavement slabs as medium-thick plates supported by an idealized subgrade. Slab models have been developed by Eberhart (28), Huang and Wang (29), Bhatia and Majidzadeh (30), and Tabatabaie and Barenberg (31). Although these models all represent the subgrade as a Winkler foundation and are therefore limited in their application, the representation developed by Tabatabaie (31) and Huang and Wang (29) contain some of the features that are considered essential for a rational analysis of concrete pavements.

In the Tabatabaie (31) model the concrete pavement system with joints is represented by up to six slabs. Six slabs are used because this is the general case with the loads applied on the middle slab when it is connected to neighboring slabs by dowel bars or keyway. The slab is treated in this model as composed of two layers which may be bonded or unbounded, but assumed to be in contact with each other. Two layers are used so that cement stabilized bases or rigid overlays may be analyzed. Rectangular plate elements with three degress of freedom per node (one vertical deflection and two rotations) are used to represent the slab and stabilized base, or slab and overlay. Dowel bars are represented by beam elements while spring elements are used to model aggregate interlock or keyway. Using this formulation, slabs with varying thickness (such as pavements with thickened edge) can be analyzed as well as pavements with tied shoulders having a thickness different from that of the slab. Also, since beams and spring elements are used, the load transfer efficiency of various joint designs can be determined, along with the effect of joint design (load transfer efficiency) on critical slab stresses. This model does not, however, consider the effects of curling, warping, voids and partial contact.

The model developed by Huang and Wang (29) represents the concrete slab as composed of a single rigid layer. The concrete pavement is represented by two or four slabs when load transfer effects are considered, and one slab when no load transfer at joints is assumed. As in the Tabatabaie (31) model, rectangular plate elements with three degress of freedom per node are used to represent the slab. Load transfer devices are not modeled; instead a load transfer efficiency is assumed. This model is also restricted to uniform thickness slabs, but it does consider the effects of curling, warping, voids, and partial contact. Despite the advantages of these models in that they are capable of evaluating all types of loading conditions, slabs of finite dimensions, and various pavement features such as joints, cracks, etc., they cannot handle multi-layered pavement support systems.

In plane-strain idealization of the concrete pavement system, changes in load or material along the centerline of the pavement is neglected. The pavement system is represented as a transverse slice of the pavement having a unit thickness. Obviously, these models, because of their simplifying assumptions, are not capable of evaluating various concrete pavement features such as joints, cracks, etc., slab action of the concrete pavement slab, and multiple loads.

In the prismatic idealization of the concrete pavement system, the pavement system is represented by a constant two-dimensional geometric shape with respect to an infinite third dimension. The greatest limitation of these models for analysis of the concrete pavement is that no variation of geometrical configuration is allowed along the longitudinal axis of the pavement system. Therefore, prismatic models cannot handle any transverse discontinuities such as joints or cracks, nor do they use a realistic representation for applied loads.

The axisymmetric model idealization considers a nearly general three-dimensional system. In this modeling technique a pavement system is idealized as a multiple layered cylindrical system loaded symmetrically at the centerline. The major limitation of these models is that they are not capable of evaluating various concrete pavement features such as joints, cracks, etc., or various loading conditions such as edge, corner, or joint.

Probably the most desired model for concrete pavement analysis is the use of the three-dimensional models, where actual geometrical configuration of whole system can be taken into consideration. There also are computer programs which employ three-dimensional finite-elements such as the SAP Program developed by Wilson (32). However, the amount of discretization and the computer cost required for solution of the problem would be high and impractical.

Coupled Models. For the purpose of this presentation coupled models are referred to as the structural models obtained by coupling two or more of the following models:

1. Finite-element or discrete-element
2. Multiple-layered system
3. Analytical (closed form)

Coupling models have been formulated by various investigators (27, 33), for structural analysis of pavement systems. In 1971, Saxena (27) coupled the discrete-element slab model developed by Hudson and Matlock (24) with the closed form solution of Boussinesq (18). In this analysis discrete-element modeling technique was used for idealization of the concrete pavement slab, and Boussinesq's equation was employed to represent the behavior of the subgrade as a semi-infinite elastic solid.

Although the Saxena (27) model with a Boussinesq solid subgrade is an improvement of the model developed by Hudson and Matlock (28), it has the following disadvantages in that the effects of voids, partial contact, and curling and warping are not considered. Furthermore, the model is only applicable to single slabs with free edges, i.e., load transfer effects are not considered.

Huang (33) has coupled a finite element model with a Boussinesq solid subgrade. This model includes all the features of the Huang and Wang (25) model discussed previously except that load transfer at joints is assumed to be either 0% for plain jointed concrete, or 100% for doweled or keyed joints.

Coupling of a finite-element program with a multi-layered elastic system was made by Prause and Kennedy (34), for analysis of railroad track structures. In this modeling technique a finite-element program representing rail and tie was coupled with a multi-layered elastic layer program representing ballast and subgrade. Although the coupling techniques developed in this program may be useful, the program itself is not directly applicable for concrete pavement analysis.

The Framework for a Mechanistic Analysis Model

A structural analysis model capable of predicting stresses in the pavement structure is an essential feature of any rational pavement design methodology. A review of pavement distresses and features indicates that the analysis model should have the following capabilities:

1. Finite element formulation
2. Ability to consider the effects of load transfer at joints due to different load transfer devices.
3. Ability to consider curling and warping effects.
4. Ability to consider the effects of voids and partial contact.
5. Ability to treat the slab as two rigid layers so that cement treated bases can be considered.
6. Ability to represent the subgrade as a 3-layer elastic continuum.
7. Ability to handle different traffic loading geometries.
8. Ability to compute stresses and strains in the slab as well as in the layered continuum.

Based on the discussion in the previous section, it can be seen that although a number of structural models are available for the analysis of concrete pavement systems, none of these contain all the features considered essential for a rational representation. Three-dimensional finite-element models which can realistically represent the whole pavement system are expensive to use and the amount of discretization required for solution of the problem will be high and impractical. The slab models which are capable of evaluating various PCC pavement features such as joints, cracks, etc., cannot be employed for analysis of the multiple layers. On the other hand, layered system models which are capable of handling multiple layers cannot evaluate the effects of the concrete pavement features.

Therefore, the solution to this problem is the development of a coupled model where a finite-element plate is coupled with a three-layered elastic or visco-elastic system. This coupled model with its unique characteristics would be capable of analyzing multiple layered systems, as well as various pavement features such as joints, cracks, load transfer, voids, curling, etc. A special case of this problem for coupling of single layer slab model with a single elastic layer system has been used successfully by other investigators (27, 33). Similar concept can be employed for the coupling of a two layered slab model with a three-layered elastic or viscoelastic system.

