A COMPUTER PROGRAM FOR SLABS WITH DISCONTINUITIES ON LAYERED ELASTIC SOLIDS

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A finite element method programmed for a high-speed computer is presented. The method is based on the classical theory of thin plate. The program is developed for a subgrade soil represented as a linear layered elastic solid. Any number of layers can be accommodated. The program is capable of analyzing stress conditions in concrete pavements with the load transfer in the joint, as well as in the supporting subgrade soil, subjected to loads and temperature warping. Multiple-wheel loads can be input and the number of wheels is not limited. Because of the large computer storage space required, the program can handle only two slabs, except for the special option where a four-slab pavement system is loaded symmetrically at the pavement's center. At the joint, the program considers only the shear transfer and assumes the moment transfer to be zero. The results computed using the program were compared with stress transfers across the joint measured in airfield test pavements. The comparisons were very favorable.

Background

The determination of stresses and deflections in concrete pavements due to wheel loads has been a subject of major concern for nearly half a century. In early 1920, Westergaard (1), using the theory of elasticity by assuming the subgrade to be a Winkler foundation and assuming the slab to be infinite in extent in all directions away from the load, first developed a mathematical method for determining the critical stresses in concrete pavements resulting from corner, edge, and interior loads. In extending the method to airport pavements, he later developed new formulas (2,3) that give the stresses and deflections at an edge point far from any corner and at an interior point far from any edge. These formulas were then employed by Pickett and Ray (4) for developing influence charts, which have been used by the Portland Cement Association (5,6) for the design of highway and airport pavements.

In spite of their wide acceptance and usages, Westergaard solutions have been subjected to many criticisms, which include:

1. The solutions are based on an infinitely large slab with a load at the corner, on the edge, or in the interior; therefore, they may not be applicable to airfield pavements for aircraft equipped with large multiple-wheel gear loads.
2. The slab and the subgrade may not always be in full contact as assumed in the solutions. Gaps are frequently observed in the subgrade near the joint because of pumping action or plastic deformation. Temperature warping can also cause the slab to curl up and lose contact with the subgrade.
3. The solutions are based on an infinitely large slab with no discontinuities and thus cannot be applied to analyze stress conditions at the joint or crack that are of most concern to pavement design engineers. A joint or crack is a structural weakness in the pavement, and the lack of attention by the engineer to such structural weaknesses creates a problem for the maintenance engineer.
4. The solutions are based on liquid foundation (or Winkler foundation); i.e., the vertical force or deflection at one nodal point in the subgrade is only proportional to the stiffness at the node and is not affected by stiffness at other nodes.

Since late 1960, a discrete element method based on the finite difference technique has been developed at the University of Texas (7) to analyze concrete slabs on a liquid foundation. The method considers the slab to be an assemblage of elastic joints, rigid bars, and torsional bars. The means of modeling in this method proved helpful in visualizing the problem and forming the solution. The values given for pavement deflections are reasonable, but there are problems in achieving accurate stress values along the edges. In fact, serious problems exist in the analysis of joints, cracks, and gaps under the slab because of the nature of the method.

The Corps of Engineers realizes that much of the maintenance of rigid pavements is associated with cracks and joints. The current Corps of Engineer rigid pavement design procedures have certain limitations that were imposed by the state of the art at a particular stage of development. During the development of the procedures, it was

necessary to make simplifying assumptions and, in many instances, to ignore the effects of cracks and joints. Since the advent of high-speed computers and the development of the finite element method, a more comprehensive investigation of the state of stress at pavement joints, cracks, and other locations in multicomponent pavement structures is allowed than was previously possible. Consequently, a better and more reasonable design procedure may be developed for rigid pavements.

The finite element computer programs originally developed by Huang and Wang (8,9) and Huang (10,11) were modified and extended at the U. S. Army Engineer Waterways Experiment Station. The two programs developed at the Waterways Experiment Station are called WESLIQID and WESLAYER. WESLIQID is for slabs on a liquid foundation, and WESLAYER is for slabs on a layered elastic solid. Both programs have the capabilities of analyzing stress conditions in a rigid pavement containing cracks and joints subjected to loads and temperature warping. Part of the pavement can be out of contact with the supporting subgrade before applying the load and the temperature gradient, and the program determines the condition of contact at each nodal point after the application of loads and a temperature gradient. Multiple-wheel loads can be handled, and the number of wheels is not limited. References 12 and 13 describe the WESLIQID computer program. This paper presents the finite element method of the WESLAYER computer program and the comparisons between the computed results and field measurements.

