MECHANISTIC DESIGN OF RIGID PAVEMENTS

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This paper deals with the development and verification of a stress analysis model (RIGMUL) and the use of this model in a program for the design of rigid pavements. RIGMUL is the result of coupling finite element plate theory with multilayer elastic layer theory and consists of a two-layer rigid slab resting on a semi-infinite three-layer elastic solid foundation. The features of RIGMUL include variable thickness of slabs, the effects of curling, warping, voids and partial contact, the effect of load transfer across joints and cracks by specifying either dowel bars, aggregate interlock, or load or shear transfer efficiency, the ability to analyze up to nine slabs in any configuration, and the ability to determine the stresses in both rigid layers as well as anywhere in the subbase and subgrade layers. Model verification includes comparison with known solutions whenever possible; however, since this model is an extension beyond the scope of existing theories, qualitative comparisons are also included. The pavement design program uses RIGMUL for stress analysis; however, the input requirements have been drastically simplified for the convenience of the user. The output includes the expected design life in fatique, along with a prediction of the amount of joint faulting, and the probability of void formation, corner cracking, and pumping.

Current Design Practices

Portland Cement concrete pavements have been constructed in this country since the early 1900's. Although more expensive to construct initially than bituminous pavements, it has generally been believed that rigid pavements require less maintenance over longer time periods than do flexible pavements. As a result of this belief, some states have traditionally built concrete pavements along high-volume arteries while using flexible design for secondary and low-volume routes. A major portion of the Interstate highway mileage in midwestern and northeastern states has been constructed with Portland Cement concrete pavements. Recent trends toward use of thicker layers and the development of rational, mechanistic design methods have enhanced the performance of flexible pavements. As a result, choosing between utilizing a flexible or rigid design has become an even more difficult decision for the highway engineer.

Initially, rigid pavement slabs were constructed directly upon the subgrade with little regard given to subgrade type or drainage conditions. Slab thickness was arbitrarily set at generally 152 to 178 mm (6 to 7 in.). In the 1920's and 1930's, analysis of the stresses in concrete pavements were made including development of the well-known Westergaard equations. It was also becoming apparent that subgrade type was related to pavement performance. States started to build pavements with thickened edges in the 1930's and 40's and by the 1950's uniform, thicker pavements (203 to 254mm (8 to 10 in.)placed on granular bases was the common practice.

Today, numerous procedures are available for the thickness design and analysis of concrete pavements. The two most commonly used are the AASHTO Interim Guide and the PCA design methods. The AASHTO Interim Guide method is based on semi-empirical or semi-rational relationships derived from the AASHTO Road Test concerning pavement serviceability. The PCA method is based on flexural stress and flexural strength relationships of the concrete with the intent of preventing fatigue cracking of the slab.

A completely mechanistic rigid pavement design method which considers multi-layer structures has not been developed. Such a design method would offer several advantages over current design methods, including:

1. The ability to predict various types of concrete pavement distress;
2. The ability to evaluate the effects of various thicknesses and types of foundation layers upon pavement distress functions and thus establish an optimized design;
3. The ability to consider various rigid pavement types (jointed, unjointed, continuous reinforced, etc.);
4. The ability to evaluate the effects of roadside characteristics such as shoulder type, drainage provision and curb construction upon the pavement;
5. The ability to make a valid cost comparison between rigid and flexible pavement structures, which consider life cycles costs based upon predicted distress functions of the respective mechanistic design methods.

A fundamental requirement of a mechanistic rigid pavement design is the ability to determine the stresses and displacements in the concrete slab as well as in the supporting layers below the slab. This paper describes the development and verification of a stress analysis model capable of considering all features of concrete pavements and the use of this model in a rigid pavement design program.

Stress Analysis Models

Existing Models

Several theories have been developed over the years for the determination of stresses and deflections in concrete slabs; however, the classical bending theory of a medium-thick plate (no deformation within the slab thickness, and no membrane action) has been the most popular one because of its simplicity and validity.

The first use of the slab theory for concrete pavement analysis was made by Westergaard (1,2,3,4)in 1926. In this analysis, subgrade was assumed to act as a set of uniformly distributed springs supporting the concrete slab. The thickness of these springs was referred to as a modulus of subgrade reaction (k), and was determined from the results of plate-load tests performed on the subgrade. Assuming the concrete pavement slab to be infinitely large and having a constant thickness, Westergaard developed closed form solutions for critical stresses and deflections under a single load for three cases of loadings, namely, interior, edge and corner.

Hogg and Hall (5,6), in their analysis, considered subgrade as a semi-infinate elastic solid. Using elastic properties of subgrade (modulus of elasticity and Poisson's ratio), they developed a mathematical model for determining the maximum stress and deflection in an infinitely large concrete slab, under a single load at the interior away from any edge and corner.

Pickett and Ray(7) extended the works of Westergaard and Hogg to include the effect of any loading configuration in forms of influence charts. The computerized version of the Pickett and Ray's influence charts were prepared by the Portland Cement Association (PCA) for concrete pavement slab analysis.

All of the above slab models are based upon the assumption that the slabs extend a considerable distance from the loaded area. However, concrete slabs are relatively narrow, and have many discontinuities in the forms of joints and cracks, with various degrees of load transfer across these discontinuities. The effect of one crack on the deflections and moments in the concrete pavement slab was studied by Niu and Pickett (8) in 1969. In this analysis, the problem of a cracked Westergaard pavement subjected to a single load was formulated and numerical integration was used to solve the problem.