Selection of Distress Models

It should be noted that although a significant amount of research has been carried out by a number of investigators for the development of various models for the prediction of some distresses, none of these models have been accepted universally. The distress models used in this design method have been selected based on compatibility with analysis theory as well as general applicability, and represent, in the opinion of the authors, the best currently available models. However, the design program has been made modular so that as new and better distress prediction models become available, they can be incorporated into the design.

Fatigue Models. Various load associated fatigue cracking distress models based on laboratory tests on flexure beams appear to be unsuitable since they fail to consider crack deterioration, and are based on uniaxial stresses whereas in-service pavements experience triaxial state of stress. Also, the predicted fatigue lives are totally inconsistent with field performance observations, independent of the analysis theory used in determining pavement stresses.

As may be seen from Figure 1, the performance based distress models all predict considerably different fatigue lives, even though the ARE (6), Vesic (8), and RISC (9) models were developed from the same performance data. Some of this difference can be explained by the degree to which variation of material properties are considered; for example both Vesic and ARE assumed that the subgrade modulus and concrete flexural strength are the same for all sections in the AASHO Road Test whereas the RISC model uses actual measured values for all variables. However, most of the difference in predicted fatigue lives is the result of using different theories in determining critical concrete stresses. It is therefore imperative that the same theory used in pavement design be used in the development of distress functions. The RISC model has been developed using a mechanistic theory and it is considered to be the most suitable model. Also, RISC model has the highest statistical significance and the lowest standard error of estimate of the performance models, and it predicted the performance of all sections (both failed and unfailed) in the AASHO Road Test very satisfactorily.

Figure 2 shows the effect of single axle load on the number of equivalent 80 kN (18 kip) axle loads, as determined from the various distress functions discussed previously. Also shown in this figure is the load equivalency derived from the AASHO design equation for Rigid pavements (35). It is interesting to note that the RISC and the Vesic models predict load equivalencies that are very similar to the AASHO relationship and the ARE model does not differ very significantly; however, the Darter (5) and PCA (4) models show load equivalencies that are totally inconsistent with general experience.

Faulting Models. The existing faulting prediction models have been primarily developed either for a particular design, geographical region or are based on limited field data. The selection of any such model as a universal method bears the limitation that it might not be applicable to all field conditions encountered. The PCA model (17) for plain jointed pavements and the Darter Model (5) for doweled pavements are considered the most promising predictive models currently available and should be used in the design procedure until better models become available. The Darter model bears the limitation that it was developed from pavement projects in Illinois only, whereas the PCA model included highways located in 5 states; however, nothing better is available.

Structural Analysis Model

General Concepts of Coupling. Consider the generalcase of two elastic bodies A and B which interact through a common border ΩAB as shown in Figure 3. If the interaction forces S(x, y, z) (the effect of body B on body A, or vice-versa) can be described as a function of coordinates and displacements at the boundary ΩAB, the solution can be simplified by separating body A from body B as shown in Figure 4. The effect of body B on body A is then described by the interaction forces S(x, y, z) applied at the boundary ΩAB. This solution could be represented as:

$$\sigma AB = \sigma A = \sigma B \qquad (1)$$

where

σAB = the stress tensor satisfying global boundary conditions

σA, σB = the stress tensor satisfying the boundary condition of bodies A and B, respectively

σA, σB = 0 at the outside boundaries of bodies A and B.

Because the size of each problem, i.e., A or B by itself, is smaller than the combined problem, it is easier and faster to find a finite element (or any other) approximation of the boundary problem.

The most difficult aspect of the solution of the boundary problem is definition of the S9x, y, z) function, which is directly related to evaluation of the Green's function for body B (or A). The development of this function is discussed in the next selection.

Coupling of a Single Slab with N-Layered Elastic Foundation. In the case of a slab resting on a multilayer elastic foundation, body A is a concrete slab and body B is a semi-infinite multilayered elastic foundation, as shown in Figure 5.

The ultimate objective of this is to fine the stress-strain distribution for the "slab-foundation couple." In order to achieve this objective, the slab can be approximated with a finite element model and the multilayer foundation can be represented by a stiffness matrix Sij, i.e., the foundation is a discrete model (finite element) will act as a "super element" connected to each nodal point of the FEM approximation of the slab, and Sij is nothing more than a discrete analog of the function S9x, y, z) discussed above.

The finite element method to represent the concrete slab is based on theory of a flat thin elastic shell with positive curvature.

After assembly process over all elements, the force-displacement relationship is written as:

$$\{F\} = (K)\ \{\delta\} \qquad (2)$$

where

(K) = global stiffness matrix of the slab

{F} = vector of all externally applied nodal forces

{δ} = global displacement vector.

As was previously mentioned the vector {F} consists of all externally applied forces (including reactions from the foundation), and can be described as

$$\{F\} = \{P\} + \{R\} \qquad (3)$$

where

{P} = actual forces applied on the slab

{R} = reactive forces from the foundation

Equilibrim equations require that:

$$\int_{\Omega A} F\ d\Omega = \int_{\Omega A - \Omega AB} P\ d\Omega + \int_{\Omega AB} R\ d\Omega = 0 \qquad (4)$$

or

$$\int_{\Omega A - \Omega AB} P\ d\Omega = - \int_{\Omega AB} R\ d\Omega \qquad (5)$$

where

ΩA is the total boundary of the slab

ΩAB is the common boundary between the slab and the foundation

Obviously, the vertical deisplacements of every common nodal point of the slab and foundation are equal, and for points not in contract, reactions from the foundation equal 0.

With the above considerations, and using Equations 3 through 5, Equation 2 can be rewritten as:

$$\{P\} = (K)\ \{\delta\} + \{R\} \qquad (6)$$

The foundation can be discribed in similar terms, i.e., the displacements of the multilayer subgrade can be determined if the applied loads are known:

$$\{\delta\} = (F)\ \{R\} \qquad (7)$$

or

$$\{R\} = (F)^{-1}\ \{\delta\} = (S)\ \{\delta\} \tag{8}$$

where
(F) = flexibility matrix of the foundations
$(S) = (F)^{-1}$ = stiffness matrix of the subgrade

Equation 7 states that the displacements of the foundation are related to the applied loads {R} through the flexibility matrix (F).

Substitution of Equation 8 into Equation 6 results in:

$$\{P\} = ((K) + (S))\ \{\delta\} \tag{9}$$

which is the general equation of the coupling process between a slab and a multilayer foundation for a discrete (FEM) model.

Of course, the foundation stiffness matrix (S) still needs to be determined.

The determination of the stiffness matrix (S) is closely related to the evaluation of the Green's function G(x,y) of the multi-layered elastic semi-infinite space, as is shown in Figure 6.

