# THICKNESS DESIGN OF PLAIN CEMENT CONCRETE PAVEMENTS ON SOILS SENSITIVE TO DIFFERENTIAL SETTLEMENTS

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As large parts of the Netherlands are situated on a soft subsoil, a new thickness design method has been developed in which differential settlements can be taken into account in a quantitative sense. Supposing a sinusoidal settlement profile of the subgrade, the bending stresses in concrete slabs have been calculated. These proved to be dependent on the quotient of settlement amplitude and the square of the half wave-length as well as on the slab dimension, slab stiffness and the modulus of subgrade reaction. To determine the fatigue life these stresses have been combined with those of the traffic loading and those following from a thermal gradient. For a practical case of a highway pavement a nomograph has been derived, relating the thickness of the slabs to the settlement amplitude and the half wave-length. Also critical values for the settlement characteristics have been calculated in order to avoid that the slabs would loose contact with the subgrade. From the calculations it followed that for concrete pavements differential settlements with half wave-lengths which are sometimes the slab's length are acceptable. However, settlements which take place over small distances (about one time the slab's length) must be avoided for they could lead very soon to premature cracking.

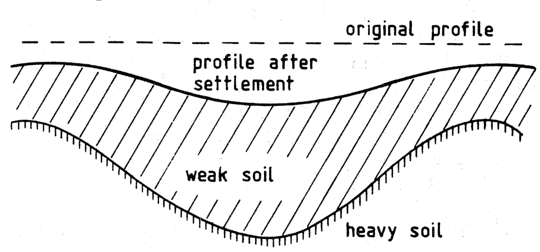
In most of the design methods for concrete pavements it is supposed that the subgrade has a homogeneous composition. In the Netherlands however, the subgrade is very often layered, while thickness and composition of the various layers can vary strongly. As a consequence the consolidation process, which mainly depends on the surplus weight of the pavement structure, differs very much from place to place (see figure 1).

Flexible pavements have the capability of following differential settlements of the subgrade, without giving raise to high stresses in the structure.

Rigid pavements, however, will less easily follow those differential settlements. Depending on the magnitude of the differential settlements, high stresses can develop, which may lead to premature failure due to cracking. It is a pity that there practically is no knowledge of how to integrate differential settlements in the design of rigid pavements in a quantitative sense.

As large parts of the Netherlands have to contend with soft soils, it was decided to develop a new design method in which differential settlements could be taken into account in the thickness design. In this paper some design principles will be mentioned first and then the relation between a supposed settlement profile and the stresses in the concrete slabs including creep effects will come to discussion. Finally a case study wil be treated, in which the surplus thickness of the slabs of a highway was calculated supposing several different settlement profiles.

Figure 1. Settlement profile caused by inhomogenities in the subgrade.




## Current Design Method

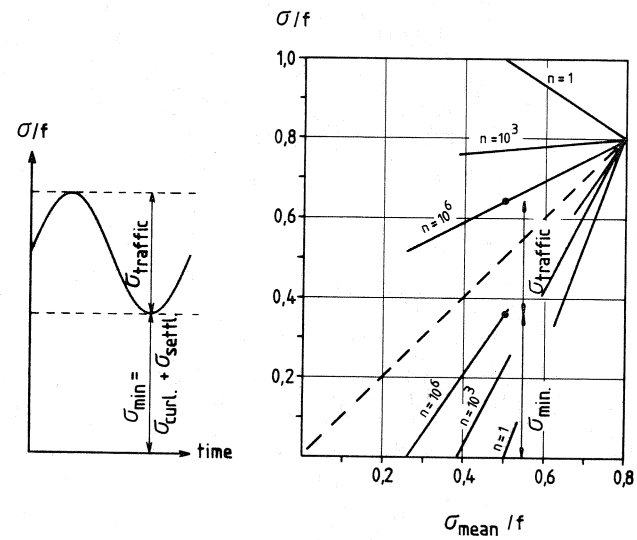
The new design method is based on the current Dutch BNC design method [1,2] which was developed by the former Foundation for Concrete Research of the Dutch Cement Industry (Stichting Betonresearch Nederlandse Cementindustrie). In this method the thickness of the slabs is determined in such a way that no fatigue cracks due to the combined action of traffic loading and a thermal gradient (curling) over the thickness of the slab wil occur during the desired service life. Besides it is investigated if the stiffness of the slabs is high enough to avoid pumping just near the joints, during the desired service life.