The use of continuously supported slab models for analysis of concrete pavement slabs have a number of shortcomings. The following is a list of the major limitations of these models:

1. Slab models are limited to pavements consisting of two layers (a rigid slab resting on a subgrade); therefore, multiple layered pavement systems cannot be analyzed;
2. Concrete pavement slabs of finite dimensions or slabs with more than one joint or crack cannot be considered;
3. It is assumed that concrete slab is continuously supported by subgrade; therefore, the effects of voids or partial subgrade support cannot be analyzed;
4. Slabs of non-uniform thickness, non-uniform material properties, and non-uniform subgrade support cannot be considered.

Generalized multiple-layered systems are also used for concrete pavement analysis. In this modeling technique, each pavement layer is represented by its modulus of elasticity, thickness and Poisson's ratio. All layers are assumed to extend horizontally to infinity with the bottom layer also extending vertically to infinity.

The first solution for a generalized elastic multiple-layered system was presented by Burmister (9, 10, 11). The work of Burmister was later extended by other investigators to include more generalized loading conditions and for viscoelastic multiple-layered systems.

Despite the capabilites of multiple-layered system models for evaluation of pavements with multiple layers and pavements with elastic and viscoelastic material properties, they have several deficiencies. The following is a list of the major limitations of these models:

1. Layered system models are not capable of analyzing concrete pavement slabs of finite dimensions; therefore, edge corner and joint loading conditions cannot be evaluated;
2. Various concrete pavement features such as joints, cracks, reinforcement, shoulders, drainage systems, voids under the slab, and loss of foundation support cannot be analyzed.

Use of discrete-element methods for structural analysis of plates was pioneered by Newmark (12). In developing a discrete-element model to simulate the slab, Newmark considered the slab as an assemblage of deformable hinges, rigid links and coil springs. An application of this model for analysis of continuous plates was made by Ang and Newmark (13).

The first use of discrete-element model for concrete pavement slab analysis was made by Hudson and Matlock (14). In this analysis, subgrade was idealized as a Winkler foundation (similar to Westergaard's analysis). The effects of joints and shrinkage cracks in this model were taken into consideration by reduction of original bending stiffness of the slab that those locations where a joint or crack exists. The effect of transverse cracks on the longitudinal bending rigidity of continuously reinforced concrete pavements was studied by Abou-Ayyash et al. (15). It was found that a significant drop of about 80 to 90 percent in the bending rigidity of the concrete slab is encountered at crack locations.

The model developed by Hudson and Matlock was later modified an improved by other investigators to include elements of different sizes (16), anisotropic skew slabs (17), and idealization of the subgrade as a semi-infinite elastic solid (18, 19, 20,).

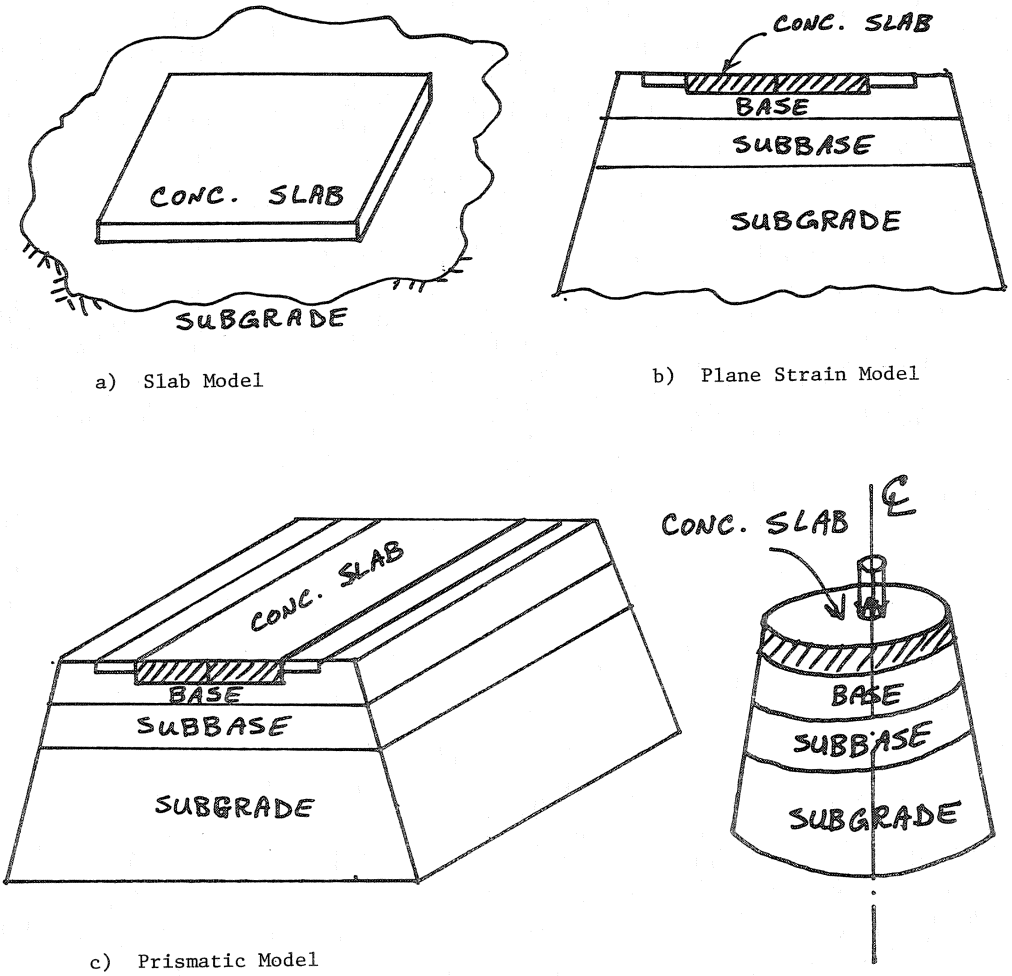
The major disadvantages of discrete-element formulations are that elements of varying sizes are not easily incorporated into the analysis and that special treatment is needed at the free edge where stresses cannot be determined uniquely.