The subgrade flexibility matrix can be determined from (for unit load q = 1)

$$Fii = \frac{1 + vi}{2\pi\ Elab} * \int_0^a dx \int_0^b dy \int_0^\infty Jo(m(x^2 + y^2)^{.5})\ F\ (m,Ei,vi,hi)dm \tag{10}$$

$$\text{and } Fij = \frac{1 + vi}{2\pi\ El} * \int_0^\infty Jo(mri)\ F(m,Ei,vi,hi)dm \qquad i \neq j \tag{11}$$

The stiffness matrix (S) is then easily determined from:

$$(S) = (F)^{-1}$$

where
$(F)^{-1}$ -is the inverse of the flexibility matrix

Since the above integrals cannot be evaluated analytically, numberical integration methds are used. Considerable care should be exercised in the numerical integration to insure that the computed values actually represent the above integrals.

It should be emphasized that both the flexibility and stiffness matrices (F) and (S) are fully-populated matrices, rather than diagonal, as in the case for Winkler foundations. The reason for this is that for an elastic solid subgrade the load at one point cuases displacements of all other points on the surface of the layered subgrade, and this influence is far from negligible, particularly if the top layer of the foundation is significantly stiffer than the underlying layers. The displacement of a point at a distance r from the load decreases approximately as the inverse of r, so that reasonable approximations are possible using limited bandwidths for large slabs (slab dimensions greater than approximately7.6 mm (25 ft), especially if the top layer of the foundation is not significantly stiffer than the bottom layers.

Y.K. Cheung and O.C. Zienkiewicz (36) were probably the first who applied this approach to solve e coupling problem for a single finite slab on a single-layer Boussinesq elastic foundation. The authors have used a generalization of their approach for a set of slabs on multilayered elastic foundation and an expansion for partial contact considerations.

Features of the Mechanistic Model

Partial Contact

Partial contact between the slab and the foundation may exist because of one or more of the following conditions:

1. An initial void or gap exists between the slab and foundation
2. A gap develops due to vertical temperature gradient in the slab
3. Positive (tensile) reactions between slab and foundation develop during the solution of the coupled problem, Equation 9.

Most existing slab analysis programs assume that the slab is tied to the foundation so that if some part of the slab wants to separate from the foundation, the foundation pulls the slab down with a force proportional to the foundation stiffness. The assumption is not valid for real slabs resting on elastic solid foundations since the only tensile forces coupling the slab and foundation are surface tension forces, which are very small even for fully-saturated cohesive soils. For real slabs the tendency of the slabs to separate form the foundation is resisted by the weight of the slab. The weight forces can be determined from:

$$Fwi = \gamma h\ Ai \tag{13}$$

where
γ = unit weight of concrete
h = slab thickness
Ai = area surrounding nodal point i

The slab weight results in constant contact pressure when the slab is in contact with the foundation and this pressure is treated as an initial stress and therefore not included in Equation 9. If partial contact occurs, however, the self-weight forces act at the nodal points out of contract and need to be included in the force vector (F) in Equation 9.

If a void exists under the slab, the points initially out of contact are assumed to remain out of contact since void sizes (depths) are generally much larger than slab delfections, but no assumption is made in case of curling and warping due to vertical temperature gradients in the slab. The amount of initial curling is determined from:

$$Wci = \frac{2\Delta T\ di^2}{2\ h} \tag{14}$$

where
σ = coefficient of thermal expansion for concrete
ΔT = difference in temperature between top and bottom of slab
di = radial distance from nodal point i to slab center
h = slab thickness

If a partial contact condition exists, the foundation does not support the slab at these points, nor, as was discussed above, does it pull the slab down. Since tensile forces cannot exist between the slab and foundations, the following additional constraint is placed on the solution to Equation 9:

$$\sigma zz \geq 0 \qquad (15)$$

In order to satisfy Equation 15, the subgrade stiffness is changed to zero at each point where tensile stresses (between slab and foundation) are developed and the slab weight forces (from Equation 13) are added to the force Vector {F}. The solution of Equation 9 thus becomes an iterative process that is repeated until Equation 15 is satisfied at all nodal points. The iteration process is automatic if vertical temperature gradients are present, but is left up to the designer when thermal loads are not present. This option is given to the designer since the iterative process is time-conusming, and in many cases useful information can be obtained without iteration; particularly if the tensile reactions are relatively small (as compared to the applied load) and acting relatvely far from the region of interest. Also, a preliminary analysis of doweled jointed pavements under edge loading can be done using one slab only with no iteration; the results of such an analysis are very similar to a complete analysis using three slabs with the iteration option. It is recommended, however, that the program be allowed to iterate for the final design.

A potential problem may arise during the iterative solution process if a large void is specified, as shown in Figure 7. If the void is wide enough, a diagonal term in the stiffness matrix may become zero, i.e., static equilibrium is no long satisfied because the slab rotates as a solid body, and the iteration stops.

Extension to Multiple Slabs

The previous discussion has centered around coupling a single slab to the multilayer foundation, which is adequate for designing plain jointed (undoweled) concrete pavements without tied shoulders. However, design of doweled jointed pavements and/or pavements with tied/keyed shoulders requires that multiple slabs be analyzed.

Two methods of extension to multiple slabs have been reported in the literature:

1. Multiple slabs are included in the finite element mesh, with dowel bars treated as beam elements (37)

2. The effect of additional slabs is determined through an indirect iterative approach (29, 38) in which the influence of other slabs is coupled to the slab being analyzed through nodal influence forces, with the requirement that the difference in deflection of the loaded and unloaded slabs be consistent with the joint load transfer efficiency.

The first approach has been successful with slabs on Winkler foundation but runs into difficulty in the case of elastic solid foundation because element aspect ratios (ratio of element length to width) at the joint are very large and can result in non-positive definite matrices. The second approach is based on somewhat shaky theoretical grounds since the effect of the solid foundation is not directly transmitted across the joint, and the iteration process converges rather slowly. Since iteration is already required to satisfy the constrant that reaction forces (between slab and foundation) may not be positive, additional iteration makes solution times very long.

The difficulty with the first approach can be overcome if the obvious assumption is made that the vertical displacements for the points along the loaded slab are greater than the corresponding points across the joints for the unloaded slab. Consequently, a void forms under the joint of the unloaded slab, as shown in Figure 8, i.e., the points along the joint of the unloaded slab are not directly coupled to the subgrade, but are treated as a special case of partial contact.

