7th International Conference on Concrete Pavements
September 9 – 13, 2001
Disney's Coronado Springs Resort
Orlando, Florida

DYNAMIC ANALYSIS OF RIGID PAVEMENTS WITH DIFFERENT SUBGRADE SUPPORT MODELS

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ABSTRACT

It has long been recognized that the accuracy of any slab-on-grade model is strongly dependent on the foundation model used. Therefore, using an accurate structural model for the concrete slab system does not guarantee an accurate response for the slab-subgrade system. Thus, characterizing the subgrade support correctly is essential in order to obtain accurate predictions of the rigid pavement response. Most existing finite-element programs for rigid pavements use the Winkler model as the primary model for subgrade characterization.

This paper describes the development of an expanded version of the DYNASLAB computer program with the addition of a frequency-dependent Winkler foundation and a layered foundation model. Using the new DYNASLAB program, several dynamic analyses were conducted to compare results from the two foundation models.

INTRODUCTION

It has long been recognized that the accuracy of any slab-on-grade model is strongly dependent on the foundation model used. Therefore, using an accurate structural model for the concrete slab system does not guarantee an accurate response for the slab-subgrade system. Thus, characterizing the subgrade support correctly is essential in order to obtain accurate predictions of the rigid pavement response.

Traditionally, the subgrade model used for analyzing Portland Cement Concrete (PCC) pavements has been the Winkler foundation. In this model, the subgrade is represented by a series of vertical springs. Most existing finite element programs for rigid pavements use the Winkler model as the primary model for subgrade characterization, including WESLIQUID (1), ILLI-SLAB (2), and JSLAB (3). This is partly due to the simplicity of the Winkler foundation, its natural ability to handle discontinuities across joints and cracks, and the relative ease of determining the spring constant, or the coefficient of subgrade reaction, *k*. The disadvantages of the Winkler foundation include its inability to correctly predict the response for edge or corner loading, and its inability to predict the response within the subgrade.

In 1943, Burmister (4) developed the two-layered foundation system, and later (1945) developed a three-layered foundation system. With the advancement of computer technology, new finite-element programs were developed with a layered foundation, including WESLAYER (1) and KENSLABS (5). These later methods either model a single slab or use a somewhat tedious iterative process for transferring forces across joints. The Layered continuum foundation model is a more realistic model than the Winkler model, although the real behavior probably lies between the two foundation models.

DYNASLAB PROGRAM

The DYNASLAB program was developed at the University of California at Berkeley for the dynamic analysis of jointed concrete pavements subjected to moving transient loads (6). The original DYNASLAB program is based on the classical theory of a medium-thick plate on a Winkler foundation and employs four noded, twelve-dof plate-bending elements. Recently, DYNASLAB was improved by the addition of the frequency-dependent Winkler foundation and the layered foundation models (7). The new DYNASLAB program has the capability of handling multiple slabs (three slabs in x-direction and two slabs in y-direction) with three different types of load transfer mechanism: 1) Dowel bar, 2) aggregate interlock and 3) combination of dowel bar and aggregate interlock. These capabilities are also applicable for both Winkler foundation options.

DYNASLAB-Winkler Foundation Models

The dynamic-Winkler option of the DYNASLAB computer program models the foundation as a series of distributed vertical springs (k) and dashpots (c). There are two main algorithms for this foundation option: (i) time-domain and (ii) frequency-domain. The details for the two algorithms are described elsewhere (6).

The main issue in predicting the dynamic response of rigid pavements accurately is which values for the parameters k, c, and β are to be used for a particular problem and how to determine them. Characterizing the subgrade response in the field has been mostly done using the Falling Weight Deflectometer (FWD). This non-destructive test involves applying a dynamic pulse load with a duration of about 30 to 40 milliseconds. The existing methods for back-calculating the modulus of subgrade reaction and the modulus of elasticity of the concrete slab are based on static analysis. While such methods can be assumed to lead to reasonable estimates of *k* for "normal" conditions, they may lead to erroneous estimates for certain conditions such as the existence of a stiff layer. The stiff layer condition can occur if a stiff soil stratum or the ground water table exists at shallow depth. Furthermore, static backcalculation is not capable of estimating the radiation (geometric) damping ratio of the subgrade. Therefore, a new closed-form solution for dynamic backcalculation that has been developed by Chatti et al (8) was used for determining the dynamic subgrade stiffness and damping coefficients. The equations for k and c are shown below:

$$k = \frac{\mathrm{Re}(P_0)\int_A \mathrm{Re}\,U dA + \mathrm{Im}(P_0)\int_A \mathrm{Im}\,U dA}{(\int_A \mathrm{Re}\,U dA)^2 + (\int_A \mathrm{Im}\,U dA)^2} + \omega^2 \rho \qquad (1)$$

$$c = -\frac{1}{\omega}\frac{\mathrm{Re}(P_0)\int_A \mathrm{Im}\,U dA - \mathrm{Im}(P_0)\int_A \mathrm{Re}\,U dA}{(\int_A \mathrm{Re}\,U dA)^2 + (\int_A \mathrm{Im}\,U dA)^2} \qquad (2)$$

where, $\mathrm{Re}(P_0)$, $\mathrm{Im}(P_0)$: Real and imaginary parts of the load
ReU, ImU : Real and imaginary parts of the deflection basins
ρ : Slab density
ω : Circular frequency

The above solution is unique for a given deflection basin. The expressions for k and c are valid for any deflection basin, including for the case of edge/corner loading. However, in standard FWD testing, the deflection basin is only measured along one axis. Therefore, the volume of the deflection basin cannot be calculated accurately from measured sensor deflections under edge/corner loading. On the other hand, the measured deflections provide enough information if an interior load test is conducted.

DYNASLAB-Layered Foundation Model

The basic approach in the layered foundation solution is to partition the rigid pavement system into two parts: The slab(s) and the foundation, as shown in Figure 1. Interaction between the slab and the foundation occurs at all nodes. The dynamic (complex) global stiffness matrix is the sum of the stiffness matrices of the slab(s), the load transfer system across joints and the foundation, respectively.

The foundation stiffness matrix is obtained directly in the global coordinate system by inverting the dynamic flexibility matrix corresponding to the vertical degrees-of-freedom at all nodes. This is done by successively applying unit amplitude disk loads at the vertical degree-of-freedom of each node and computing the corresponding vertical displacement amplitudes at all nodes. These displacement amplitudes are computed using the SAPSI computer program, which solves for the steady-state response of a multi-layered system subjected to surface disk loads (9). The radius of the disk loads is chosen to be equal to the distance between two adjacent nodes in order to establish near-compatibility with the displacements of the plate elements representing the slab system. The computational effort required to construct the subgrade flexibility matrix is small because the axisymmetric loading problem has to be solved only once. A displacement basin is computed (using the SAPSI program) at discrete points, and the nodal displacements are interpolated. The discretization of the deflection basin needs to be fine enough so that the interpolation is accurate, and the extent of the deflection basin needs to be such that it is at least equal to the longest diagonal distance of the entire slab system.

The basic assumptions that are satisfied within the layered system are:

1) The material for each layer is assumed to be homogeneous, isotropic and (damped) elastic.
2) Each layer is assumed to have a finite thickness and to be infinite in extent in the horizontal direction. The lowest layer is modeled as a semi-infinite layer by using a series of variable thickness sublayers such that the total thickness corresponds to the depth at which Rayleigh wave motion decays.
3) A uniform pressure is applied on the surface over a circular area.
4) Continuity conditions: Two consecutive layers are continuously in contact with shearing resistance fully active between them.

Modeling multiple slabs that are separated by zero (or near zero) width joints on top of a continuous (layered) foundation translates mathematically to having off-diagonal coefficients in the flexibility matrix that are equal (or nearly equal) to the diagonal coefficients. This leads to an ill-conditioned flexibility matrix. To avoid the problem at the joint, the solution in DYNASLAB assumes that the end-nodes across the joint are not connected through the subgrade. This is done by imposing low values of deflection for calculating the off-diagonal flexibility coefficients at the joint, thus creating hypothetical gap (see Figure 2). The reasonableness of this assumption is checked later in this paper.

ANALYSIS

Two profiles were used in the DYNASLAB analysis (Figure 3) : (1) a “thin” PCC slab over a “stiff” foundation composed of an asphalt-treated base, a granular subbase and uniform subgrade; (2) a “thick” PCC slab over a “soft” subgrade. The corresponding foundation parameters are shown in Figure 4.