Finite-element methods for analysis of concrete pavement slabs have been used by a number of investigators. A number of finite-element models have been developed for analysis of the concrete pavement system. These may be divided into one of the following major classes:

1. Slab Models;
2. Plane Strain Models;
3. Prismatic Models;
4. Axisymmetric Models;
5. Three-Dimensional Models.

The slab models idelized concrete pavement slabs as medium-thick plates supported by an idealized sub-

Figure 1. Idealizations of PCC pavements.



a) Slab Model

b) Plane Strain Model

c) Prismatic Model

d) Axisymmtric Model

grade (Figure 1a). Slab models have been developed by Ebnhard (21,22), Huang and Wang (23,24), Bhatia and Majidzadeh (25), and Tabatabaie and Barenberg (26). Although these models all represent the subgrade as a Winkler foundation and are therefore limited in their application, the representations developed by Tabatabaie (26) and Huang and Wang (24) contain some of the features that are considered essential for a rational analysis of concrete pavements.

In the Tabatabaie model, the concrete pavement system with joints is represented by up to six slabs. Six slabs are used because this is the general case with the loads applied on the middle slab when it is connected to neighboring slabs by dowel bars or keyways, and up to nine slabs may be necessary when considering non-symmetrical loading of the middle slab if tied concrete shoulders are used. The slab is treated in this model as composed of two layers which may be bonded or unbonded, but are assumed to be in contact with each other. Two layers are used so that cement stabilized bases or rigid overlays may be analyzed. Rectangular plate elements with three degrees of freedom per node (one vertical deflection and two rotations) are used to represent the slab and stabilized base, or slab and overlay. Dowel bars are represented by beam elements while spring elements are used to model aggregate interlocks or keyways. Using this information, slabs with varying thickness (such as pavements with thickened edges) can be analyzed as well as pavements with tied shoulders having a thickness different from that of the slab. Also, since beams and spring elements are used, the load transfer efficiency of various joint designs can be determined, along with the effect of joint design (load transfer efficiency) on critical slab stresses. This model does not, however, consider the effects of curling, warping, voids and partial contact.

The model developed by Huang and Wang (24) represents the concrete slab as composed of a single rigid layer. The concrete pavement is represented by two or four slabs when load transfer effects are considered, and one slab when no load transfer at joints is assumed. Load transfer devices are not modeled, a load transfer efficiency is assumed instead. This model is also restricted to uniform thickness slabs, but it does consider the effects of curling, warping, voids and partial contact. Despite the advantages of these models in that they are capable of evaluating all types of loading conditions, slabs of finite dimensions, and various pavement features such as joints, cracks, etc., they cannot handle multi-layered pavement systems.

In plane-strain idealization of the concrete pavement system, changes in load or material along the centerline of the pavement is neglected (Figure 1b). The pavement system is represented as a transverse slice of the pavement having a unit thickness. Obviously, these models, because of their simplifying assumptions, are not capable of evaluating various concrete pavement features such as joints, cracks, etc., slab action of the concrete pavement slab, and multiple loads.

The prismatic idealization of the concrete pavement system was shown in Figure 1c. In this modeling technique, the pavement system is represented by a constant two-demensional geometric shape with respect to an infinite third dimension. The greatest limitation of these models for analysis of the concrete pavement is that no variation of geometrical configuration is allowed along the longitudinal axis of the pavement system. Therefore, prismatic models cannot handle any transverse discontinuities such as joints or cracks, nor do they use a realistic representation for applied loads.

The axisymmetric model idealization considers a nearly general three-dimensional system (Figure 1d). In this modeling technique, a pavement system is idealized as a multiple-layered cylindrical system loaded symmetrically at the centerline. Despite the advantages of these analysis of layered systems, they are not capable of evaluating various concrete pavement features such as joints, cracks, etc., or various loading conditions such as edge, corner or joint.

Probably the most desired model for concrete pavement analysis is the use of three-dimensional models, where actual geometrical configurations of a whole system can be taken into consideration. There are also computer programs that employ three-dimensional finite-elements such as the SAP Program developed by Wilson (27). However, the amount of discretization and the computer costs required for solution of the problem would be high and impractical.

For the purpose of this paper, coupled models are referred to as the structural models obtained by coupling two or more of the following models:

1. Finite-element or discrete-element;
2. Multiple-layered system;
3. Analytical (closed form).

Coupling models have been formulated by various investigators (18, 28, 29, 30) for structural analysis of pavement systems. In 1971, Saxena (18, 19, 20) coupled the discrete-element slab model developed by Hudson and Matlock (14) with the closed-form solution of Boussinesq. In this analysis, discrete-element modeling techniques were used for idealization of the concrete pavement slab, and Boussinesq's equation was employed to represent the behavior of the subgrade as a semi-infinite elastic solid.