Transverse Joint and Dowel Bars

The dowel bars in a finite element model are presented by 3-dimensional beam elements as shown in Figure 9, with 6 degrees of freedom at each nodal point. One end of the dowel bar is free to move horizontally, which simulates the behavior of real dowel bars. Since the placement of dowel bars may not necessarily correspond to plate nodes, the bending stiffness of the dowel bars is recomputed based on the size of the plate elements (mesh size) and the total number of bars per transverse joint:

$$I = K \cdot I \qquad (16)$$

$$K = n/N \qquad (17)$$

where

I = dowel bar moment of inertia about local axes 2 and 3 (Figure 14)

$$= \pi d^4/64 \qquad (18)$$

d = adjusted dowel bar diameter

$$= K^{.25} \pi d \qquad (19)$$

d = actual dowel bar diameter

N = number of nodal points (of the slab) along the transverse joint

n = number of dowel bars along the transverse joint

This approximation shifts the position of the dowel bars from their actual location (Figure 10a) to an equivalent location (Figure 10b) which simulates the effect of dowel bars on load transfer from slab to slab but results in a relatively crude approximation of stresses in the dowel bars. Therefore, Friberg analysis (39) is used to calculate the bearing pressure in concrete (σzz):

$$\sigma zz = \frac{KP}{Y^3 EI}(2 + \beta c) \qquad (20)$$

where

K = modulus of dowel support in psi (Figure 11)

E = elastic modulus of dowel bar

t = joint width (in inches)

β = relative stiffness of the dowel bar embedded in concrete

$$= \left(\frac{Kb}{4EI}\right)^{.25} \qquad (21)$$

P = slab forces calculated by the program

b = dowel bar diameter

I = dowel bar moment of inertia

In the RISC program the dowel bar modulus has been assumed as 207 GPa (30,000,000 psi) and K as 10.3 GPa (1,500,000 psi) as suggested by Friberg. Of course, the above calculations are all done internally in the program; the user needs to specify only bar diameter, spacing, and joint width.

Dowel Bar Looseness

Since the free end of the dowel bar does not precisely fit the hole in the concrete, the load transfer efficiency of dowel bars is generally lower than would be expected if no looseness is assumed. To take into account the effect of dowel bar looseness, analyses were carried out to relate dowel bar looseness to effective bar diameter with no looseness. The following assumptions were made in this analysis:

1. Looseness is uniform for all dowel bars.
2. The loaded slab has to deflect by the amount of looseness before dowel bars become effective in load transfer, i.e., the loaded slab behaves as a single slab with a free edge until the deflection exceeds the amount of looseness, at which time the free edge is transformed into a joint with fully effective dowel bars and without voids.

A void with depth equal to the amount of looseness forms under the joint of the loaded slab due to the high stress concentration (at the slab-foundation interface) under an undoweled joint.

4. All dowel bars come into contact at the same time (deflection value) independent of the distance from the applied loads.

The above assumptions were made in order to simplify the analysis and make it possible to analyze looseness inside a linear elastic model.

The determination of effective bar diameter as a function of dowel bar looseness is a multistep process:

1. For no dowel bars and a void under the loaded slab transverse joint (assumption 3), determine the magnitude of load (standard single axle configuration) required to bring the slab in contact with the foundation. Maximum displacement differences across the joint must be equal to the amount of looseness. Determine maximum stress in the slab produced by the required load.
2. Determine the maximum stress in a fully-supported slab with actual dowel bars with no looseness as a result of wheel load L = 40 kN (9000 lb) - load from step (1), standard axle configuration.
3. Sum corresponding stresses from steps (1) and (2).
4. For a fully-supported slab and no dowel bar looseness, find an effective bar diameter that results in the same stress in the slab as was obtained in step (3).

A parametric study carried out by Majidzadeh et al. (37) has determined the effective bar diameter. The following additional conditions were used in order to minimize the number of variables involved:

1. The bar diameter is a function of slab thickness - standard construction practice is to use a bar diameter that is approximately 1/8th of the slab thickness; therefore dowel bar diameter was assumed to be 1/8 of slab thickness.
2. The granular subbase modulus is related to subgrade modulus through

$$Eb = 11.06\ Es^{0.837} \qquad (22)$$

which states that granular subbases can only be compacted to a maximum density (modulus) that depends on the subgrade strength.

3. Concrete modulus was kept fixed at 34.5 GPa (5,000,000 psi) since a priliminary sensitivity analysis has shown that concrete modulus (over the range of values expected for highway concretes) had an insignificant effect on the results.

The parametric study resulted in the following relationship:

$$d'=d(1-10.5L-12.58L^{.7}\log(Es/1000)) \qquad (23)$$

where
- d' = effective dowel bar diameter
- d = actual dowel bar diameter
- L = amount of looseness (in inches)
- Es = subgrade modulus, psi

Equation 23 is incorporated into the RISC program so that the user needs only to specify actual bar diameter and the amount of looseness-the User Manual (37) provides guidelines for selecting appropriate values of looseness. It should be emphasized, however, that the bearing stress and bending stress in dowel bars is computed based on the acutal bar diameter.

Longitudinal Joint

The longitudinal joint is generally constructed as one of the following types:

1. plain butt joint
2. tied butt joint
3. tied, keyed joint

The untied joint has no effect on stresses or deflections; some load transfer is obtained through the use of tie bars, and tied, keyed joints provide additional load transfer, helping to reduce both edge and corner stresses futher. However, keyed joints cannot directly be modeled as a set of plate elements with bending stiffness per unit length to the bending stiffness per unit length of the tie bars. This unit stiffness can be obtained from, for tie bars:

$$K = Eb\ Ib\ m \qquad (24)$$

where
- Eb = modulus of tie bar
- Ib = moment of inertia of tie bars
- m = number of tie bars per linear foot

and, for plate elements:

$$K = h^3\ Ec/12 \qquad (25)$$

where
- Ec = concrete modulus
- h = plate thickness

By substitution of Equation 24 into Equation 25,

$$h = (12\ Eb\ Ib\ m/Ec)^{1/3} \qquad (26)$$

Equations 25 and 26 are used to determine the effective stiffness of the plate element used to simulate tie bars. The analysis is somewhat conservative for tied, keyed longitudinal joints since the increased shear transfer of the key is ignored; however, since a key is generally assumed as being incapable of transmitting moments, the degree of conservation is small.

Required Input Parameters

The inputs in this model include traffic in terms of equivalent 80 kN (18 kip) axle loads, material characteristics (layer thicknesses, moduli, Poisson's ratios, concrete flexural strength), and environmental effects are incorporated through the AASHTO regional factor (40). The output includes a summary of the input data, maximum displacement data, critical stresses and their location, predicted fatigue life (both in number of equivalent axle loads and in years), the predicted faulting during the predicted life, and dowel bar stresses, if appropriate.