Convergence Studies with Winkler Foundation Models

The accuracy of the program for the Winkler foundation models (both time-domain and frequency domain solutions) was verified by using varying element sizes, slab sizes, and time

steps. The analysis showed that accurate slab response can be obtained using 0.3 m-elements and a time step equals to 0.1 T, where T is the duration for the load to traverse two elements. The details of this analysis are shown elsewhere (7).

Effects of Radiation Damping and Speed

The damping ratio can significantly affect the response of rigid pavements. Chatti (6) indicated that for over-damped systems, rigid pavement response will decrease with increasing speed, especially at higher speed values. On the other hand, for under-damped systems, the response will increase with increasing speed. Several DYNASLAB runs were done to verify the effect of radiation damping ratio, β. Three damping ratios were selected to determine this effect (0.1, 1.0 and 2.0), and speed was varied from creep speed to 320 km/hr (applicable to an aircraft on a runway). The results are shown in Figure 5. These results are very similar to those reported by Chatti (6) using a semi-closed-form solution for a beam of finite length supported with a damped Winkler foundation. It should be noted, however, that a vehicle speed of 96 km/hr causes only a 9% reduction in the response for the over-damped case.

Convergence Studies with Layered Foundation Model

In order to verify the accuracy of the layered foundation solution in the new DYNASLAB program, several runs were performed for different mesh and slab sizes. For the effect of mesh size, profile 1 was used, and for the effect of slab size, both profiles1 and 2 were used. (see Figure 3).

The Effect of Mesh Size

The mesh (element) size was varied from 0.3 m to 1.2 m to determine the convergence and accuracy of the program. The results shown in Table 1, indicate that bending stress is much more sensitive to mesh fineness than deflection. Similar results were found in the case of Winkler foundation and in ILLI-SLAB analysis with elastic solid subgrade, as reported by Ioannides et al (10). The results showed that accurate predictions can be obtained using 0.3 m-elements. However, for computational efficiency, 0.6 m-element can be used with sufficient accuracy. This element size was used for further analyses.

TABLE 1. Effect of Mesh Size on the Responses of a Slab Resting on a Layered Foundation

Mesh size (m)	Deflection (μm)	Normalized Deflection*	Stress (MPa)	Normalized Stress*
0.3	184.0	1.00	0.610	1.00
0.6	182.5	0.99	0.589	0.96
0.9	182.8	0.99	0.483	0.79
1.2	182.0	0.99	0.435	0.71

* Normalized to the results of a 0.3 m-element

The Effect of Slab Size

To verify the behavior of the visco-elastic layered foundation solution in the DYNASLAB program, the slab size was varied from 0.9 m to 7.2 m in the longitudinal direction, while for the transverse direction, 3.6 m was used for all analyses. The results are shown in Tables 2 and 3. As expected, both deflection and stress converge for longer. The results also show that as slab size increases, deflection converges from a high value to a low value, while stress converges from low to high. Also, deflection is less sensitive to changes in slab size. The difference between the two profiles can be seen for the cases of small slab size. The first profile shows that at a small slab size (0.9 m) there is not much decrease in deflection. However, the second profile shows more difference in deflection. Similar results were reported by Ioannides et al (10) using a uniform elastic solid foundation. Note that the results from modeling the concrete slab as an infinite layer (using SAPSI) match well with the finite element results in terms of deflection; however, this is not the case for bending stress. This can be expected since the assumptions (in terms of shear deformation) are different in the plate and layered theories.

TABLE 2. Effect of Slab Size on the Responses with a Layered Foundation – Profile 1

Slab size (m)	Deflection (μm)	Normalized Deflection*	Stress (MPa)	Normalized Stress*
0.9	231.0	1.255	0.343	0.38
1.8	192.3	1.045	0.466	0.52
3.0	184.8	1.005	0.577	0.64
4.8	182.5	0.992	0.586	0.65
6.0	181.5	0.987	0.585	0.65
7.2	180.0	0.978	0.585	0.65
SAPSI-M(∞)	184.0	1.000	0.897	1.00

* Normalized to the SAPSI result

TABLE 3. Effect of Slab Size on the Responses with a Layered Foundation – Profile 2