Although Saxena (18)model with a Boussinesq solid subgrade is an improvement of the model developed by Hudson and Matlock (14), it has the following disadvantages in that the effects of voids, partial contact, and curling and warping are not considered. Furthermore, the model is only applicable to single slabs with free edges, i.e., load transfer effects are not considered.

Huang (29)has coupled a finite element model with a Boussinesq solid subgrade. This model includes all the features of the Huang and Wang (24) model discussed previously except that load transfer and joints is assumed to be either 0% for plain jointed concrete, or 100% for doweled or keyed joints.

Coupling of a finite-element program with a multi-layered elastic system was done by Prause and Kennedy (30), for analysis of the track structures. In this modeling technique, a finite-element program representing rail and tie was coupled with a multi-layered elastic layer program representing ballast and subgrade. Although the coupling techniques developed in this program may be useful, the program itself is not directly applicable for concrete pavement analysis.

Proposed Structural Analysis Model

A review of pavement distresses and features indicates that the analysis model should have the following capabilities:

1. Finite element formulation;
2. Ability to consider the effects of load transfer at joints due to different load transfer devices;
3. Ability to consider curling and warping effects;
4. Ability to consider the effects of voids and partial contact;
5. Ability to treat the slab as two rigid layers so that cement-treated bases can be considered;
6. Ability to represent the subgrade as a 3-layer elastic or viscoelastic continuum;
7. Ability to handle different traffic loading geometries;
8. Ability to compute stresses and strains in the slab as well as in the layered continuum.

Based on the discussion in the previous section, it can be seen that although a number of structural models are available for the analysis of concrete pavement systems, none of these contain all the features considered essential for a rational representation. Three-dimensional finite-element models which can realistically represent the whole pavement system are expensive to use and the amount of discretization required for solution of the problem will be high and impractical. The slab models which are capable of evaluating various PCC pavement features such as joints and cracks, etc., cannot be employed for analysis of the multiple layers. On the other hand, layered system models which are capable of handling multiple layers cannot evaluate the effects of the concrete pavement features.

Therefore, the solution to this problem is the development of a coupled model where a finite-element plate is coupled with a three-layered elastic or viscoelastic system (Figure 2). This coupled model with its unique characteristics is capable of analyzing multiple layered systems as well as various pavement features such as joints, cracks, load transfer, void, curling, etc.

The coupled analysis model is an extension and combination of two existing finite element programs for slab analysis:

1. ILLI-SLAB developed by Tabatabaie (26); and,
2. Huang program (29).

ILLI-SLAB currently has features 1, 2, 5 and 7, and Huang's program has features 1, 3, 4 and 7. The inclusion of a multi-layer representation of the subgrade thus meets all the stated requirements.

Figure 2 shows a two-layer slab resting on a 3-layer semi-infinite continuum. At the interface between the slab and the layered subgrade, static equilibrium conditions require that the forces exerted by the slab on the subgrade be equal to the forces exerted by the subgrade on the slab, i.e., the reaction forces in the slab model.

The first step in formulation of the combined model requires evaluation of the reaction forces at the node points between the slab model and three-layered system. This can be achieved by means of the following matrix equations:

1. For three-layered system:

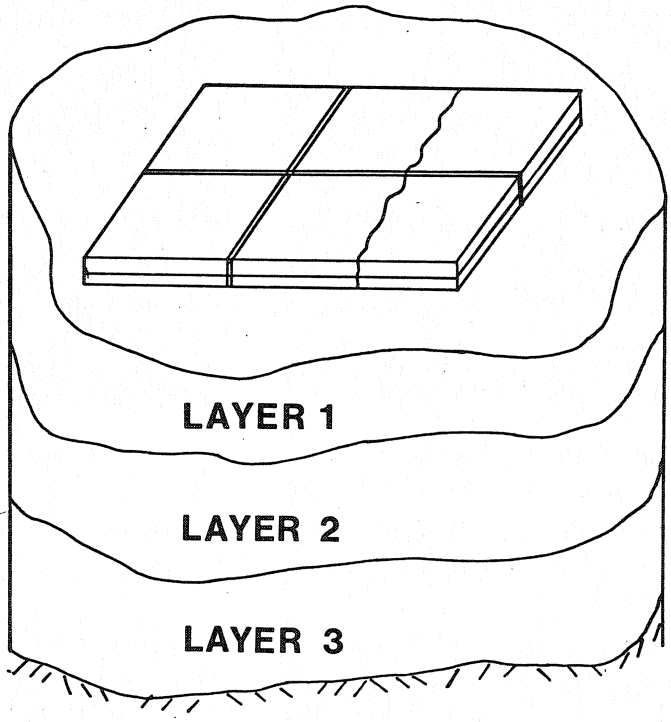
$$\{ U \} = [F_{3LS}] \{ R \}$$

where

$[F_{3LS}]$ = Flexibility matrix of the three layered system;

$\{ R \}$ = reaction force vector between slab

Figure 2. Two-layer rigid plate coupled with three-layer elastic solid subgrade.



model and three-layered system; and,

$\{ U \}$ = displacement vector of the three-layered system.

which states that the displacements {U} are proportional to the applied forces {R} , related by the proportionality constants $[F_{3LS}]$. Although no analytical expression exists for $[F_{3LS}]$, these constants can be determined using any of the layer programs with unit applied loads; it is only necessary to compute the deflection at each node point resulting from a unit load at a nodal point. The present layer programs are capable of dealing with a maximum of 10 load configurations; however, the same superposition principle used in programs such as BISTRO (31) or ELSYM5 (32) can be used to determine the resulting deflections due to any number of applied loads. The above programs are, however, relatively slow since they are intended for general conditions. Therefore, an unpublished program developed by Ilves is used which is specifically written for three layers and takes advantage of the behavior of functions in layer analysis in order to reduce the required computer time significantly.

