THE ANALYTICAL DESIGN OF CONCRETE PAVEMENTS

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This paper considers fundamentally based design procedures for concrete roads. The literature has been reviewed for procedures relevant to the analysis of concrete roads, for structural performance models, and for data relating to material performance. It was concluded that further development of analysis techniques, particularly in relation to pavements containing cracked layers is necessary. Also, further information relating laboratory measurements of material performance to field performance is necessary. If this information is forthcoming, and by drawing on information from other research fields, it will be possible to produce a detailed fundamentally based design package.

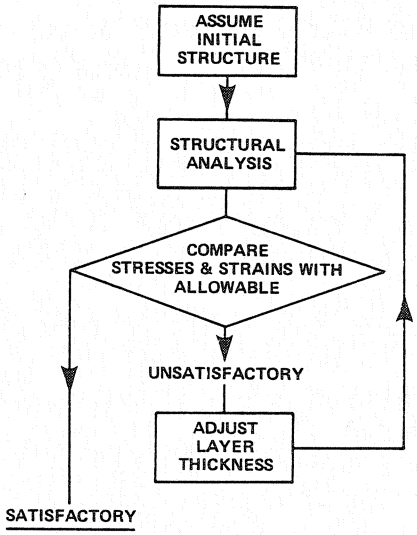
The objective of this paper is to arouse interest in, and provide impetus for the development of design procedures for concrete pavements, based on fundamental engineering principals. This process, outlined in Figure 1 involves analysis to determine critical parameters within a trial structure and a knowledge of the behaviour and performance of constituent materials. This information is used to ensure that the structure is neither over nor under stressed. Use of this type of design procedure is common in most branches of engineering and it will be shown that the basic information necessary to develop such a procedure for concrete roads is available, though some significant effort is still required to produce an implementable, design from it.

With this objective in mind, this paper will discuss the need for analytically based design procedures and review the literature for information which can be used to develop analytical design procedures.

The Need for Analytical Procedures

Before reviewing some of the information available it is appropriate to discuss the development of pavement design methods based on structural analysis. In the author's view development in this area really started in 1926 with a paper by Westergaard entitled, 'Stresses in Concrete Pavements Computed by Theoretical Analysis' (1). This work was developed further by Westergaard (2) and by Teller and Sutherland (3) but the impetus in the application of analytical techniques to concrete pavements provided by this early work has waned. In 1943 Burmister (4) published a paper on the calculation of stress and strain in layered systems which provided the foundation for the development of analytically based design procedures for flexible roads. In 1962 the International Conference on the Design of Flexible pavements (5) was held at the University of Michigan with the principal objective of providing a forum for the discussion of the results of the AASHO road test and developments in pavement design techniques for flexible pavements. This was followed by the publication by Shell (6) of design charts based on analytical procedures for flexible pavements. Research relating to the development of analytical design procedures for flexible pavements has continued and in 1977 the Fourth International Conference on the Design of Flexible Pavements (7) was devoted to papers describing imprementable design packages. Some of these packages are now in use, either substituting

Figure 1. Flow chart of analytical design procedure.



for or supplementing existing empirically based design methods.

The implementation of analytically based design procedures should be hastened by the need to cut expenditure on highway construction and maintenance since the versatility of analytical techniques offers a vehicle for the assessment of new and cheap materials within pavement structures. Mathematical modelling of the performance of pavement structures is an essential part of pavement maintenance, and thus will assist authorities who are under pressure with respect to maintenance budgets to make rational and economically justifiable decisions.

Unfortunately the state of the art with respect to this analytically based approach to concrete pavements is far behind that related to the rival flexible pavement even though analytically based design packages have been available from the Portland Cement Association for several years. Thus user agencies are faced with the prospect of operating an analytically based design and maintenance package for the flexible pavements in parallel with the empirically based package for concrete roads, an inefficient and sad state of affairs. It should be noted that flexible pavements and concrete (or rigid) pavements are both layered structures, consisting of at least three layers, frequently using similar materials in all layers except one. Thus a fundamentally based design procedure will be able to cope with both. In the U.K. a common form of pavement construction includes a layer of lean concrete with an asphalt surfacing. This is not a concrete pavement, but since lean concrete has a modulus approaching that of plain concrete it can hardly be described as flexible. It is also increasingly common to find old concrete pavements being overlain with asphalt. Thus a fundamentally based design procedure must be of increasing value to the engineer who has to deal with pavements which do not conform with classical simplifying definitions, and will offer him one 'tool' which will cover the full range of structures which he may have to deal with.