Description of the Finite Element Program

Plate Bending Model

The finite element method employed in this study is based on the classical theory of thin plate by assuming that the plane before bending remains a plane after bending. Because the slab is modeled as a thin plate, there is no variation in vertical deflection along the thickness of the plate; i.e., the deflection at the top of the plate is the same as that at the bottom. Where the slab is divided into rectangular finite elements, the division is made only in the longitudinal and transverse directions, because the vertical direction (the direction along the thickness of the slab) is not needed. The model is thus two dimensional. The other advantage of the plate bending model is that the application of the finite element method is not involved in the subgrade soil. Only the subgrade reactive forces acting at the nodes are important. The subgrade reactive forces can be evaluated readily by other procedures.

Figure 1 shows a rectangular finite element with nodes i, j, k, and l. Each node has three fictitious forces and three corresponding displacements. The three forces are a vertical force, F_w ; a moment about the X-axis, M_x ; and a moment about the Y-axis, M_y . The three displacements are the deflection in the Z-direction, w ; a rotation about the X-axis, θ_x ; and a rotation about the Y-axis, θ_y . These forces and displacements are related by

$$\{F\} = \left([K] + [H]\right)\{\delta\} \qquad (1)$$

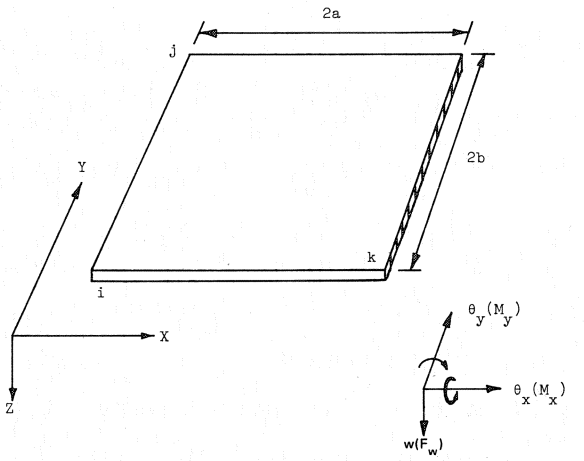
in which

$$\{F\} = \begin{Bmatrix} F_i \\ F_j \\ F_k \\ F_P \end{Bmatrix}, \quad \{\delta\} = \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_k \\ \delta_\ell \end{Bmatrix}, \quad F_i = \begin{Bmatrix} F_{wi} \\ M_{xi} \\ M_{yi} \end{Bmatrix}$$

$$\delta_i = \begin{Bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix}$$

and {F} = externally applied nodal forces; [K] = stiffness matrix of the slab, the coefficients of which depend on the finite element configuration and the flexural rigidity of the slab; [H] = stiffness matrix of the subgrade, the coefficients of which depend on the nodal spacings and the Young's modulus and Poisson's ratio of the subgrade; and {δ} = nodal displacements, each consisting of a vertical deflection and two rotations.

Figure 1. Rectangular plate element.



FORCES AND CORRESPONDING DISPLACEMENTS

The characteristics of equation 1 are such that the stiffness matrix of the slab [K] is banded, but the stiffness of the subgrade matrix [H] is not banded. When the two matrices are added, the composite matrix [P] , i.e., [P] = [K] + [H] , is not banded. An iterative scheme was developed by Huang (10) to make [P] a banded matrix to save computer storage space.

To illustrate the iterative scheme, consider a simple example for a slab divided into only two finite elements with a total of six nodes. Because each node has three unknown displacements, there are 18 simultaneous equations:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & 0 & 0 \\ & K_{22} & K_{23} & K_{24} & 0 & 0 \\ & & K_{33} & K_{34} & K_{35} & K_{36} \\ \text{Symmetric} & & & K_{44} & K_{45} & K_{46} \\ & & & & K_{55} & K_{56} \\ & & & & & K_{66} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix}$$

$$+\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ \text{Symmetric} & & & H_{44} & H_{45} & H_{46} \\ & & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}$$

in which K_{ij} = 3 by 3 submatrix; H_{ij} = 3 by 3 submatrix; i = nodal numbers from 1 to 6; and j = nodal numbers from 1 to 6.

$$\delta_i = \begin{Bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix} \quad ; \quad F_i = \begin{Bmatrix} F_{wi} \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

in which θ_{xi} = rotation about x-axis; θ_{yi} = rotation about y-axis; and F_{wi} = equivalent vertical force determined by statics.