Equation 1 above may be solved for the reaction forces:

$$\{ R \} = [F_{3LS}]^{-1} \{ U \} \quad (2)$$

so that when the displacements are known, the resulting forces can be determined.

2. For slab model:

$$\{ P \} - \{ R \} = [K_{slab}] \{ U \} \quad (3)$$

where

$\{ P \}$ = traffic load vector;

$\{ R \}$ = reaction force vector between slab and subgrade;

$[K_{slab}]$ = stiffness matrix of the slab model, which is completely specified by the particular finite element formulation used in representing the slab.

Equation 3 states the general force-displacement relationship of a slab and is independent of the type of subgrade model used.

The second step in formulation of the problem is to couple the slab model with the three-layered system. This can be achieved by replacing the reaction force vector {R} in equation 3 by its equivalent $[F_{3LS}]^{-1}$ {U} from equation 2:

$$\{ P \} - [F_{3LS}]^{-1} \{U\} = [K_{slab}] \{ U \} \quad (4)$$

or

$$\{ P \} = ([K_{slab}] + [F_{3LS}]^{-1}) \{ U \} \quad (5)$$

Equation 5 is the generalized force-displacement relation for the whole pavement system. After this equation is solved for the unknown displacement vector {U}, stresses, strains and deflections in the three-layered system as well as in the slab model can be determined. If Tabatabaie (26) slab formulation is used in the above equation, the problem of a 2-layered slab on a layered solid subgrade with variable load transfer across joints can be solved, but the effects of voids, partial contact, and curling and warping are not included. On the other hand, Huang's slab model (29) which includes the latter effects cannot consider a 2-layered slab nor does it consider variable load transfer. However, the methodology developed by Huang (29) can be applied directly to the coupled Tabatabaie slab. Using, this information, equation 5 can be written as:

$$\{ P \} = [K_{slab}] \{ U \} + [F_{3LS}]^{-1} \{ U \} \quad (6)$$

where

$\{ U' \} = \{ U \} - \{U''\}$, and

$\{ U'' \}$ represents the deflection due to curling and warping, or voids and partial contact, or both.

In the finite element slab theory, each node has three degrees of freedom, i.e., the displacement:

$$U_i = \begin{matrix} W_i \\ \theta_{xi} \\ \theta_{yi} \end{matrix}$$

where W_1 is the vertical displacement at node i, and θ_{xi}, θ_{yi} represent rotations about the x and y axes, respectively. If curling or voids are present:

$$U''_i = \begin{matrix} C_i \\ 0 \\ 0 \end{matrix}$$

i.e., C_i represents the space between the slab and subgrade. When voids or partial contact exist, it is assumed that the magnitude of C_i can be specified a priori, and for temperature-induced curling or warping:

$$C_i = \frac{\alpha t\, d_i^2}{2h} \qquad (9)$$

in which C_i = amount of curling at node i in an unrestrained slab, α = coefficient of thermal expansion of concrete, t = temperature differential between the top and bottom of slab, positive when the slab warps up with a temperature at the top smaller than that at the bottom, d_i = distance from node i to the center of slab where curling is zero, and h = thickness of slab. When both curling and voids are present, then C_i represents the combined value of curling (from equation 9) and the void spaces.

The stresses in the elastic layer below the rigid layers can be determined from the following scheme:

1. At the time when the subgrade flexibility (F_{3LS}) is obtained from the unit loads, the virtual stresses $\{S_i\}$ can also be calculated for the virtual loads, at any point in the subgrade. These are column vectors, one column for each point in the multi-layered subgrade and for each type of stress (i.e., horizontal or vertical)
2. If [K] is the total stiffness of the plate and subgrade combined and d is the displacement of the top surface of the subgrade, then the passive reaction {R} is as shown in Figure 3.

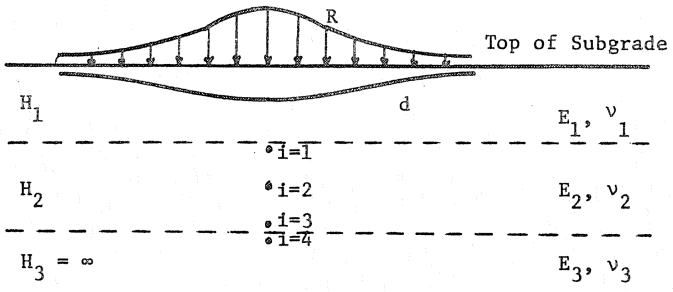
$$\{R\} = [K]\,\{d\} \qquad (10)$$

Since [K] has been assembled for the system and the displacement solution {d} for the pavement has been obtained, the reactions {R} on the subgrade are obtained from equation 10.

3. The stresses in the subgrade are obtained by multiplying the passive reactions and the virtual stresses, for all points i:

$$\sigma_{i=1,\,2..n} = \{R\}^T \{S_i\} \qquad (11)$$

Figure 3. Displacement of the top surface of multilayer subgrade as a result of reaction forces.