Techniques for Determining Wheel Load Stresses

Westergaard Analysis

In order to use the Westergaard method of analysis (1, 2, 3) a pavement must be reduced to a two layer structure. The upper layer which may be considered to represent the concrete slab, must be homogeneous, have uniform elastic properties, and be finite in extent. The reaction of the subgrade, the lower layer, is assumed to be vertical only, and to be proportional to the deflection, i.e. the support is similar to that provided by a dense fluid, and hence the subgrade has no shear strength. The original formulation (1) assumed uniform contact between the slab and the subgrade and was modified (2, 3) to take account of the non-uniform support conditions which result from slabs warping.

The two layer system which can be analysed by the Westergaard method is not truly representative of a concrete pavement. Subgrades invariably have shear strength and the literature on the behaviour of soils indicates that the state of stress and moisture condition have highly significant effects on its performance. However the difference between the assumption and reality may not be significant in a pavement consisting of a well designed uncracked concrete slab and a reasonable foundation. Also concrete pavements are very rarely two layer structures there usually being at least one and commonly two layers between the subgrade and the concrete slab itself. If the intervening layer is not a stiff material it may be reasonable to assume that it can be considered as similar to the subgrade. However in the United Kingdom it is not unusual to find a layer of relatively stiff material beneath the concrete slab, which definitely cannot be considered as part of the subgrade. In this case a method proposed by Fukuda (8) for combining two layers into one could be useful. This method derives an equivalent thickness for a composite slab H' as follows:

$$H_1' = \frac{1 + 2\alpha\beta + \alpha\beta^2}{1 + \alpha\beta} H_1 \qquad (1)$$

where

$$\alpha = \frac{E_2}{E_1} \frac{1 - \nu_1^2}{1 - \nu_1^2} \qquad (2)$$

$$\beta = \frac{H_2}{H_1} \qquad (3)$$

H_1 = thickness of the top layer (concrete slab)

H_2 = thickness of the second layer

E_1, ν_1, E_2 and ν_2 are elastic properties of the two layers

The tensile stress at the lower face of the concrete slab can be calculated by

$$\sigma = \left(2 \frac{H_1}{H_1'} - 1\right)\sigma' \qquad (4)$$

where σ' is the tensile stress calculated for H_1' by the Westergaard equations.

The Portland Cement Association has played a leading role in developing design methods for concrete pavements for both airfields and highways. These methods are based on the computation of stresses in roads and the Westergaard analysis technique has been used by them for the analysis. Unfortunately no quantitative information appears to be available on the performance of these pavements which can be used to evaluate the design procedure and help identify areas in which it is unsatisfactory.

Elastic Layer Analysis

This method of analysis treats a pavement as a multi-layered structure based on the following assumptions:

1. The material in each layer is homogeneous and behaves in a linear elastic manner.
2. All layers are infinite in lateral extent.
3. The subgrade is infinite in depth.

Solution of a pavement structure consisting of elastic layers requires the use of a computer, BISTRO (9) and CHEVRON (10) being two programs commonly used for solving layered systems.

One of the disadvantages of the elastic layer model for concrete pavements is that as a consequence of assumption 2. above, only interior stresses can be calculated, where as the Westergaard analysis allows computations to be made for edge and corner loading conditions, which may be more critical.

Elastic layer programs have been modified to

treat layers as non-linear elastic (11) and thus can use the stress dependant models which have been developed for unbound granular materials and for soils, thereby increasing the versatility of this analysis technique.

Parker et al (12) in their design package for rigid airfield pavements have used the elastic layer analysis, but have modified the assumption of a semi-infinite subgrade by including a relatively stiff layer at a depth of approximately 6 metres. This has been done to obtain agreement between measured and computed surface deflections.

McCullogh and Boedecker (13) have concluded that it is reasonable to use elastic layer theory for the design of continuously reinforced concrete pavement overlays, and recommended its use.