Slab size (m)	Deflection (μm)	Normalized Deflection*	Stress (MPa)	Normalized Stress*
0.9	162.3	1.38	0.265	0.45
1.8	144.5	1.23	0.334	0.57
3.0	130.8	1.11	0.369	0.63
4.8	121.3	1.03	0.390	0.67
6.0	122.3	1.04	0.389	0.66
7.2	120.8	1.03	0.386	0.66
SAPSI-M(∞)	117.3	1.00	0.586	1.00

* Normalized to the SAPSI result

COMPARISON OF SLAB RESPONSES USING WINKLER AND LAYERED FOUNDATIONS

The two profiles in Figure 3 were used to calculate the deflection and bending stress responses using the different foundation models. Both weak and strong aggregate interlock at the joint were

used, and the responses were sought for interior loading (Figures 6 through 8), edge loading (Figures 9 and 10), and transverse joint loading (Figures 11 and 12).

Interior Loading Cases

Figures 6 (a) and (b) show dynamic influence deflection functions with weak and strong aggregate interlock, respectively, for Profile 1 ("thin" slab on "stiff" foundation), due to the load moving across three consecutive slabs at a speed equal to 65 km/hr. The results show that the peak deflections using Winkler and layered foundation models are very close, with the layered foundation giving about 8% lower peak deflection. As expected, the effect of aggregate interlock on the response is negligible. However, the shapes of the influence functions from the two foundation models are different. The layered foundation shows a smoother function, reflecting the effect of the transfer of shear stresses across the continuous foundation system. On the other hand, the Winkler foundation, being a discontinuous system, shows a clear break in the deflection function at the transverse joint.

Figure 7 is similar to Figure 6 except that it shows the bending stress influence lines. The figure indicates that the effect of the foundation model on the peak stress is more significant than on deflection. The layered foundation model gives a peak stress that is about 30% lower than that using the Winkler foundation. The compressive bending stress (when the load is away from the response point) is also lower than that predicted using the Winkler model.

Figure 8 shows the dynamic deflection and bending stress influence functions using both foundation models for Profile 2 ("thick" slab on a "soft" foundation). The results clearly show the effect of slab thickness leading to a much smaller response. The influence functions become more asymmetric under a layered foundation, especially for the deflection, indicating a viscoelastic behavior on the part of the foundation system. On the other hand, the peak responses using both foundation models become closer (about 7% for both deflection and bending stress).

Edge Loading Cases

Figures 9 and 10 show the dynamic deflection and bending stress influence lines for Profiles 1 and 2, respectively. As expected, the Winkler model leads to a much higher response at the (unsupported) edge. The difference is much more significant for the "thin" slab (140% increase for deflection and 100% increase for bending stress) as compared to the thick slab response (50% increase for deflection and 20% increase for bending stress).

Transverse Joint Loading Cases

Figures 11 (a) and (b) show dynamic influence deflection functions with weak and strong aggregate interlock, respectively, for Profile 1. The results show that the difference in peak deflections using Winkler and layered foundation models becomes greater as compared to the interior loading case, with the layered foundation giving about 18% lower peak deflection. As expected, the effect of aggregate interlock on the response is significant, and it is more significant in case of a Winkler model (45 % difference) as compared to the layered foundation (10 % difference). Moreover, the shapes of the influence functions from the two foundation

models are different, with the layered foundation showing a smoother function and a higher response away from the load. Again, this reflects the effect of the transfer of shear stresses across the continuous foundation system. The Winkler foundation, being a discontinuous system, shows a larger jump in the deflection function at the transverse joint.

Figure 12 is similar to Figure 11 except that it shows the bending stress influence lines. The figure shows that the effect of the foundation model on the peak deflection is more significant than on bending stress. The layered foundation model gives a peak stress that is about 50% higher than that using the Winkler foundation in the case of weak aggregate interlock. For strong aggregate interlock, the peak bending stress is the same using both foundation models. However, if the difference between compression and tension were used as a measure of stress amplitude, then the Winkler model would lead to a 40% increase in amplitude. In fact, Figure 12 (b) shows that the layered foundation with strong aggregate interlock at the transverse joint does not cause a compressive stress as the load approaches the joint.