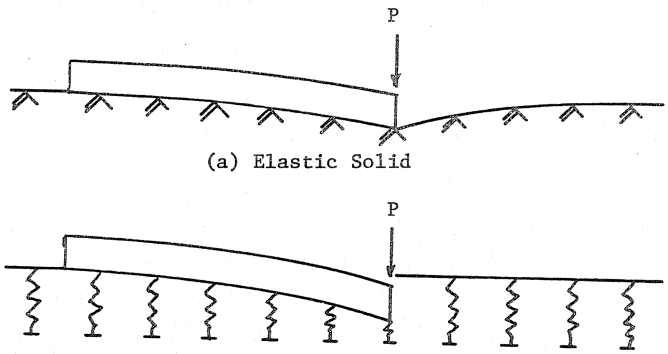
With the above formulation, it can be seen that equation 6 is the generalized force-displacement relation for the pavement system and includes all the effects enumerated above. The general solution of this equation is an iterative process, however, since traffic loading will change the contact area between slab and subgrade. In the iterative procedure, the slab is assumed to be in full contact with the subgrade and deflections computed due to applied load. Next, the deflections at each nodal point are compared with C_i; if W_i is smaller than C_i, the slab is not in contact with subgrade at these points. In this case, a new subgrade stiffness matrix $[F_{3LS}]^{-1}$ is computed, assuming the load is zero at the points of no contact, and new deflections computed from equation 6. The interative procedure is continued until the contact area remains unchanged.

Another difficulty in solving the general problem is that $[F_{3LS}]$ is not banded, nor is it generally a symmetric matrix; therefore, considerable more computer storage is required. However, Huang (29) has suggested a scheme that allows a non-banded matrix to be approximated by a banded matrix and solved by iterative means. Since the solution of equation 6 is iterative anyway, it is possible to incorporate this concept to reduce the amount of storage without undue increase in computer time.

Model Verification

A coupled FEM-elastic multilayer subgrade program (RIGMUL) has been developed using the concepts described above. In cooperation with Resource International, Dr. Huang has improved and modified his plate program (24)to incorporate the features of the Tabatabaie Model (28), and this program has been coupled to multilayer subgrade to form RIGMUL. The program has been checked for a number of cases with existing methods and solutions, wherever available. Table 1 shows a comparison of displacements as determined from RIGMUL and other theories. It can be seen that for interior loads, the results from the coupled program match very closely with Burmister's multilayered theory (CHEVRON) but are considerably different from Westergaard's plate which assumed a Winkler foundation. For the corner load, the deflections obtained from the elastic subgrade (using RIGMUL) are almost one-fourth of those predicted by Westergaard (with K-value estimated). The elastic subgrade would definitely behave much stiffer at the corner due to continuity as can be seen in Figure 4, since the subgrade outside the plate deforms as well. In the above comparion, full contact is assumed between the slab and subgrade.

Figure 4. Foundation displacement under a loaded slab



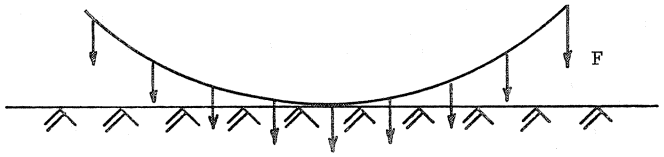
In Westergaard's analysis of thermal stresses (curling and warping), the slab is assumed to be weighless and connected to the springs (Winkler foundation) so that when the edges of the slab curl up as a result of vertical temperature gradient, the Winkler springs pull it down, as is shown schematically in Figure 5. Figure 6 shows the deflections and stresses as determined from Westergaard theory and from RIGMUL. As may be seen from these figures, the same assumptions lead to the same results (the differences in Figure 6 are the result of a coarse mesh representation in RIGMUL). However, Westergaard assumptions are far from realistic; slabs are not

Table 1. Comparison of RIGMUL with existing solutions.

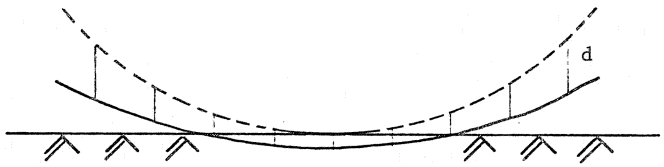
	DISPLACEMENT - mm											
	WESTERGAARD WINKLER			RIGMUL FINITE ELEMENT			CHEVRON BURMISTER			ELASTIC SUBGRADE HOGG AND HALL		
LOAD POSITION	Ks - MN/m^3 9.5	54.3	81.5	Es - MPa 17.2	89.6	113.8	Es - MPa 17.2	89.6	113.8	Es - MPa 17.2	89.6	113.8
CORNER	.787	.302	.241	.373	.086	.069	*	*	*	*	*	*
CENTER	.101	.042	.034	.165	.056	.043	.163	.053	.043	.254	.089	.076

NOTE: 1mm = .039 in. 1MPa = 145 psi. $1MN/m^3$ = 9.82 pci.
Not available from method