Finite Element Analysis

This is the most versatile of the methods currently available for the analysis of pavement structures. In principle no assumptions with regard to the extent of the layers in either horizontal or vertical directions are required, and materials can be described by an appropriate mathematical model. In practice the application of the method to complex material models can be difficult due to the inability of some of the computational processes to cope with the equations resulting from these models, and the tendency in these cases for the size of the problem to exceed the capacity of the computer. Also in comparison with the other analysis techniques, finite elements can be very costly because of the large amount of computer time required to complete the analysis.

Whilst finite element methods are available they do not appear to be widely used. However Tabatabaie and Barenberg (14) have applied the method to jointed or cracked concrete pavement slabs on the Westergaard type of subgrade. Also Macleod and Monismith (15) have used the technique in developing a performance model for plain, jointed portland cement concrete pavements.

Techniques for Determining Thermal Stresses

Temperature stresses in concrete slabs can be conveniently divided into three categories:

1. End-restraint compressive stresses.
2. Foundation restraint compressive and tensile stresses.
3. Partially or completely restrained warping stresses.

Depending on the details of any particular structure under consideration, these stresses can be highly significant, and so techniques for their determination are of importance.

End Restraint Compressive Stresses

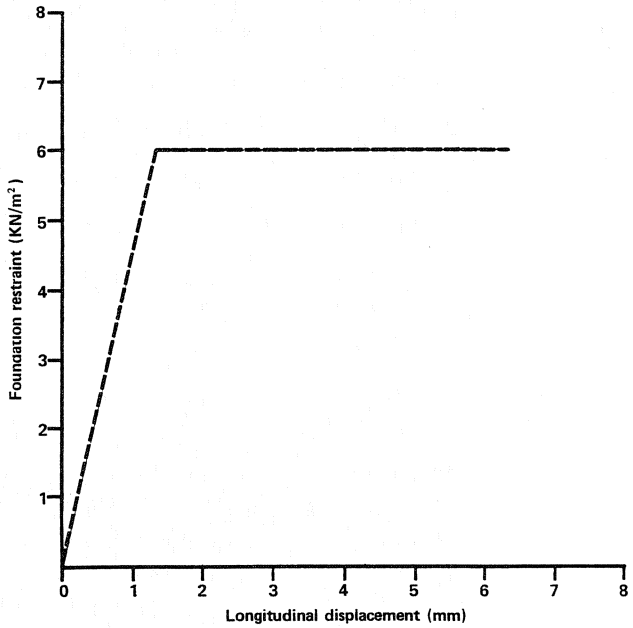
This stress results from the restrained expansion of a concrete slab. For conditions in the United Kingdom in which construction of concrete road slabs is not permitted at a temperature below 3° C and with relatively low air temperatures even in the summer, the maximum compressive stress likely to be generated is 10 MN/m^2. Since the minimum 28-day strength required by British specifications (16) is 27.6 MN/m^2 end restraint compressive stresses should not be a problem. However in areas with a greater temperature range, or which use a lower specified strength, compressive failure could occur. The more likely form of failure due to restrained compressive failure is buckling of the slab, this kind of failure generally being described as a 'blow up'.

Foundation Restraint Stresses

Thermally induced changes of dimension in concrete road slabs which are not restrained mean that the slab will attempt to slide across its foundation. The friction between the slab and its foundation will generate compressive stresses as the temperature increases, and tensile stresses as it decreases. These stresses will be greatest at the mid point of the slab.

Sparkes (17) and Stott (18) have reported experimental investigations designed to determine the force required to initiate slip between concrete slabs and various subgrades. Reviewing this information Croney (19) has proposed the relationship between foundation restraint and longitudinal displacement shown in Figure 2. Extending this work, Croney (19) has developed the relationships shown in Figure 3 between foundation restraint stress and slab length. These relations are independent of slab thickness and were derived assuming an E value of 35 GN/m^2 and a coefficient of expansion of $10^{-5}/^{o}$ C which represent a normal strength concrete used in United Kingdom pavements.

Figure 2. Relationship between foundation restraint force and slab displacement for 150 mm thick slab (19).