CONCLUSION

The dynamic rigid pavement analysis program called DYNASLAB has been successfully improved with the addition of the frequency-dependent Winkler foundation as well as the layered foundation option. The model can handle multiple slabs on either foundation, and takes into account moving loads. The effects of the foundation model on the slab responses were investigated for several loading cases and combinations of subgrade conditions, slab thickness and aggregate interlock at the joint. The results show that the foundation model affects the responses significantly at the slab edge and near the transverse joint.

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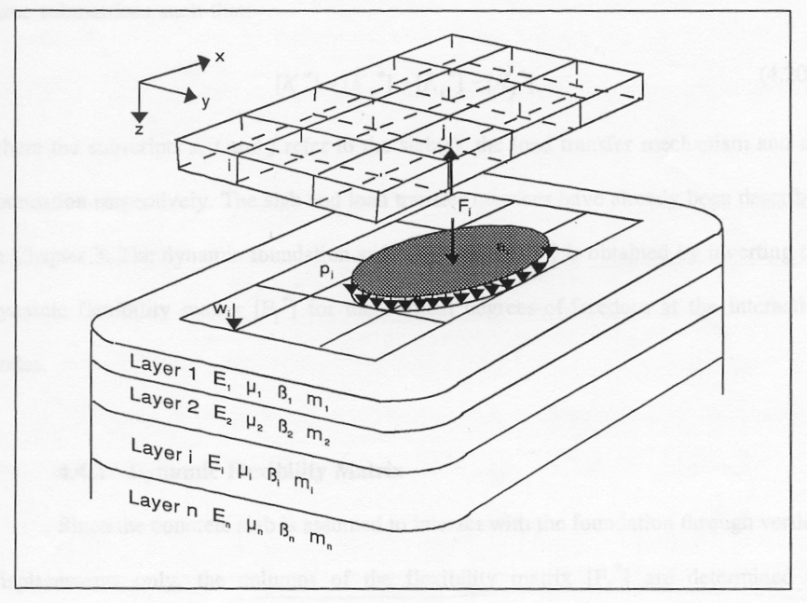

FIGURE 1. Interaction Between Concrete Slab and Multi-layered System

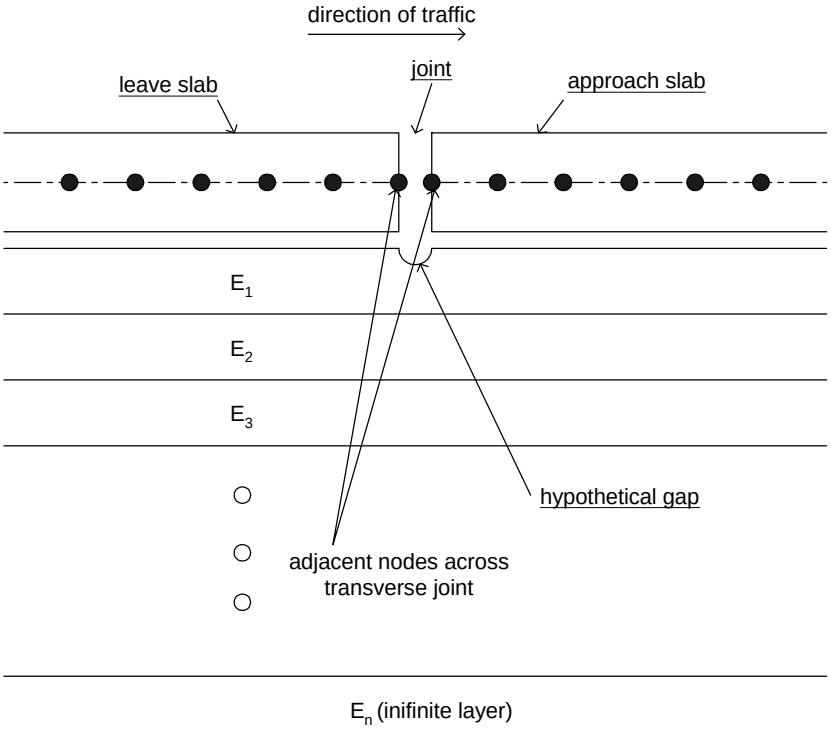

Figure 2. Modeling Joint in the Layered System

(a) Profile 1

PCC 200 mm
E = 690 MPa μ = 0.3 β = 0.02 150 mm
E = 207 MPa μ = 0.35 β = 0.03 150 mm
E = 34 MPa μ = 0.4 β = 0.05 infinite