As no angular continuity is assumed between the slab and the subgrade

$$H_{ij} = \begin{bmatrix} h_{ij} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

in which h_{ih} is the vertical force at node i due to the vertical displacement at node j .

The developed scheme is to transfer part of the [H] matrix to the right side of equation 2, or

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & 0 & 0 \\ & P_{22} & P_{23} & P_{24} & 0 & 0 \\ & & P_{33} & P_{34} & P_{35} & P_{36} \\ \text{Symmetric} & & & P_{44} & P_{45} & P_{46} \\ & & & & P_{55} & P_{56} \\ & & & & & P_{66} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix} \tag{5}$$

$$= \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & H_{15} & H_{16} \\ 0 & 0 & 0 & 0 & 0 & H_{26} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ H_{15} & 0 & 0 & 0 & 0 & 0 \\ H_{16} & H_{26} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix}$$

First, assume the displacement $\{\delta\}$ on the right side of equation 5 to be zero, and compute a set of nodal displacements $\{\delta\}$ by solving equation 5. Enter the displacements thus computed into the right side of equation 5, and find a new set of displacements. The process is repeated until the displacements converge to a specified tolerance.

In separating the [H] matrix, any half bandwidth may be used. The coefficients within the half bandwidth are placed on the left of equation 5, whereas those outside the half bandwidth are transferred to the right. Furthermore, when the number of equations is large while the half bandwidth is small, the displacements may not converge and a larger bandwidth should be used.

Program Logic

Two cycles of iterations are involved in the program. One checks the subgrade contact condition, and the other checks the convergence of deflections along the joint. At the outset of the computation, the program first assumes a full contact between the slab and the subgrade except at the nodes where gaps are preassigned. The gaps can be caused by pumping or plastic deformation of the subgrade. The deflection at each node along the joint is computed for each slab in a sequential order by a method of successive approximations until the convergence criterion is met.

In solving equation 5, each slab can be considered separately. First, consider the left slab only; then determine the first set of nodal displacements from equation 4. Next, consider the right slab. The existence of the left slab has two effects on the right slab: (a) the vertical deflections along the joint in the right slab must be set equal to those in the left slab; and (b) the vertical nodal forces in the right slab must include those caused by the deflections of the left slab. Because the deflections of the left slab have been previously determined, the nodal displacements of the right slab can be computed. Then return to the left slab. The existence of the right slab also has two effects on the left slab: (a) the displacements of the right slab will induce a set of nodal forces along the joint, which must be transferred to the left slab; and (b) the deflections of the right slab will induce vertical reactive forces in the left slab. By taking these effects into consideration, a new set of displacements for the left slab is determined. The process is repeated until the nodal displacements converge.

When the deflections have all converged, the deflection at each node is checked if the contact condition has changed. If not, the computed deflections are the final values; otherwise, a new subgrade contact condition is assigned, and the iteration for deflections starts again until the subgrade contact condition ceases to change. To assure proper convergence, an underrelaxation factor, which is used in the programs, is changed automatically.

Once the nodal displacements are computed, the nodal moments are then computed using the stress matrix tabulated in Table 7.2 (14, p. 97). The nodal stresses are then computed from the nodal moments. Because the stresses at a given node computed by means of one element might be different from that computed by the neighboring elements, the stresses in all adjoining elements were computed and their average values obtained.

The detailed procedure of the plate bending model can be found in many textbooks and papers, such as References 14 and 15, and will not be presented herein.

Stresses, Strains, and Deflections in the Supporting Subgrade Soil

Once the subgrade reactive forces between the subgrade and the slab at each node are determined, stresses and strains in the supporting subgrade soil can be computed. The stresses and strains are induced by the nodal reactive forces, but the forces are acting in the direction opposite to those when the stress condition in the slab was computed. Burmister's layered elastic solution is used to compute the stresses and strains. Since the stresses and strains in the subgrade soil under the concrete slabs are very small, the principle of superposition is valid and is used to compute the stresses and strains in the soil induced by all the nodal forces. It should be pointed out that at the subgrade surface, the deflection at the subgrade at a node is the same as that of the concrete slab at the same node; the vertical stress at the node is equal to the reactive force acting at the node divided by the affected area.