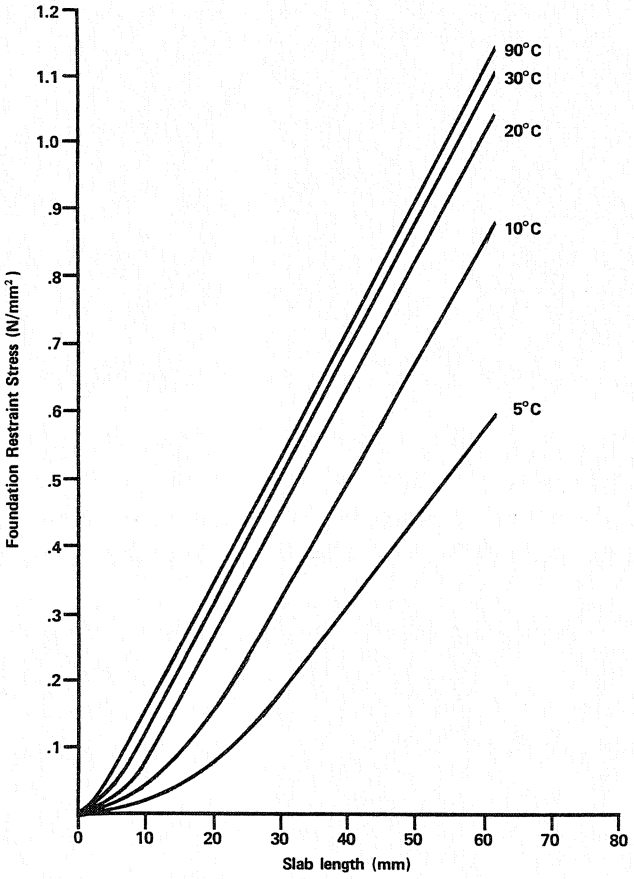


Warping stresses

During the normal 24 hour thermal cycle temperature gradients are set up within the pavement. When a concrete pavement slab is subjected to a temperature gradient it will tend to warp. This tendency is restrained by the self weight of the slab which results in stresses being induced within it.

As already indicated the original analysis proposed by Westergaard (1) has been modified to take

Figure 3. Relationship between foundation restraint stresses at the centre of the slab and slab length (19).

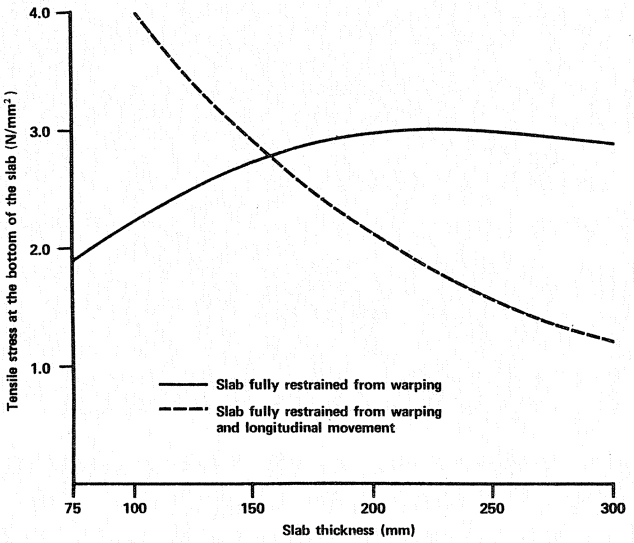


account of warping (2). In addition Bradbury (20) has developed a procedure for estimating temperature stresses and Thomlinson (21) has made a detailed analysis of the warping stresses in concrete slabs, taking into account the effects of internal stresses. Figure 4 reproduced from Croney (19) shows the variation of combined internal and restrained warping stress with time of day calculated by Thomlinson's (21) method.

Summary

Analytical techniques exist which can be used to determine wheel load stresses in concrete pavements, and the Westergaard technique has been in use for some years. Also methods exist which permit the determination of temperature stresses, thus there is no reason why development of analytically based concrete pavement performance models cannot continue. Also use can be made of improved knowledge of the performance of all paving materials. The analysis of cracked slabs has not been mentioned. Some efforts to treat this problem have been made, Niu and Pickett (22) and Tabatabaie and Barenberg (14) describing analytical procedures. However both papers describe two layer analyses, the subgrade being modelled in the same manner as for the Westergaard approach. The most pressing need is for an analytical technique which permits the analysis of cracked multi-layer pavement structures so that the effect of the crack on all layers in the

Figure 4. Tensile stresses at the bottom of a slab due to clayline temperature gradients (19).



pavement can be assessed. This is particularly important when considering maintenance options for a cracked pavement, when a reasonable estimate of the stresses in a layer on top of the cracked one is important.