For determining the stresses in the slab due to traffic loading the original formulae of Westergaard are applied, whereas the magnitude of the curling stresses is calculated according to formulae of Eisenmann [3].

In the BNC method it is discerned, as also stated by Domenichini and Marchionna [4], that when traffic stresses appear in the slabs, they add themselves to the existing curling stresses. To take this into account a fatigue law is adopted which depends on both the curling stresses ($\sigma_{min}$) and the sum of curling and traffic stresses ($\sigma_{max}$). In figure 2 this approach is shown schematically by means of a Smith-Goodman diagram, which differs only slightly from the one used by Eisenmann [3].

As opposed to most current design methods the full spectrum of the traffic loading is used to determine the cumulative damage value, using Miners rule. Recently the adopted fatigue law has been compared with experimental fatigue results on uniaxial loaded cylinders (see Cornelissen and Leewis [5] and Cornelissen [6]). It showed that the adopted fatigue law was conservative for nearly all combinations of $\sigma_{min}$ and $\sigma_{max}$.

Figure 2. Smith-Goodman diagram as adopted in the current BNC design method related to the characteristic flexural tensile strength.




## General Principles

In the newly developed design method the fatigue life is based on two principles:

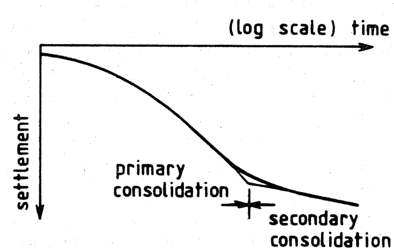
- the slabs must be able to follow the settlement profile of the subgrade by deflection due to their dead load, so contact with the subgrade must be guaranteed;
- the service life is calculated by taking into account not only the traffic stresses and the curling stresses, but also the stresses induced by differential settlements of the subgrade.

The latter principle implies that the value of $\sigma_{min}$ in the fatigue relation (see figure 2), is the sum of both curling stresses and stresses due to the differential settlement. Consequently for $\sigma_{max}$ the sum of all three mentioned stress components is taken.

On purpose of calculating the stresses in rigid pavements due to differential settlements and to control whether the contact of the slab with the subgrade is guaranteed, the following suppositions have been made:

- the settlement profile in the longitudinal direction of the pavement is shine-shaped;
- the slab is considered as a thin plate resting on an elastic foundation with a modulus of subgrade reaction k (see figure 3);
- although the differential settlement is a function of time (see figure 4), it is idealized in the method as being constant in time;
- the influence of stress relaxation of the concrete slab due to a constant imposed deformation can be taken into account by a reduction of the Young's modulus of the concrete.

Figure 4. General consolidation behaviour of the subgrade.




## Characteristics of the Settlement Profile

Figure 5 shows a general settlement profile of the subgrade, induced by inhomogenities in the subgrade. It is supposed that this profile does not vary in the transversal direction of the pavement. Due to the dead load, the slab in position 1 will try to follow the settlement profile, thus causing tensile stresses on top of the slab. The slab in position 2 will also try to follow the settlement profile, thus causing tensile stresses at the bottom of the slab.

In order to determine the curvature of the settlement profile near the slabs in position 1 or 2, sine-shaped approximations of the profile have to be made (see figure 5). Thus resulting in values for the settlement amplitude $\delta_m$ and the half wave-length T for each critical position.

Figure 3. Mechanical modelling of the initial situation (on the left) and the "loaded" situation (on the right).