Subgrade Contact Options

Complete subgrade contact condition was assumed in the Westergaard solution. The slab always has a full contact with the subgrade soil, and gaps are not allowed between the slab and the subgrade no matter how much the slab has warped upward because of the temperature change or the applying load. In other words, the slab is supported by a group of springs and the springs are always connected to the slab. In reality, the pavement can lose subgrade support at some part because of temperature warping, pumping, and plastic deformation of the subgrade.

An iterative scheme is developed in the computer program to determine the subgrade contact condition. A node that is not in contact before loading may become in contact after loading, and vice versa. For pavements, if certain nodal points are known to be not in contact with the subgrade due to pumping, the magnitude of gaps at those nodes can be specified and the reactive forces at those nodes are then deleted in the computations. If certain nodal points near pavement edges and corners are first assumed to be in full contact but become not in contact later due to temperature warping, the stresses in the concrete slab are computed based on the subgrade support conditions determined at the last cycle of the iterative process.

Symmetry

The application of the finite element method for analyzing rigid pavements involves solving a large set of simultaneous equations. However, the use of symmetry can greatly reduce the number of simultaneous equations, for only one half or one quarter of the slabs have to be considered. Consequently, the computer time and storage can be greatly reduced, and the results yielded are practically the same as those when symmetry is not used.

Stress Transfer Along the Joints and Cracks

Joints are placed in rigid pavements to control cracking and to provide enough space and freedom for movement. Load is transferred across a joint or crack principally by shear forces and, in some cases, by moment transfer. Shear force is provided by either dowel bars, key joints, or aggregate interlock. Moment transfer, on the other hand, is provided by the strength of the concrete slab. When a joint or a crack has a visible opening, however, the transfer of moment across the joint or crack becomes negligible. It is, therefore, justified to assume that there is no moment transfer across a joint or a crack, except in such cases as a tied joint where some moment transfer may be expected if the joint remains tightly closed.

If moment transfer across a joint is neglected, the amount of stress transfer at a joint is governed by the difference in deflection between the two slabs along the joint. In other words, shear transfer is 100 percent if deflections at both slabs are equal. This difference in deflection depends on the shear deformation of the dowel bar and the dowel-concrete interaction. The analyses presented later indicate that the effect of the dowel-concrete interaction is more predominant than the shear deformation of the dowel bar. The negligence of the deformation of concrete surrounding the dowel bar can make the bar more effective than in reality.

Shear and Moment Transfer Across Joints and Cracks

The program provides two options for specifying shear transfer and assumes moment transfer to be zero. The two options are: (a) efficiency of shear transfer, and (b) diameter and spacing of dowels.

Efficiency of Shear Transfer. The efficiency of shear transfer is defined as the ratio of vertical deflections along the joint between the unloaded, or less heavily loaded, slab and the more heavily loaded slab. By assigning an efficiency between 0 (no shear transfer) and 1.0 (complete shear transfer), reasonable results can be obtained with a minimum number of iterations. The efficiency can be easily checked on the printout by comparing each pair of deflections along the joint. However, the use of a given efficiency for all nodes along a joint is not realistic, because the deflection ratios in an actual pavement should vary along the joint, with the smallest ratio at a point where the deflection is the largest.

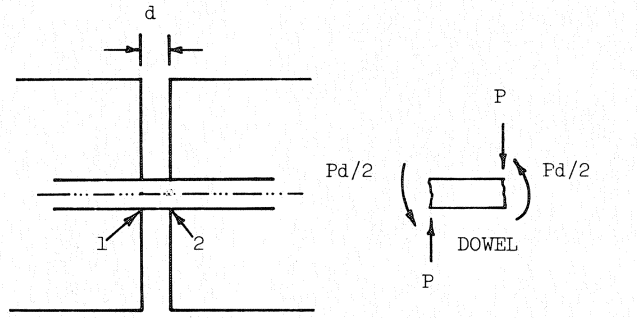
Diameter and Spacing of Dowels. This option is most straightforward and does not require any field test to determine such parameters as the efficiency of shear transfer. This method also takes into consideration the diameter and spacing of dowels or steel bars. However, the steel bars are assumed to be the sole means of shear transfer that may not be applicable to a tied joint which remains closed.

Computations for Dowel Bars

For stress transfer using the method of diameter and spacing of dowel bars, the derivation of the equations is presented in the following paragraphs.