Material Properties for Analytical Design

A wealth of information is available in the literature regarding the behaviour of soils and unbound granular materials, though effort is required to translate this into a form which is of use to the pavement engineer. Conveniently the development of analytical techniques for designing flexible pavements has resulted in the derivation of models for these materials which can also be applied to concrete pavements. The proceedings of the Fourth International Conference on the Design of Flexible Pavements (7) is a useful source of information on models as applied to pavements, and will provide references to the original research behind these models.

Since for structural purposes, concrete is regarded as an elastic material, then values of Young's modulus and Poisson's ratio for concrete are necessary as input parameters for the analysis of pavements. Croney (19) has presented the following equations which relate the cube strength (C) at 28 days with the dynamic modulus of elasticity E_d:

For concrete 28 days old

$$E_d = 42900 + 93C \quad (5)$$

For concrete 1 year old

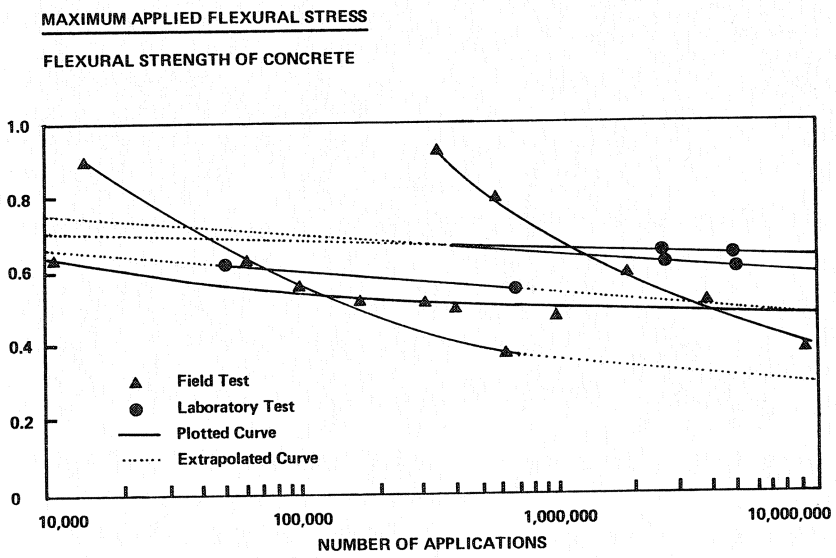
$$E_d = 45600 + 130C \quad (6)$$

For concrete 5 years old

$$E_d = 48000 + 135C \quad (7)$$

(The units for E_d and C are MN/m^2)

Figure 5. Comparison of fatigue tests for concrete (23)



Anson and Newman (22) have reviewed the information available with regard to Poisson's ratio for concrete and from their work a value of 0.19 appears to be appropriate.

Having obtained material descriptions for use in analysing a trial pavement structure, hence obtained a picture of the state of stress within it, performance criteria are required so that the pavement can be designed. Since traffic loading is repetitive in nature, it is reasonable to consider the performance of concrete under repeated loading conditions. Yimpraset and McCullough (23) have compared a number of sets of repeated load tests on concrete, which include both laboratory and field tests. Figure 5. is reproduced from this work. The Transport and Road Research Laboratory (TRRL) has also studied the performance of pavement quality concrete in repeated loading (24, 25, 26), examining the effects of rate of loading, moisture changes and age in this study. Whilst no generalized performance model has, as yet, been published by the TRRL, it is possible to draw two useful conclusions from the range of tests undertaken. The first is that rate of loading does not affect the measured performance under repeated loading, and the second is that changes in fatigue performance follow, fairly closely, changes in flexural performance. The latter conclusion suggests that performance in repeated loading could be predicted from simple flexural tests.

The Portland Cement Association has provided a table which gives numbers of allowable repetitions of stress for given ratios between the actual stress in the pavement and the modulus of rupture of the concrete (27). This data is reproduced in Table 1. When using this data for design purposes, the design loading is usually divided into axle-load groups and the number of allowable repetitions for each group determined from a knowledge of the stress that it will induce in the slab and a knowledge of the strength of the concrete. By comparison with the predicted vehicle loading the proportion of life used by each axle-load can be determined. If the summation of the proportions of life used by each axle group is unity or less then the pavement will not fail. It is reported (27) that a value of 1.25 can be approached in highway pavements and 1.1 in airfield pavements without serious problems occurring.