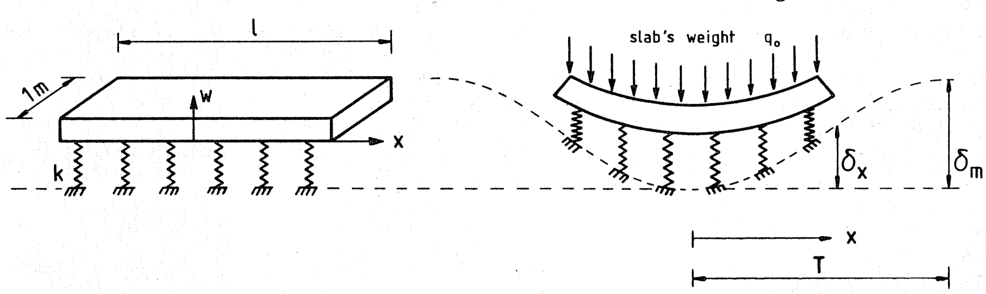
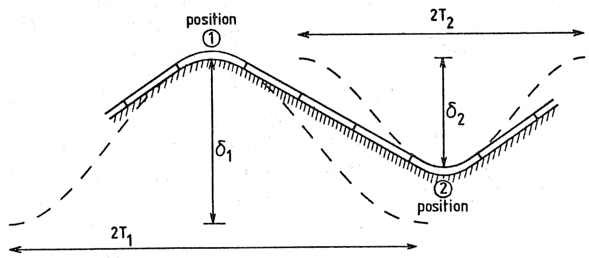


Figure 5. Sinusoidal approximation of the critical points in a settlement profile.




Provided the slab in position 1 does not loose contact with the subgrade, in general the slab in position 2 will be the critical one. This can be explained by examining the combination of the stresses due to differential settlements and those caused by a thermal gradient (curling) and the traffic loading. As can be seen in figure 6, only for the slab in position 2 the combination with stresses due to traffic loading must be considered. Besides that, the curling stresses in case 2 (warming up in day time) will be much higher than those of case 1 (night time cooling), see Eisenmann [3].

## Mechanical Modelling

The stresses in every cross section can directly be calculated because they are linearly related to the curvature in every cross section, postulated that the slabs will follow exactly the settlement profile caused by the inhomogenities in the subgrade. In reality the slab will offer resistance to the imposed curvature, which will result in pushing away the subgrade at the edges of the slab and lifting a bit of the slab in the middle (figure 3). This will reduce the maximum curvature of the slab and in that way the maximum tensile stress.

To bring this effect into account in the calculation of the tensile stresses due to differential settlements, the model as geven in figure 3 is considered. For a simplification, the slab has been positioned symmetrically in the settlement profile. Because of the fact that it is supposed that the settlement profile does not change in a transversal direction of the pavement, only a strip with a width of e.g. 1 m has to be considered.

## Derivation of the Differential Equation

As initial situation it is supposed that the x-axis is along the bottom of the slab (see figure 3). The settlement profile of the subgrade is described by the displacement $\delta_x$:

$$\delta_x = \delta_m \; (1-\cos(x\;\pi\;/T)) \qquad (1)$$

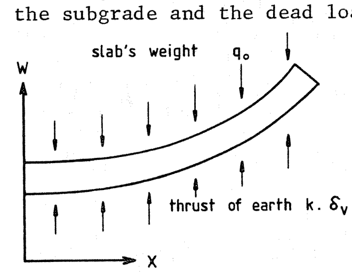
in which $\delta_m$ is the settlement amplitude and T the half wave-length. For the sake of convenience the positive direction of the vertical displacement of the slab w and the displacement $\delta_x$ is chosen in the upward direction.
The shortening $\delta_v$ of the springs, which simulate the reaction of the subgrade is equal to:

$$\delta_v = \delta_x - w \qquad (2)$$

As illustrated in figure 7 the slab is loaded by an upward pressure q, being the difference of the subgrade reaction and the dead load $q_o$:

$$q = k\;\delta_v - q_o \qquad (3)$$

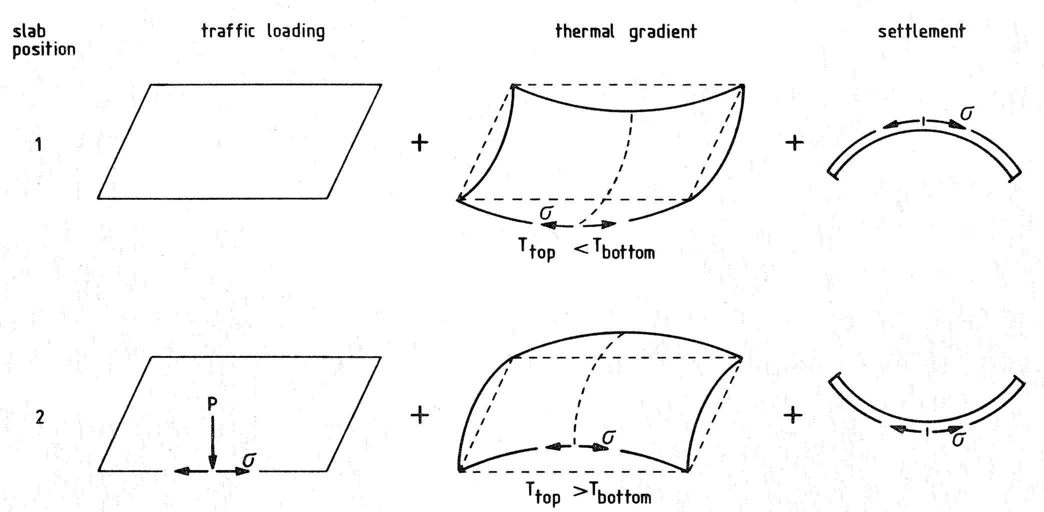
Figure 7. The loading acting on the slab by pressure of the subgrade and the dead load.




The vertical displacement w of the slab can be calculated by resolving the following differential equation which is known from the theory of elasticity (see Timoshenko [7]):

$$w'''' = q/E'I = 12\;(1-\nu^2)q/(E'h) \qquad (4)$$

Figure 6. Two different loading combinations.



in which E' is the modulus of elasticity of the concrete taking account of creep effects, I the moment of inertia, $\nu$ the Poisson's ratio, h the thickness of the slab and w'''' the fourth differential quotient of w to x.

In [8] the complete resolution of the differential equation 4 is presented.

## Resolution of the Differential Equation

Using the condition that at the end of the slab (joints) both moment and shearing force are zero, the general shape of the resolution is as follows:

$$w = 2\,A\,\mathrm{ch}(\beta\,x)\,\cos(\beta\,x) + 2\,B\,\mathrm{sh}(\beta\,x)\,\sin(\beta\,x) + 0.5\,\delta_m - q_o/k + C\,\cos(x\,\pi\,/T) \qquad (5)$$

in which the parameter $\beta$ is equal to:

$$\beta = \sqrt[4]{k/(4E'I)} \qquad (6)$$

The tensile stress at the bottom of the slab can be calculated using the equation:

$$w'' = -\,M/E'I = \sigma_{settl.}\,(1-\nu^2)/(E'0.5h) \qquad (7)$$

The maximum tensile stress wil be in the middle of the slab where x=0:

$$\sigma_{settl.} = \frac{\pi^2\,E'h}{4(1-\nu^2)}\,\frac{\delta_m}{T^2}\,\alpha \qquad (8)$$

The constants A, B, C and $\alpha$ (see [8]) prove to be dependent on the wave-length of the settlement profile, the length of the slab and the quotient between the modulus of subgrade reaction and the bending stiffness of the slab.

## Stress Distribution and Stress Reduction Factor $\alpha$

Inspecting formula 8 for the maximum tensile stress due to differential settlements, it is found that the tensile stress is linearly related to the thickness of the slab, the modulus of elasticity of the concrete, the settlement amplitude, the inverse of the square of the half wavelength and the factor $\alpha$.

In order to get some idea of the magnitude of $\alpha$, a number of calculations have been carried out. In these calculations a half wave-length T = 10 m was supposed. The value of the static modulus of elasticity of the concrete was set on 34000 MN/m$^2$. In order to bring into account the effect of stress relaxation, the effective modulus method (see Neville [9]) was followed.
This means that the static modulus of elasticity E is replaced by a sustained modulus E' based on the total deformation at the given time:

$$E' = \frac{E}{1+\phi\ (t)} \qquad (9)$$

in which $\phi$ (t) is the creep factor at time t. Since stress relaxation is a relatively fast process with respect to the obtained service life of pavements, the ultimate creep factor can be chosen. In this study for $\phi$ (t), a value of 2 has been chosen, which seems to be realistic for this type of concrete [10,11].