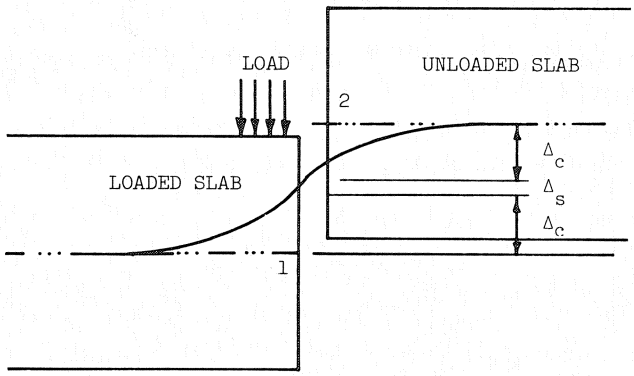
Figure 2 shows an exaggerated view of a doweled joint with a width d. In the finite element method, the slabs are divided into a number of elements. Suppose that 1 and 2 are a pair of the many nodal points. If a vertical shear force P

Figure 2. Difference in deflection between two slabs.



(a) EXAGGERATED VIEW OF DOWEL JOINT

(b) VERTICAL SHEAR FORCE TRANSMITTED TO DOWEL



(c) ILLUSTRATION OF DIFFERENCES IN DEFLECTION

exists at node 1, it will be transmitted to the dowel, as shown in Figure 2b. After deformation, the deflections at nodes 1 and 2 will not be the same because of the shear deformation of the dowel Δ_s, and the deformation of concrete under dowel Δ_c, which can be determined by considering the dowel as a beam and the concrete as an elastic foundation (16). Consequently, the difference in deflection Δ between nodes 1 and 2, as shown in Figure 2c, is

$$\Delta = \Delta_s + 2\Delta_c \tag{6}$$

The shear deformation of the dowel can be determined approximately by

$$\Delta_s = \frac{Pd}{GA} \tag{7}$$

in which G and A are the shear modulus and the area of the dowel, respectively. By considering the dowel as a beam on an elastic foundation, Δ_c can be determined by (16,17)

$$\Delta_c = \frac{P}{4\beta^3 EI}(2 + \beta d) \tag{8a}$$

$$\beta = \sqrt[4]{\frac{Kb}{4EI}} \tag{8b}$$

in which E = modulus of elasticity of the dowel; I = moment of inertia of the dowel; K = modulus of dowel support; and b = diameter of the dowel. By using realistic values for the parameters in equations 7 and 8, it can be shown that Δ_s is one or more orders of magnitude smaller than Δ_c and can usually be neglected; thus, the difference in deflection depends principally on the dowel-concrete interaction. Because the amount of stress transfer across the joint is governed by the difference in deflection, the negligence of the dowel-concrete interaction will result in a stress transfer that is too large.

Modulus of Dowel Support

The stress analysis of dowels is based upon work by Timoshenko and Lessels (16). They modeled a dowel bar encased in concrete as a beam on a Winkler foundation. The ratio between the bearing pressure and the deflection of the dowel bar was termed as the modulus of dowel support K. Studies have shown that K is not a constant quantity but varies with the concrete properties, the dowel bar diameter and length, the slab thickness, and the degree of dowel looseness in the concrete. Yoder and Witczak (17) suggested that values of K range between 81,000 and 405,000 MPa/m (300,000 and 1,500,000 lbf/in.^{3}) and that the use of 405,000 MPa/m (1,500,000 lbf/in.^{3}) appears to be warranted.

Comparison with the Strain Measurements from the Corps of Engineers

During the 1950's, a series of full-scale static loading tests on rigid pavements was conducted at eight U. S. Air Force bases and at the Ohio River Division Laboratories. The purpose of such tests was to evaluate different methods of dowel installation in terms of the load transfer achieved at the joint and to check if any of the methods failed to provide a joint meeting the minimum requirement of 25 percent load transfer as assumed in the design and evaluation of rigid pavements.

Doweled joints investigated for the study were constructed by five different methods: (a) cast in place, (b) remove and replace, (c) split dowels (screw-type only), (d) dummy half dowel, and (e) oversize dummy half dowel (cement grout only). Aircraft used in the loading tests included the B-47, B-52, and KC-97. Joint performance was evaluated on the basis of strain measurements in the direction parallel to the joint. Test procedures included the "no load," "load," and "rebound" measurements. The description of test sites, instrumentation, test procedures, and test results of the study are documented in Reference 18.