Table 1. Stress ratios and allowable load repetition (27)

Stress[a] Ratio	Allowable Repetitions	Stress Ratio	Allowable Repetitions
0.51[b]	400,000	0.63	14,000
0.52	300,000	0.64	11,000
0.53	240,000	0.65	8,000
0.54	180,000	0.66	6,000
0.55	130,000	0.67	4,500
0.56	100,000	0.68	3,500
0.57	75,000	0.69	2,500
0.58	57,000	0.70	2,000
0.59	42,000	0.71	1,500
0.60	32,000	0.72	1,100
0.61	24,000	0.73	850
0.62	18,000	0.74	650

[a] Load stress divided by modulus of rupture

[b] Unlimited repetitions for stress ratios of 0.50 or less

Parker et al (12) have developed an alternative relationship from studies of test pavements subjected to controlled accelerated simulated

aircraft traffic. The relationship thus derived is

$$DF = 0.58901 + 0.35686 \log (cov) \quad (8)$$

where DF = design factor

$$= \frac{R}{\sigma} \quad (9)$$

R = concrete flexural strength

σ = maximum principal tensile stress at the bottom of the concrete slab, calculated by an elastic layer model

Log (cov) = logarithm of the traffic in terms of coverages

This relationship allows the design factor to be computed for a given number of coverages, and thus, for concrete of a known strength, the allowable principal tensile stress can be determined. To design the pavement, trial structures with various slab thicknesses are analysed and a graph plotted relating the principal tensile stress to thickness. It is then a simple matter to read off the thickness for the allowable stress which has already been calculated.

Concrete Pavement Performance Models

The section above has considered pavement performance as the number of load cycles which can be sustained by the concrete slab prior to cracking. However a realistic assessment of the performance of a concrete pavement involves factors such as crack width, which affect the possible ingress of water to the subgrade and vertical pavement deformations of cracks and joints. Thus there is a need for performance models which cover wider aspects of the performance of concrete pavements.

Much attention is currently being focussed on continuously reinforced concrete pavements. Ma and McCullough (28) have reported the development of a computer program, known as CRCP-2 which produces predictions of the final values of crack spacing, crack width, and steel stress, which can be used as a design tool for the continuously reinforced concrete pavement.

Macleod and Monismith (15) have reported the development of two performance models for plain jointed concrete pavements covering pavement roughness and faulting and also for cracking. This work has involved study of the performance of pavements in California and encouraging corelation between predictions and observation has been reported and emphasizes the importance of realistic modelling of base layers. The models thus derived are a valuable contribution to the further development of analytically based design methods for concrete roads and indicate the type of study which is of great value with respect to future developments.

The major limitation of both research programs discussed above is that they are relevant only to the particular type of concrete pavement construction. The models, whilst of considerable use with respect to the performance of new pavements, do not help the engineer who is faced with a distressed pavement to undertake an analytical study of his maintenance alternatives.

Concluding Comments

It has been possible to design concrete pavements by the use of analytical techniques for some time. Therefore it would seem reasonable to expect some validitory evidence for these design methods. As far as the author is aware, there are no long term performance measurements on pavements constructed to an analytically based design to indicate whether they are successful or not. This indicates that the design techniques, if used at all, are not monitored and thus, perhaps, explains why there is relatively little interest in using the improved methods for characterising other layers in the pavement which have become available in recent years.

It is the author's view that two major areas need further study. Firstly the analysis of cracked multilayer structures. Secondly to relate laboratory measured material performance characteristics to the performance of full scale pavements. Application of existing analytical design procedures combined with monitoring, which should be carried out as part of the maintenance program, will provide valuable information for the second area of study. Initially the derivation of analytical techniques for cracked multilayer systems is a job for the theoretician, but performance data is essential to allow the suitability of the technique to be judged.

None of this work will be of any value unless it is used in the design of pavements. The technique of analysis is one of the most powerful available to the engineer for assisting him to make decisions. Only by using it will deficiencies be recognized. Effort can then be directed towards rectifying these deficiencies, and a healthy atmosphere in which the available technology is utilized to the full in designing and building concrete roads will result.

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