In table 1 for several values of the length of the slabs, the thickness and the modulus of subgrade reaction k, the calculated values of $\alpha$ are given. It can be seen that the value of $\alpha$ tends to one for high values of k and/or high values of slab length. From the formula of $\alpha$ (see [8]), it can be derived that for $k \to \infty$, which means that the slab will follow the settlement profile, the value of $\alpha$ is exactly one. For a short length of the slabs and for low values for k, it can be seen that $\alpha$ can drop to values lower than 0.3.

Table 1. Values of the stress reduction factor $\alpha$

| dikte (m) | k (MN/m$^3$) | $\beta$ (m)$^{-1}$ | $\alpha$ with a slab's length (m) of 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 0.16 | 7 | | | | | | |
| 0.20 | 13 | 0.81 | 0.27 | 0.59 | 0.83 | 0.97 | 1.01 |
| 0.25 | 26 | | | | | | |
| 0.16 | 13 | | | | | | |
| 0.20 | 26 | 0.95 | 0.45 | 0.78 | 0.97 | 1.03 | 1.04 |
| 0.25 | 50 | | | | | | |
| 0.16 | 26 | | | | | | |
| 0.20 | 50 | 1.13 | 0.65 | 0.94 | 1.04 | 1.05 | 1.03 |
| 0.25 | 98 | | | | | | |
| 0.16 | 50 | | | | | | |
| 0.20 | 98 | 1.33 | 0.84 | 1.03 | 1.06 | 1.03 | 1.01 |
| 0.25 | 190 | | | | | | |

As an illustration, in figure 8 the deflection profile of the slab for several values of k, and in figure 9 the tensile stress distibution at the bottom of the slab are given. These figures clearly show that for low values of k, the slab is less curved in comparison with the settlement profile, and consequently less higher tensile stresses will occur.

Calculations with other values of half wavelength T or modulus of elasticity E' of the concrete show that the value of $\alpha$ does not change significantly.

Figure 8. Deflection profile of the slab itself for several values of k in MN/m$^3$.

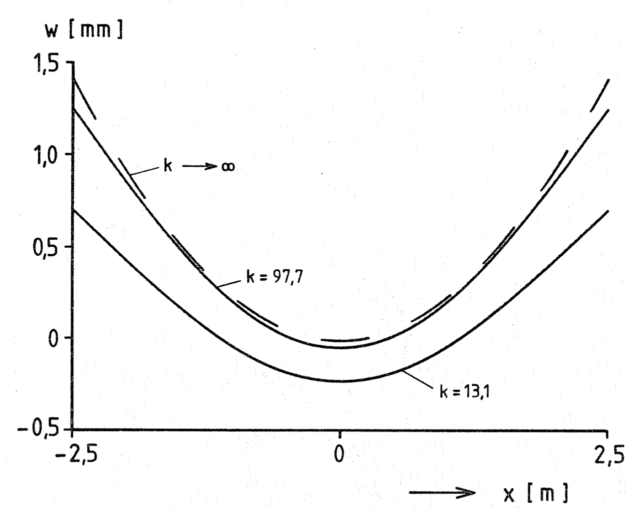
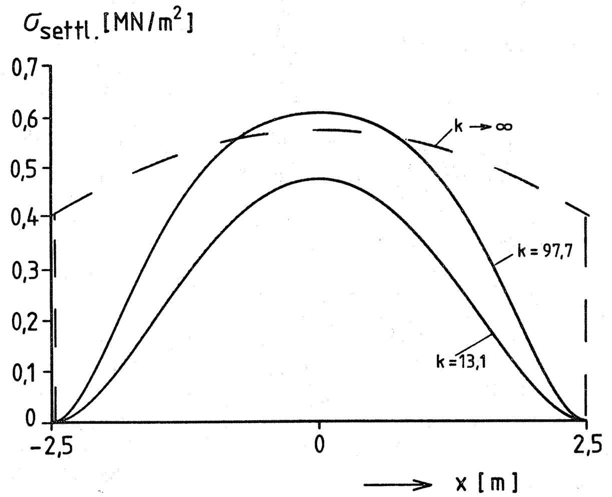


Figure 9. Settlement stresses over the length of the slab for several values of k in MN/m³.




Concludingly it can be stated that the coefficient $\alpha$ is to be considered as a stress reduction factor. It brings into account the fact that due to bending stiffness of the slab, the slab will not exactly follow the settlement profile. Near the edges of the slab the subgrade will be pushed away, whereas in the middle the slab will be lifted a little. With the exception of some critical cases in which the slab is more curved in the middle, this will lead to a reduction of the tensile stresses induced in the slabs by the imposed deformations of the subgrade.