The WESLAYER program was used to analyze the test sites. The computations were carried out with assumed values for some variables. The modulus of the concrete E, the joint spacing d, and the modulus of dowel support K were assumed to be 41,364 MPa (6,000,000 lbf/in.^{2}), 0.79 mm (1/32 in.), and 405,000 MPa/m (1,500,000 lbf/in.^{3}), respectively. To convert the modulus of subgrade support k values (measured at test sections) to modulus of elasticity E values (used at the WESLAYER computer program), efforts were made to establish correlations between k and E values for a given value of Poisson's ratio ν for the test sections analyzed. These correlations were made by computing the stresses and deflections for a given pavement with a range of subgrade k values (using WESLIQID program for slabs on liquid foundation) and for the same pavement with a range of subgrade E values (using WESLAYER program assuming the subgrade is homogenous and $\nu = 0.4$). The corresponding E and k values were determined by matching the

maximum stress or deflection at a certain nodal point in the two pavements that have the same loading and geometrical conditions but have different subgrade conditions. The reason for selecting the maximum values is that the shapes of the stress and deflection basins under the liquid foundation (represented by the WESLIQID program) and the elastic foundation (represented by the WESLAYER program) are different.

Table 1 shows the computed (WESLAYER program) and measured percentages of stress transfer for five test pavements. The stress transfer is computed to be the ratio of the strain on the unloaded side of the joint to that of the total strains (the sum of the strains on both unloaded and loaded sides) in percentage. Pavement information for each test site is also included in the table. All the pavements were tested under the B-47 aircraft loads. For completeness, the percentages of stress transfer computed for liquid subgrade using the WESLIQID programs are also listed. Note that the computed values are very close to the measured ones and are also close to those computed by the WESLIQID program.

As mentioned earlier, the shapes of the stress and deflection basins under the liquid foundation (represented by the WESLIQID program) and the elastic foundation (represented by the WESLAYER program) are different. The difference is due to the nature of the assumption. In a liquid subgrade, the deflection at a node depends solely on the modulus of subgrade reaction k at the node and not elsewhere. In an elastic subgrade, however, the deflection at a node depends not only on the elastic modulus E of the subgrade but also on the deflections at other nodes. Consequently, the deflection basin should be steeper in the liquid subgrade than in the elastic subgrade.

The results of the analysis show that for the conditions of various percentages of shear transfers across the joint, the basins are much flatter when the subgrade is elastic than when it is liquid and when the joint has better load transfer capability. Contrary to the deflection basins, the stress basins for the elastic subgrade are slightly steeper than those for the liquid subgrade.

Effect of Subgrade Elastic Modulus E on Stress Transfer Across a Joint

In References 12 and 13, the effects of a number of important parameters on stress transfer across a joint were studied (by the use of WESLIQID program for pavements on liquid subgrade). The parameters studied were joint spacing d, modulus of concrete E, modulus of dowel support K, dowel bar diameter b, concrete pavement thickness t, and modulus of subgrade reaction k. The percentage of stress transfer across a joint was found to be mostly affected by K, t, and b and much less by the other parameters. It is believed that the conclusions derived are also applicable to pavements on elastic subgrade soils (WESLAYER). Efforts were thus made to investigate only the effect of subgrade elastic modulus E on stress transfer across a joint. Figure 3 presents the results computed for the Lockbourne Air Force Base test pavement.

The results presented in Figure 3 for elastic subgrade soils (E) are very similar to those for liquid subgrade (k). The percentage of stress transfer decreases as the subgrade modulus increase. This result is reasonable because the load is transferred to the other slab through dowel bars when the loaded slab is deflected. In the extreme case when the subgrade modulus is increased until infinitely large, the loaded slab ceases to deflect and the load is not transferred to the other slab through the joint; the percentage of stress transfer should thus approach zero.

Figure 3 shows that for E values ranging from 27,576 to 1,378,800 kPa (4,000 to 200,000 lbf/in.^{2}), the percentage of stress transfer varies from 12.9 to 29.4. However, the subgrade elastic modulus of the five test pavements analyzed (Table 1) ranges only from 51,016 to 103,410 kPa (7,400 to 15,000 lbf/in.^{2}), and the corresponding variation of stress transfer is only 3 percent (Figure 3), which is not significant at all.