Figure 5. Curling of a Westergaard slab



(a) Initial curl without correction for vertical pull



(b) Final deflected shape of slab

Figure 6. Comparison of stresses and deflections due to temperature

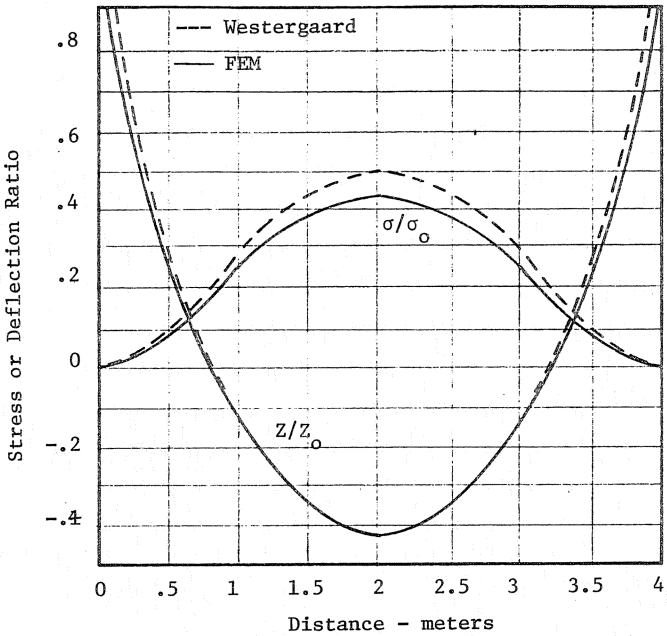
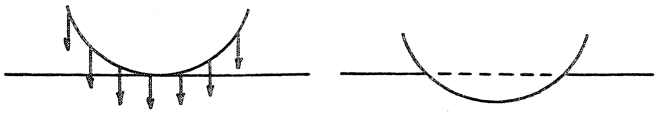
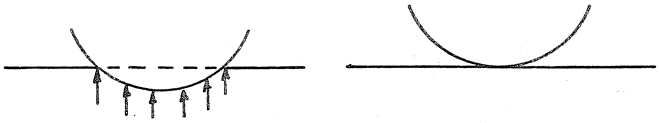


Figure 7. Slab curling with full and partial contact



(a) Full Contact: Initial and Final Positions



(b) Partial Contact: Initial and Final Positions

weightless, nor are they coupled to the subgrade; i.e., an elastic solid subgrade is not able to resist the vertical lifting of the slab. For a weightless slab, forces F (Figure 45a) are zero and the final deflected shape is the same as the initial position; furthermore, there are no temperature stresses. An analysis based on partial contact iterates to this result, as is shown in Figure 7. The partial contact condition assumes that the subgrade is only capable of exerting upward forces (when it is in compression), as indicated in Figure 7b.

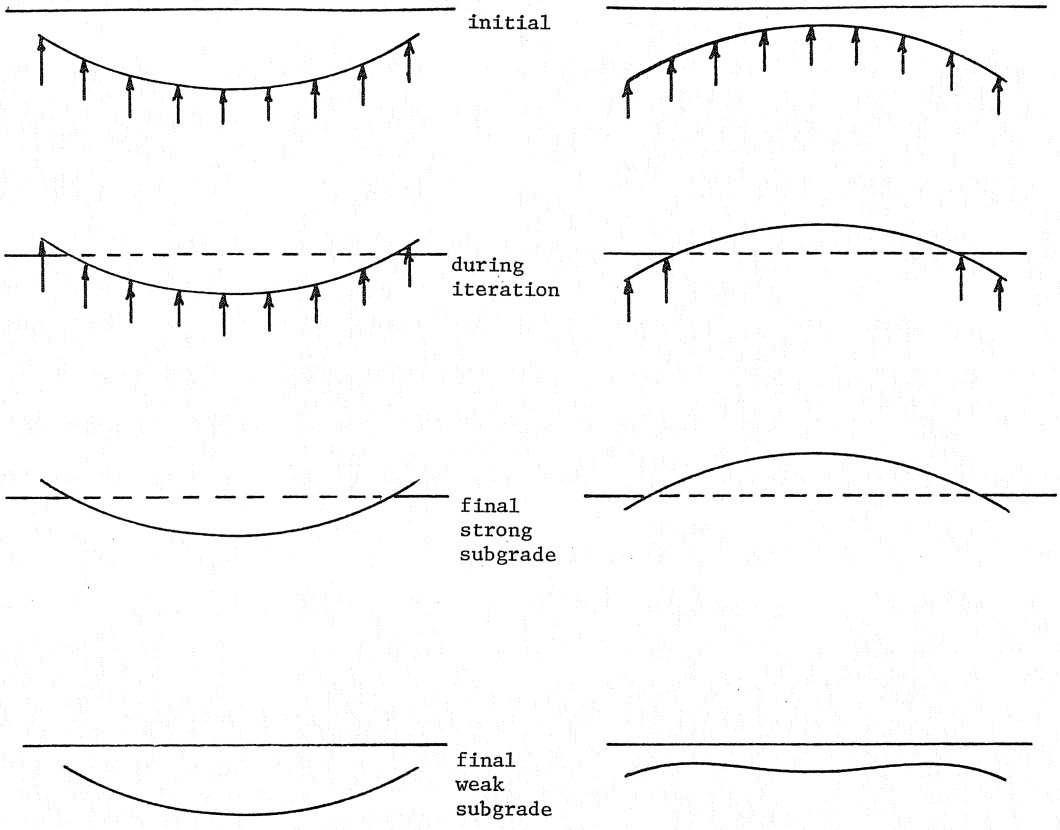
A more realistic model of slab curling and warping includes the slab weight, as well as the assumption of partial contact (upward forces exerted only when subgrade is in compression). When self-weight of the slab is introduced, along with temperature differentials, a case of partial contact occurs. Figures 8a and 8b show how the solution would iterate and converge to the final solution, and although fairly fast convergence is achieved to the final deflection profile and contact condition, an initial guess as to the contact condition is possible in the program and would be helpful. For example, in case of the slab in Figure 8a, for high values of subgrade support, if the edges and corners had been assumed to be out of contact initially, the solution time would have been reduced considerable. Figure 8 shows that the stiffness value of the subgrade changes the deflection profile and the final contact condition. However, no appreciable changes were observed in the stresses for changes in subgrade support conditions.