## Critical Values of $\delta_m/T^2$

From equation 2 and 3 it can be seen that when $w > \delta_x$ the pressure of the subgrade will become negative. This is of course not realistic. Instead of that the slab will be lifted a little from the subgrade in the middle. This means that due to the traffic loading, higher stresses will develop than were calculated with the formulae of Westergaard, which assume a homogeneous support. This could lead to premature cracking and must therefore be prevented.

For several cases the critical values of $\delta_m$ have been calculated for which contact is still guaranteed, using equation 1 and 5. From the calculations it proved that the critical value of $\delta_m$ was almost linear dependent on $T^2$. For that reason only critical values for the quotient $\delta_m/T^2$ are given in table 2, both for a slab thickness of 0.2 m and 0.25 m.

The given values can directly be interpreted as critical values for $\delta_m$ in mm when a half wave-length of 10 m is considered.

As a general trend, the calculations show that the critical value for $\delta_m$ increases with increasing slab length and decreases with increasing value of the modulus of subgrade reaction or the slab thickness.

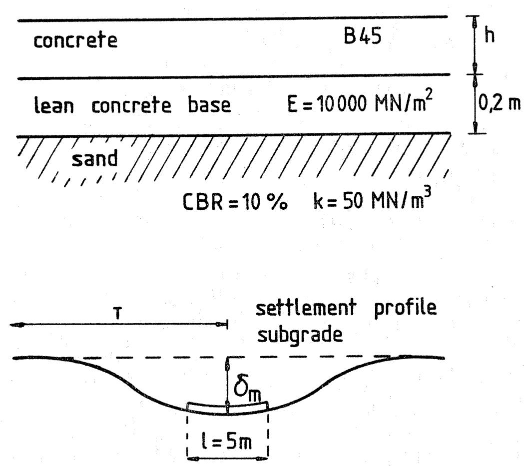
Caroti et al [12], have developed a simplified approach to determine the allowed deflection before the slab looses contact. In this approach, in which the subgrade is considered to be fully rigid, $k \to \infty$, the same general trend is found both for a variation in slab length and slab thickness. The absolute values of $\delta_m$ as given by Caroti, are slightly lower than the values calculated by the author of this paper. However, this is partly a drawback of the higher value for the modulus of elasticity as adopted by Caroti.

## Thickness design example of an ordinary highway

Some calculations have been carried out using the current BNC thickness design method, extended with the feature to incorporate in the fatigue law also the influence of stresses due to differential settlements. The main purpose of this exercise, which is completely described in [13], was to find a relation between the settlement characteristics $\delta_m$ and T and the surplus thickness needed, compared with the case in which no differential settlements occur.

In figure 10 a scheme of the structure is given. It shows that the slabs with a length of 5 m rest on a lean concrete base layer, which is quite common in the Netherlands. In the calculations of the (dynamic) stresses due to traffic loading, the lean cement concrete base layer is taken into account by an increase of the (dynamic) modulus of subgrade reaction k of 50 to 173 MN/m³. This is according to the nomograph given in the AASHTO Interim Guide for Design of Pavement Structures 1972 [14].

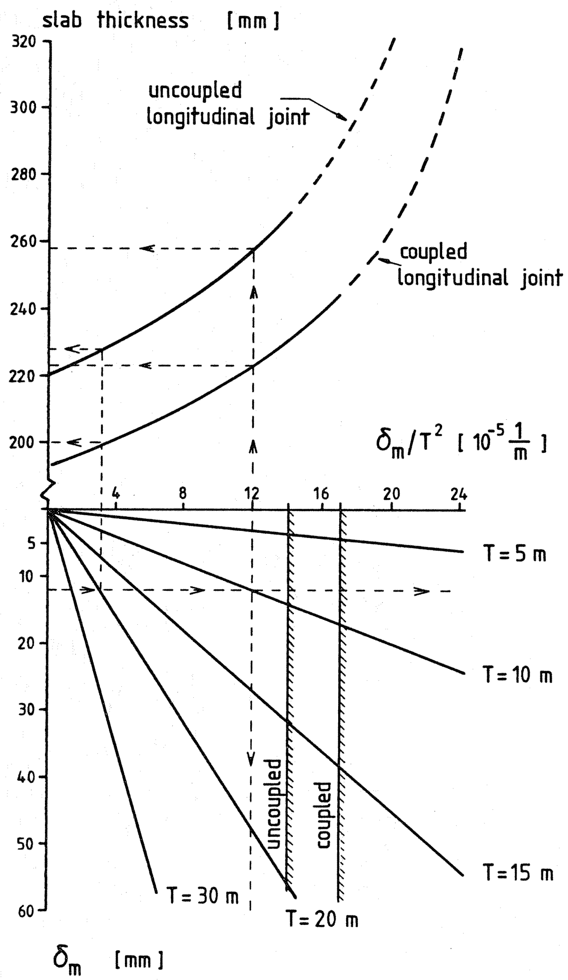
Figure 10. Calculation example.