The design method is not intended to replace the design engineer; rather, it provides the user a powerful tool for a rational analysis of the adequacy of proposed designs. The program realistically considers such parameters as joint spacing, joint width, effect of tie bars, dowel bars, dowel looseness, load location and magnitude, voids and partial contact between slab and supporting layers, thermal stresses resulting from vertical termperature gradients, and support layer properties. One, two, or three slabs in a row may be considered, with or without tied shoulders, and the slabs may be of different lengths, widths and thicknesses; however, joints must be aligned and continuous. Because of the expanded capability of the design programs, somewhat more input data is required than for most other design methods; however, the development of this data is not difficult, and the User Manual (40) give detailed guidelines for the development of the required data

Traffic Model. The nations highways are used by vehicles of varying sizes and axle configurations, number of axles, axle loads. Additionally traffic placement is not confined to a well-defined channel; rather, the distance from the pavement edge to the centerline of the tire varies with traffic volume, number of lanes, vehicle type and width, shoulder type, etc. A rigorous analysis would consider the damage resulting from each load and axle configuration at different placement positions separately and sum these damages using Miner's law (41) or some other procedure. There are three major difficulties with such an approach:

1. The precise placement and axle load magnitudes are generally not known; furthermore, estimates of traffic growth rate are relatively imprecise.
2. A detailed analysis including the variables listed above would increase the computation time dramatically (even if only 3 axle configurations, 3 load levels/axle and the 3 placement positions were considered, the computation time would increase by a factor of 27).
3. A rational damage function with the required sensitivity is not currently available.

In view of the above difficulties, the AASHTO rigid pavement load equivalency equation (40) is used to convert mixed traffic to equivalent 80 kN (18 kip) axles. It can be seen from Figure 2 that the load equivalency cracking distress function (RII-OAR) (9) used in RISC, i.e., the models are compatible. The load placement is assumed to be 457 mm (18 in) from the edge to the centerline of the dual tire which reflects actual variation in load placement and takes into consideration the increased damage from loads that are closer to the edge, i.e., the 457 mm (18 inch) position is a weighted average of typical placement hystograms and the resulting damage at each placement position. The development of this concept is discussed in detail in Reference 9.

Materials Model. A rational analysis of stresses and displacements in a rigid pavement requires that the pavement component layers (slab, base/subbase and subgrade) be considered as realistically as possible. The following material models are available for materials characterization:

1. linear elastic
2. non-linear elastic (stress-dependent)
3. elasto-plastic
4. viscoelastic
5. viscoplastic
6. non-linear viscoplastic

The most realistic of these models is the non-linear viscoplastic model; however, it is also the most complex of the models and requires a large number of input parameters which are generally not available except for a few materials. Additionally, all but the linear elastic model require additional iterative techniques, thus making solution times unreasonably long. Furthermore, the only available distress functions have been developed using linear elastic models; therefore the linear elastic model was chosen for materials characterization. It should be emphasized, however, that since the foundation is treated as a 3-layer elastic solid, laboratory test results can be used to develop moduli as a function of stress, temperature and loading rate, so that the analysis could be performed with non-linear viscoelastic material properties, if desired. However, the distress functions would no longer be applicable and new models would need to be developed.

Environmental Effects. The environmental effects could be divided into the following broad categories:

1. Temperature
2. Moisture
3. Freeze-thaw cycles
4. Depth of frost penetration

The pavement layer moduli used in rigid pavements are not particularly temperature dependent, (unless A.C. bases are used) except during the frozen state. Excluding freezing conditions, temperature affects primarily joint/crack spacing (average concrete temperature) and the amount of temperature curling (verticle temperature gradient). The maximum seasonal temperature change governs the amount of reinforcement required in CRC pavements, and the joint design for jointed pavements. Large annual temperature changes result in large joint openings which in turn decrease the laod transfer efficiency of dowel bars, increase dowel bars, increase dowel bar stresses and may result in sealant failure; it is therefore desirable to select shorter joint spacing in regions with high annual temperature changes. Although the RISC program can be used to compute joint opening as a function average concrete temperature, mechanistic models for determining maximum allowable joint movement are currently not available; therefore the design engineer's judgement in selecting appropriate joint spacing for his climate is invaluable.

Vertical temperature gradients can alter critical stresses significantly and may change predicted fatigue lives by a factor of 10 or more. Vertical temperature gradients undergo a daily cycle but are also greatly influenced by the average temperature and the time of the year. In order to incorporate temperature gradient effects into the design, a day would have to be divided into at least 6, and more likely, 12 periods and the seasonal effects on temperature gradients and on moisture variation (which affects foundation layer moduli) require that the year be divided into at least 3, and probably 6 representative periods. Additionally, the depth of frost penetration and the number of freeze-thaw cycles alter support layer properties, requiring additional divisions into representative periods. Thus, a rigorous analysis of temperature and climatic effects requires that the year be divided into something like 48 to 72 representative periods, each requiring separate traffic and stress analyses. The damage sustained during each of these periods could then be summed using Miner's law (41) to arrive at an average damage during the year. While such an undertaking is possible using the RISC program, it is impractical for the following reasons:

1. The amount of computer time would be prohibitive.
2. The effort required to generate the required input data would be astronomical, particularly since much of the required data is unavailable and would have to be determined through detailed field and laboratory measurements.
3. Compatible distress functions are unavailable.

Therefore, the design program uses average annual conditions for support analysis and assumes that temperature gradients do not exist. Since the distress functions have been developed using the same assumptions, compatibility is assured. It should be emphasized, however, that RISC is capable of computing the stress due to vertical temperature gradients along with load-induced stresses. The designer is encouraged to check his design at the critical location by including thermal effects as well in order to prevent immediate failure.

The environmental effects are incorporated into the design through the regional factor (40) which is used to modify the traffic, i.e., the design traffic for a particular climate is the estimated equivalent 80 kN (18 kip) axle loads times the regional factor. Since better models are not available, the use of regional factor at least incorporates the concept that roads deteriorate faster in some climates than in other regions under similar traffic conditions.

Model Verification

Since RISC represents a significant advance of the state of the art in rigid pavement analysis and design, only a very limited comparison with existing theories is possible. The following steps have been carried out for model verification:

1. Convergence of solution.
2. Comparison with multilayer theory.
3. Comparison with Pickett and Ray theory (17).
4. Qualitative comparion.
5. Comparison with field performance.

Steps 1 through 4 evaluate the validity of the stress analysis procedure and Step 5 is used to verify the design method, i.e., the combined effects of stress analysis and distress models. Each of the steps are described in the sections that follow:

Convergence

One of the most important steps in the verification of any finite element solution is the question of mesh convergence, i.e., as the number of elements is increased, does the solution converge to a limiting value? Detailed investigations were carried out for various load configurations and different material properties. These analysis showed that the rate of convergence is dependent both on load configuration and support layer properties - smaller element sizes are required for analysis of pavements on stabilized bases than for pavements on granular bases; therefore, 457 mm (18 inch) standard element size has been selected for use in RISC. Although 457 mm (18 inch) element size is somewhat smaller than required for convergence, it is convenient in that most slab dimensions are divisible by this number.

Comparison with Multilayer Theory

Multilayer theory assumes that slabs extend to infinity in the horizontal direction whereas only finite size slabs can be analyzed using RISC. Also, multilayer theory assumes that the load is applied over a circular area but because of rectangular elements rectangular areas are required in RISC, therefore, the comparisons are not strictly valid.