Table 2. Effect of dowel bars on slab stresses and displacements.

	DOWEL DIAMETER AND SPACING (mm)									LOAD TRANSFER	
	25.4 152.4	25.4 304.8	25.4 457.2	12.7 152.4	12.7 304.8	12.7 457.2	12.7 609.6	12.7 762.	6.4 152.4	100%	1%
SLAB 1 DISP. (mm)	.331	.391	.391	.371	.381	.381	.424	.483	.472	.320	.635
SLAB 2 DISP. (mm)	.310	.249	.249	.269	.259	.259	.213	.157	.177	.320	.005
SLAB 1 STRESS (kPa)	700	772	793	795	830	837	869	910	896	990	979
SLAB 2 STRESS (kPa)	283	210	190	186	152	144	110	73	83	490	0

NOTE: 1 mm = .039 in. 1 kPa = 0.145 psi

Figure 8. Slab curling and warping with partial contact condition, weak and strong subgrade.



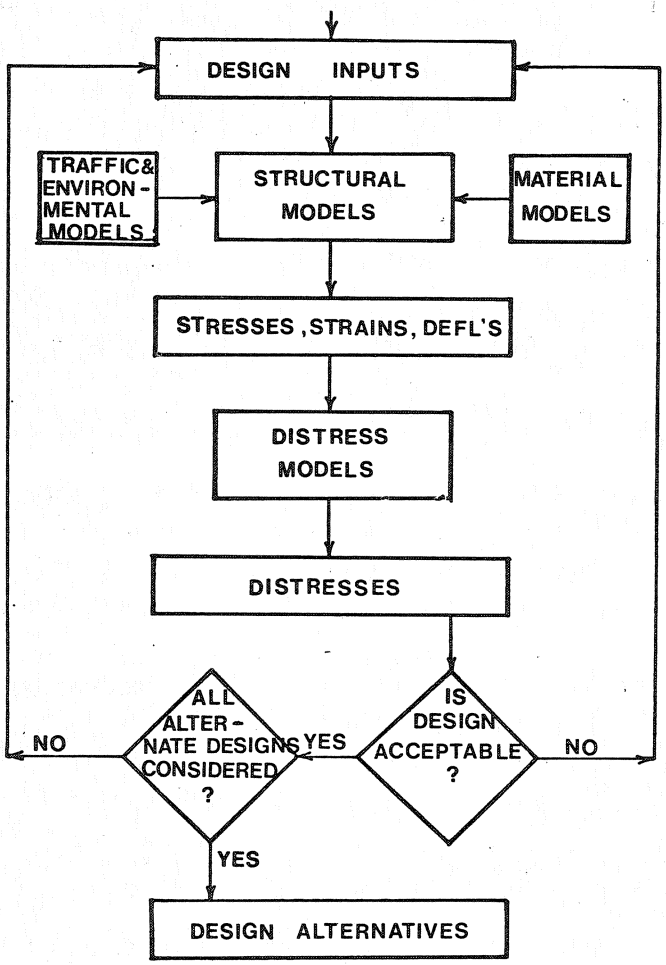
(a) Positive temperature difference (b) Negative temperature difference

As was stated earlier, RIGMUL is capable of analyzing load transfer effects as a result of dowel bars between slabs. Two methods are available: by specifying a percent load transfer from one slab to the other (as defined by the ratio of deflections of the loaded and unloaded slab), or by specifying dowel bar diameter, spacing, modulus, and modulus of dowel support. Thus it is also possible to consider the effects of dowel bar looseness by specifying a low support modulus up to some deflection values, followed by high support modulus for higher deflections.

The results of a preliminary analysis of the effects of dowel bars are shown in Table 2, but these results have not been validated independently; it is planned to use solid SAP for this verification.

Although detailed quantitative verification of RIGMUL is not possible since many of the features of this program represent extensions beyond the scope of existing theories, several qualitative verifications are possible. For example, two slabs with 100% load and moment transfer across the joint are theoretically equivalent to a single slab with the

Figure 9. Flow chart for design program.



same dimensions as the two-slab system. RIGMUL analysis shows that these systems are indeed equivalent. Similarly, the effects of dowel bar diameter and spacing on slab stresses and displacements (Table 2) are consistent with experience. Various other considerations of this nature (such as the effect of additional slabs of varying thicknesses around the loaded slab) show that RIGMUL results are consistent with theory.

Pavement Design Program

Work is currently underway to assemble the results of the stress analysis system into a pavement design package. Figure 9 shows the flow chart for this program. Since rigid pavement design involves a fairly large number of variables, special effort is directed toward making the input of information as simple as possible.

The ultimate aim of the design program is thickness design, taking into consideration effects of load transfer devices, subbase and subgrade support, shoulders, slab dimensions, etc. Since distress models at the present time are limited to fatigue fracture and joint faulting, the thickness design will be based on these considerations. However, other critical stresses, strain and displacements are available as output so that when mechanistic models for pumping, permanent deformation, etc. become available, these considerations can easily be incorporated into the design program.

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