The value of the modulus of subgrade reaction k, which must be used to determine the stress reduction coefficient $\alpha$ (see table 1), for determining the (static) stresses due to differential settlements however, calls for some discussion. Since the analysis of the stresses in the slabs due to differential settlements is related to a long term (static) process, the value of the modulus of subgrade reaction k has to deal also with a long term loading. For that reason the (static) k value, to determine the stress reduction factor, may be chosen lower than the (dynamic) value of 173 MN/m³. Probably a value of one third or even lower is realistic. This results for the given slab length in a stress reduction factor which is about one. This implies that there is no stress reduction caused by the weakness of the support. This administers justice to the fact that it is not very probable that the lean concrete base layer will allow the slabs to rotate much in the joints!

The results of the calculations are presented in figure 11. In the lower part of this figure, supposing a value of the settlement amplitude and the wave-length, the parameter $\delta_m/T^2$, which is a measure for the curvature, can be determined. In the upper part, the thickness of the slab can be found dependent on the parameter $\delta_m/T^2$.

Figure 11. Nomograph relating slab thickness to settlement amplitude and half wave-length for 5 m long slabs.




For example, the figure shows that when the joints in the direction of the axis are coupled, the theoretical thickness is 194 mm for $\delta_m/T^2 = 0$. For a sinusoidal settlement profile with a half wave-length of 10 m and a settlement amplitude of 12 mm, a surplus thickness of 29 mm is needed.

The figure also clearly shows the significant influence of the (estimate) wave-length. When changing the half wave-length from 10 m to 20 m, the surplus thickness drops to only 7 mm. However, when changing the half wave-length to only 5 m, no value for the surplus thickness can be found. This means that whatever thickness is chosen, the concrete will always crack.

Because of the fact that the value of the parameter $\delta_m/T^2$ determines the surplus thickness, it can be concluded that a surplus thickness of only 29 mm is also necessary for e.g. a settlement amplitude of 192 mm and a half wave-length of 40 m!

This conclusion, that from the point of view of structural design for concrete pavements differential settlements of about 0.2 m, in combination with a half wave-length of 40 m are acceptable, is also confirmed by Wilk [9]. He mentions a case of a settlement amplitude of 0.4 m and a half wave-length of 40 m on the highway Flums-Sargans N3. Even after a service life of 10 years, no cracks in the slabs nor compression damages in the joints were found due to the rotations of the slabs.

In the lower part of figure 11, the critical value for $\delta_m/T^2$ for the loss of contact of the slabs with the subgrade is also shown. It can be seen that for the cases of $\delta_m$ = 12 mm and T = 10 m or $\delta_m$ = 192 mm and T = 40 m, the slabs will follow the differential settlements by bending caused by their dead load. From the values given in table 2, it shows that for a slab length of 4 m instead of 5 m, the admissable $\delta_m/T^2$ value is lower, which means that this criterion will become more critical.