The stresses and displacements derived from RISC were compared to those computed by ELSYM5 (21). This comparison showed that the deviation between the methods was less than 2.5 percent in stress and less than 2 percent in deflection over a wide range of material properties (thicknesses and layer moduli). Of course, this comparison only shows the the validity of the model for interior loading condition, which is not of interest in rigid pavement design. It does, however, demonstrate that the assumptions and solution methods are valid.

Comparison with Pickett and Ray Theory (17)

As was discussed in the review of existing stress analysis methods, the only available models that consider the subgrade as an elastic solid are Pickett and Ray theory (17) and finite element slab programs developed by Huang (33), by Zienkiewicz and Cheung (36) and by Saxena (27). Since Pickett and Ray influence charts have widespread acceptance, this method was chosen to verify stresses computed by the RISC program as a result of edge loading. The comparisons showed a discrepancy of less than 6 percent over a wide range of material properties. Of course, since Pickett and Ray assumed that the slab is very large, i.e., semi-infinite, the comparison is not totally valid; however, the comparisons show excellent agreement, particularyly since the use of the influence charts results in some error.

Qualitative Comparison

The comparisons shown in the previous two sections show the validity of stresses and displacements in a single slab, but have not checked the concepts used to extend the theory to multiple slabs. Since there are no theoreis that can be used to check stresses in multiple slabs (available theories that treat multiple slabs use Winkler foundations), direct verification is not possible. Some qualitative comparisons can be carried out, however. Consider for example, a two-slab system as shown in Figure 12 load case A. Common sense shows that as the dowel bar stiffness increases, the stresses and displacements should decrease, and when the dowel stiffness becomes equal to the concrete stiffness, the resulting stresses and displacements should be the same as those computed from interior loading of a slab with dimensions equal to the 2-slab system. The results in the following table show that this is indeed the case. These results therefore show that the assumptions used in extending to multiple slabs are valid in the limiting case and are most probably valid generally.

Similar arguements apply to corner loading of a 2-slab system (load case B, Figure 12), except that the limiting case of corner loading should be equivalent to edge loading of a single slab with dimensions equal to the 2-slab system. The following table shows that this is also true.

Next consider the case of thermal gradients in the two-slab system shown in Figure 12. For a negative temperature gradient and no dowel bars, the maximum stress occurs at the center of each slab. With dowel bars, however, the maximum stress should increase and move closer to the joint between the slabs since dowels provide additional restraint against vertical movement. In limiting case of dowel stiffness equal to slab stiffness, the maximum stress should be at the joint, i.e., it should be the same as that derived for a single slab with dimensions equal to the 2-slab system. The results in Table 1 again shows this to be the case.

While the above comparisons do not prove the validity of the assumptions conclusively; they show that the trends are according to expectation, and that the assumptions are valid in the limiting case.

Table 1. Effect of Dowel Stiffness on Stresses.

BAR in	DIA mm	STRESSES psi	MPa	LOAD TYPE	NO. SLABS
0	0	250	1.724	Case A	2
1	25	197	1.358	Case A	2
4	102	162	1.117	Case A	2
-	-	162	1.116	Case A	1
0	0	403	2.778	Case B	2
1	25	312	2.151	Case B	2
4	102	251	1.730	Case B	2
0	-	250	1.724	Case B	1
0	0	60	0.414	T.G.	2
1	25	88	0.607	T.G.	2
4	102	131	0.903	T.G.	2
-	-	132	0.910	T.G.	1

Temperature Gradient = -1^0 F/in (0.22^0 C/mm
Slab Dimensions 12x15 ft. (3.66x4.57 m)

Comparison with Field Performance

Probably the most significant step in validating a theory is how well this theory predicts the performance of actual pavements. The RISC program was used to analyze the performance of the 221 unfailed sections of the AASHO Road Test; the results of that analysis showed excellent agreement with the observed performance (9). However, since the distress function was developed using the 99 failed sections of the Road Test, this agreement is to be expected.

To further verify the validity of the design procedure, performance data was compiled from some 90 projects located in 5 states: Florida, Louisiana, Maryland, New York and Ohio. The data collected for these projects included pertinent information regarding concrete strength (both compressive and flexural), slab thickness, subbase and subgrade type, pavement type (i.e., CRC or jointed), joint spacing and type, as well as pavement age and estimated cumulative equivalent 80 kN (18 kip) axle loadings.

In general very little data was available for concrete modulus, so this value was derived from compressive or flexural strength, the subgrade modulus was derived from CBR values, and the subbase modulus was assured to be related to the subgrade modulus through equation.

Since PSI data was not available for these projects, the pavement condition was rated using the PCR procedure developed for ODOT (42). This PCR procedure rates, among other factors, the amount of faulting, and the severity and extent of corner breaks and transverse and longtudinal cracking, which are factors related to load-induced damage. The failure criteria used was if a particular distress deduct exceeded 15% of maximum, the pavement was considered as failed. Corner cracking criteria was used for undoweled jointed concrete pavements, and the amount of transverse and longitudinal crack deterioration for CRC and doweled jointed pavements.

The pertinent information concerning these project can be found in Reference 37 and the analysis results are shown in Figure 13. This figure shows excellent agreement between predicted and actual performance as measured by the PCR procedure. It can be seen from Figure 13 that while some projects failed earlier than predicted, about an equal number lived longer than predicted, and that the grouping of projects is very close to the line of equality. The most significant deviation occurs for the Florida projects, which were all undoweled jointed pavements and would thus be more sensitive to variations in support modulus. Even for these projects, the deviation is less than might be expected from the normal scatter in fatigue data. Nevertheless it must be concluded that RISC predicted the performance of these pavements exceedingly well, and since the projects cover a wide range of climatic conditions, the universal validity of the procedure is demonstrated.

It should be noted that the actual traffic experienced represents the estimated cumulative 18 kip (80 kn) axle loads applied to the pavement, multplied by the regional factor for the project (123). RF of 1.0 was used for Maryland, southern Ohio and northern Louisiana; and 1.1 for northern Ohio, 1.3 for northern Louisiana; and 1.5 for New York.

Summary and Conclusions

It was shown previously that the existing rigid pavement analysis and design procedures all have severe shortcomings since they either treat pavement support conditions unrealistically and/or fail to take into account the effects of rigid pavement features, and that a new rational methods for the analysis and design of rigid pavements is needed. Therefore, a new rational stress analysis model was formulated. The essential features of this model are:

1. Pavement support is treated as a multilayer elastic solid composed of a base layer, a subbase layer and a semi-infinite subgrade layer. One, two or three layers may be used for support analysis.
2. Concrete slab is modeled using finite element methods with thin shell elements.
3. One, two or three slabs in a row can be analyzed, and these may have tied shoulders, so that up to six slabs can be analyzed.
4. Dowel bars are analyzed realistically as beam elements and dowel looseness effects are included.
5. Both load-induced and thermal stresses can be computed, either singly or in combination, although the distress function include load-induced stresses only.
6. Thermal stresses are computed as a result of vertical temperature gradients using realistic concepts that include slab weight, rather than the simplistic Westergaard analysis that assumes these stresses to arise as a result of the foundation pulling the slab down.
7. The effect of voids and partial slab support are considered.
8. All essential rigid pavement features such as slab size, joint width, load locations, absence or presence of shoulders.