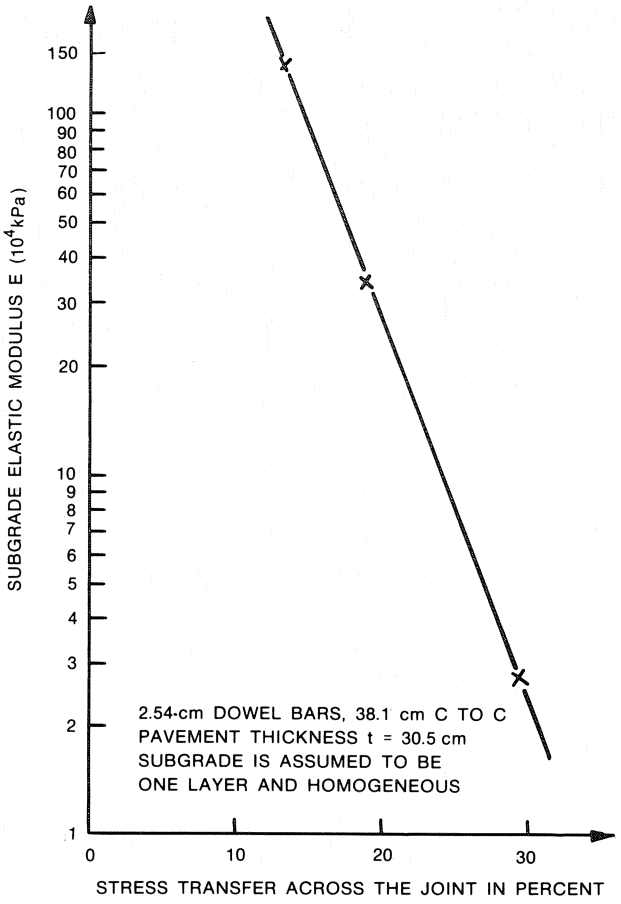
Table 1. Comparison of stress transfer between theoretical solutions and experimental measurements.

Location	Pavement Thickness (cm)	Modulus of Subgrade Reaction k (kPa/cm)	Equivalent Subgrade Elastic Modulus E (kPa)	Dowel Diameter (cm)	Dowel Spacing (cm)	Stress Transfer (%)	Subgrade Type	Method
Lockbourne AFB, OH	30.48[a]	202.5	53,733	2.54	38.1	21.1	--	Measured
						27.0	Liquid	WESLIQID
						26.5	Elastic	WESLAYER
Lincoln AFB, NE	53.34	175.5	51,016	3.81	25.4	36.8	--	Measured
						27.8	Liquid	WESLIQID
						31.4	Elastic	WESLAYER
Hunter AFB, GA	45.72	427.5	86,173	3.81	45.72	27.1	--	Measured
						23.0	Liquid	WESLIQID
						26.9	Elastic	WESLAYER
McCoy AFB, FL	45.72	607.5	103,410	3.81	45.72	23.1	--	Measured
						22.0	Liquid	WESLIQID
						26.0	Elastic	WESLAYER
March AFB, CA	40.64	270.0	60,667	3.81	43.18	32.5	--	Measured
						28.0	Liquid	WESLIQID
						28.8	Elastic	WESLAYER

Note: 1 cm = 0.3937 in.; 1 kPa/cm = 0.369 lbf/in.^{3}; 1 kPa = 0.145 lbf/in.^{2}

[a]Reinforced.

Figure 3. Results computed for Lockbourne Air Force Base test pavement.



Note: 1 kPa = 0.145 lbf/in.^{2}; 1 cm = 0.3937 in.

Conclusions

A finite element method programmed for a high-speed computer (WESLAYER) was developed for analyzing concrete pavements with a load transfer at the joint on layered elastic solid and for analyzing stress and strain distributions in the supporting subgrade soil. The comparisons between the computed and the measured stress transfers across the joint were favorable. The percentages of stress transfer across the joint computed by the WESLAYER program (slabs on layered elastic subgrade) were very close to those computed by the WESLIQID program (slabs on liquid subgrade). Computed results show that the dowel bar is not a very efficient load transfer device in a rigid pavement. The range of 25 to 30 percent stress transfer that was computed is in line with the experimental data from the Corps of Engineers. Computed results showed that the deflection basins are much flatter when the subgrade is elastic than when it is liquid and when the joint has better load transfer capability.

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