Table 2. Critical values for $\delta_m/T^2$ in $10^{-5}$ $m^{-1}$ for which contact between the slabs and the subgrade is just guaranteed.

| k ($MN/m^3$) | h (m) | $\delta_m/T^2$ with a slab's length (m) of 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 13 | 0.2 | 26 | 23 | 26 | 36 | 52 |
| 26 | 0.2 | 16 | 17 | 25 | 38 | 48 |
| 50 | 0.2 | 11 | 16 | 26 | 32 | 38 |
| 98 | 0.2 | 10 | 17 | 21 | 24 | 30 |
| 13 | 0.25 | 29 | 21 | 20 | 24 | 31 |
| 26 | 0.25 | 17 | 14 | 17 | 23 | 33 |
| 50 | 0.25 | 10 | 11 | 16 | 24 | 31 |
| 98 | 0.25 | 7 | 10 | 17 | 20 | 24 |

## Conclusions and final considerations

It appeared that the rule, which is commonly used in (utility) construction practice, stating that a differential settlement of e.g. 1 at 500 (this means 1 cm settlement over 5 m length) can be allowed, fails where pavement constructions are concerned. The main parameter which determines the stresses induced in the slab by differential settlements, is the settlement amplitude $\delta_m$ divided by the square of the half wave-length T.

The length of the slabs and the stiffness of the subgrade (including the base layer!) proved to be important too. The smaller they are, the lower the stresses induced in the slabs by differential settlements will be. This advocates for base layers with low stiffness, e.g. granular base layers, instead of lean concrete ones.

The case study concerning a highway section, learned that a surplus thickness of about 0.03 m is necessary in case of a settlement amplitude $\delta_m$ of e.g. 0.2 m and a half wave-length of 40 m. Whether such an increase in thickness is really a must or not, depends also on the number of slabs that probably lie in a critical position. When this is the case for only a few slabs, there is a small chance that just in these, the tensile strength equals the lowerbound value of the tensile strength, which is used in the fatigue law. In such cases there is a need for a more probabilistic approach.

Finally, it can be concluded that progress has been made by incorporating differential settlements in the thickness design of concrete pavements in a quantitative way. The major advantage is that there is a solid base now for taking an economical decision on:

a) adding a certain surplus thickness to the slabs dependent on stiffness of the subgrade, the base layer and the slab length;
b) making geotechnical provisions in order to eliminate the differential settlements as much as possible;
c) applying a reinforcement in the slabs.

References


1. Leewis, M., Most, H.E. van der, De dikte van een ongewapende betonverharding, Betonwegen-Nieuws 43, 1980, pp. 12-16.
2. Leewis, M., Most, H.E. van der, Lastoverdracht bij voegen in een ongewapende betonverharding, Betonwegen-Nieuws 44, 1981, pp. 10-14.
3. Eisenmann, J., Betonfahrbahnen, Wilhelm Ernst und Sohn, 1979.
4. Domenichini, L., Marchionna, A., Influence of stress range on plain concrete pavement fatigue design, Proc. 2nd Int. Conf. on Concrete Pavement Design, Purdue University, West Lafayetta, 1981, pp. 55-65.
5. Cornelissen, H.A.W., Leewis, M., Vermoeiing van ongewapende betonverhardingen; dimensionering en onderzoek, Betonwegen-Nieuws 55, 1984, pp. 2-7.
6. Cornelissen, H.A.W., Constant-amplitude tests on plain concrete in uniaxial tension and tension-compression, Report 5-84-1, Delft University of Technology, 1984.
7. Timoshenko, P., Theory of Plates and Shells, McGraw-Hill Int. Book Comp., 2nd edition, 1983.
8. Vos, E., Internal report Road Engineering Division COO-N-83-53 (in Dutch).
9. Neville, A.M., Creep of concrete: Plain, Reinforced and Prestressed, North Holland Publ. Comp.-Amsterdam, 1970, p. 367.
10. Wilk, W., Cement concrete pavements on soft soils sensitive to differential settlement, Proc. 2nd Int. Conf. on Concrete Pavement Design, Purdue University, West Lafayetta, 1981, pp. 201-210.
11. Voorschriften Beton VB 1974, Part A, NEN-3861, p. 23.
12. Caroti, L., et al, Concrete pavements and compressible subgrade soils compability, PTRC Summer Annual Meeting, Warwick, 1981.
13. Vos, E., De dikte van een ongewapende cementbetonverharding bij een ongelijkmatige zetting van de ondergrond, Wegen 6, 1984, pp. 220-227.
14. AASHTO Interim Guide for Design of Pavement Structures 1972, American Association of State Highway and Transportation Officials (AASHTO).