This stress analysis model was translated into a pavement design program, as discussed in the preceeding sections. The primary emphasis in this task was to make the program easy to use and to minimize the amount of data entry required, with the result that the user does not need an understanding of the theory involved; he merely needs to understand the fundamental concepts of rigid pavement design. Detailed instructions on how to generate the required input data as well as a summary of pavement design concepts and methods to minimize those distresses that currently lack mechanistic predictive models are presented in Reference 37.

The RISC program predicts the amount of fatigue cracking and faulting based on mechanistic distress prediction models and is modular so that as predictive models for other pavement distresses become available, these can easily be incorporated into the design method.

Since the RISC program represents a significant advance in the concepts of rigid pavement design, extensive investigations were carried out to verify the design model. These verifications are discussed in the preceeding sections and show that the model predicts the performance of some 90 actual pavement projects exceedingly well.

The design model currently lacks a mechanistic predictive model for pumping distress; however, a model is currently being developed under an FHWA contract and it is hoped that this model can be incorporated into RISC when it becomes available. The revised PCA design procedure (43) contains a pumping model of tests using erision criteria; however, this model has been developed using Winkler representatives for the foundation layers and is therefore inconsistent with the RISC program.

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Figure 1. Summary of PCC Fatigue Data with Various Fatigue Models

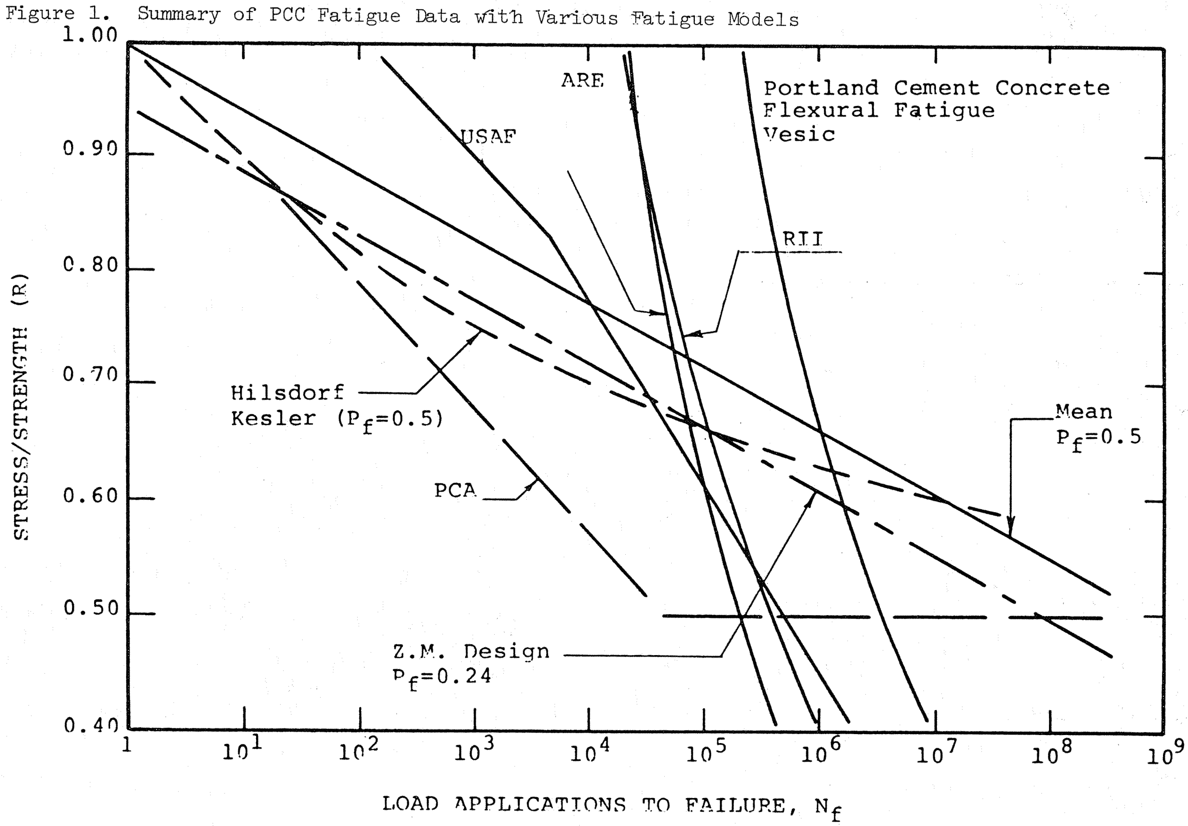

Figure 2. Effect of Axle Load on E18

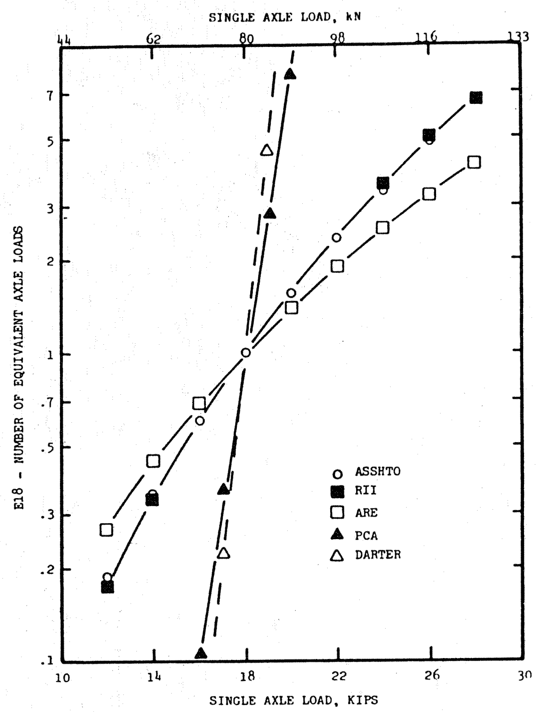

Figure 3. Two Bodies Joined at a Common Boundary.

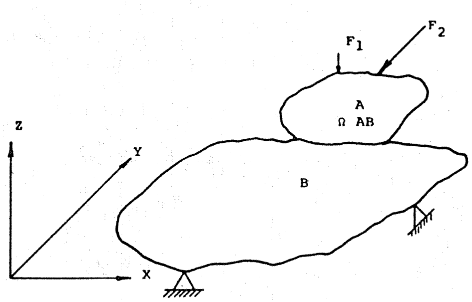

Figure 4. Separation of Two Bodies at the Common Boundary.

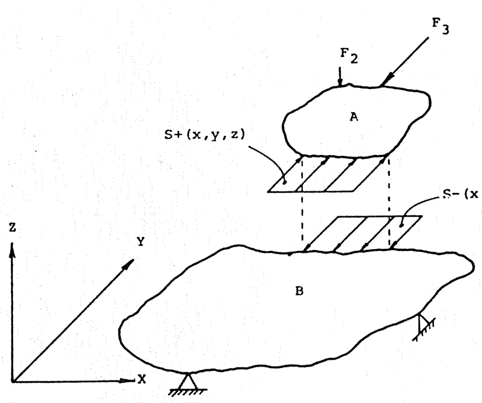

Figure 5. Slab Resting on a Multilayer Elastic Solid Foundation.

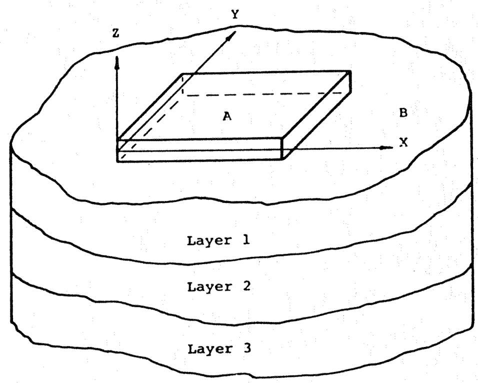

Figure 6. δfunction representation of the unit force.

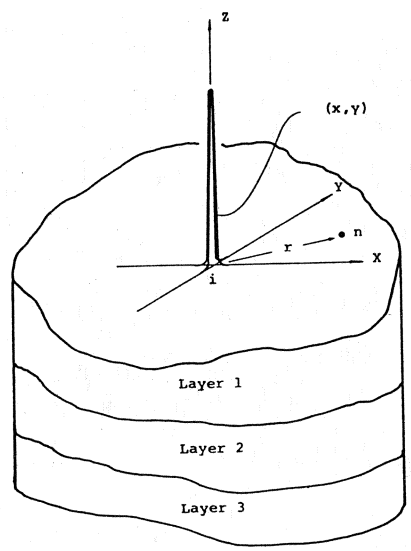

Figure 7. Example of Statically Unstable Slab

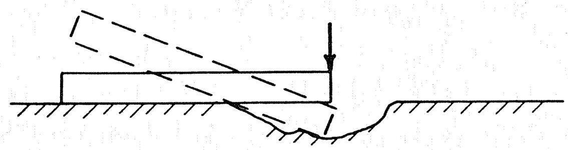

Figure 8. Formation of a Void Under Edge of Unloaded Slab.

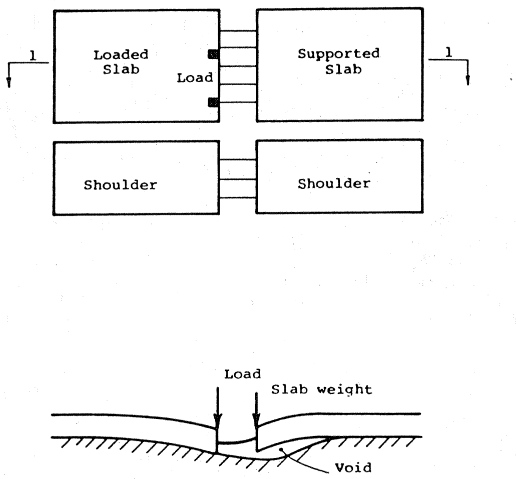

Figure 9. Dowel Bar Modeled as a 3-Dimensional Beam Element.

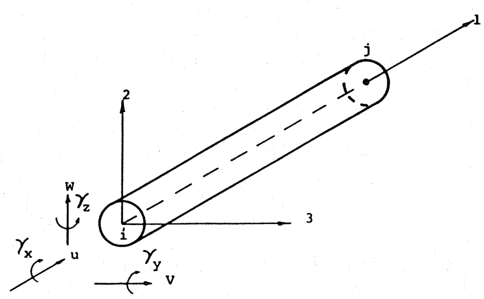

Figure 10. Comparison of Dowel Bars in a Real Pavement with a FEM Model.

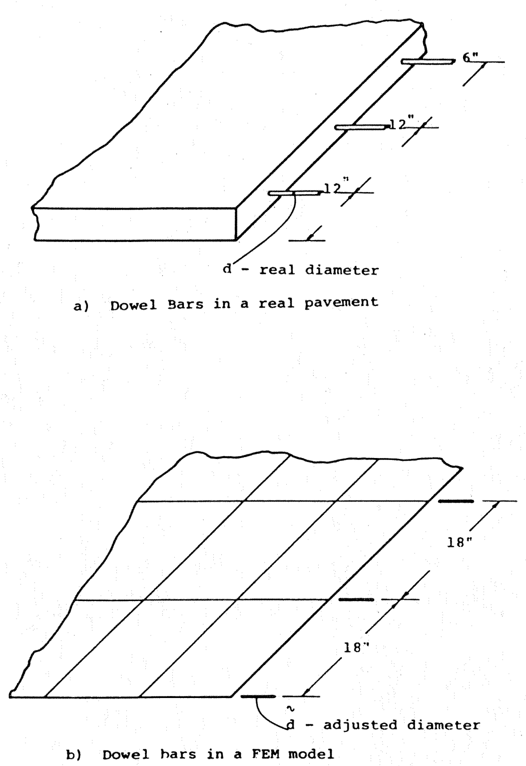

Figure 11. Friberg Analysis of Dowel Bar Support.

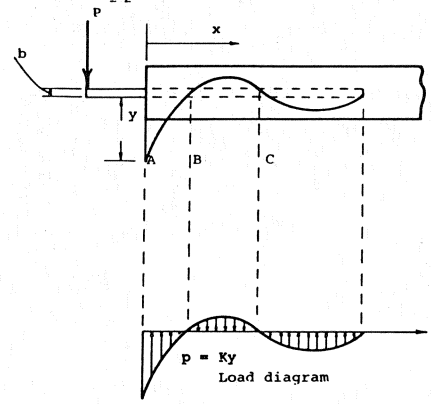

Figure 12. Slabs Used in Qualitative Comparisons

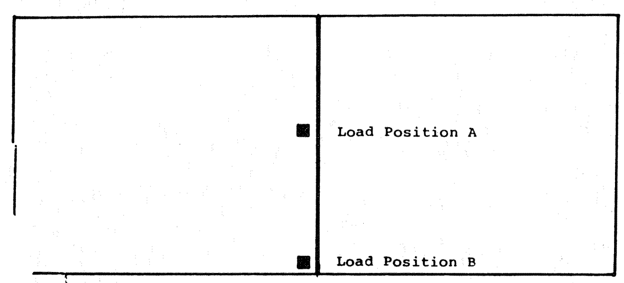

Figure 13. Failure Analysis of In-Service Pavements.