(b) Profile 2

PCC 300 mm
E = 34 MPa μ = 0.45 β = 0.05 infinite

Figure 3. Profiles Used in the Analysis

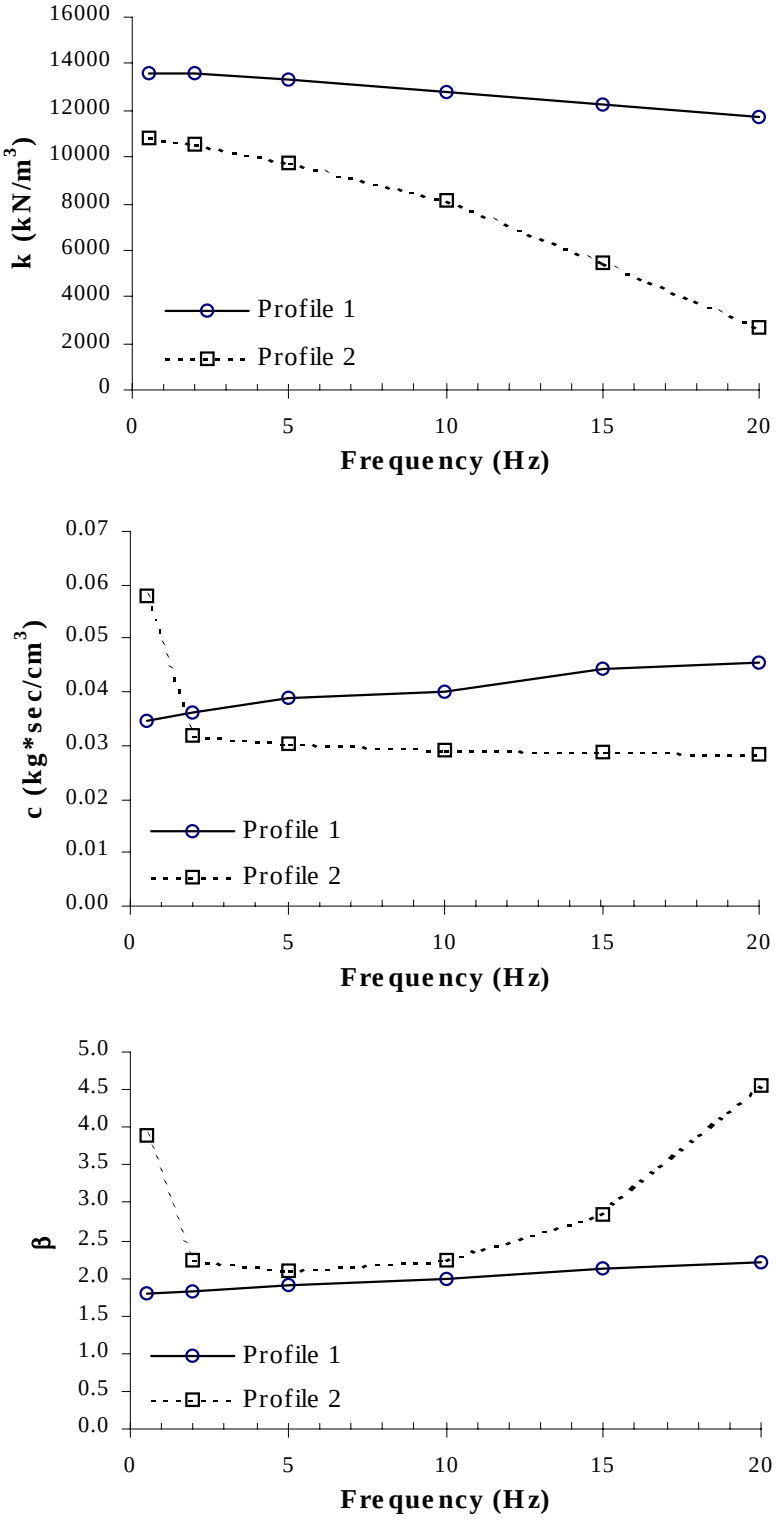

Figure 4. Foundation Parameters for Profiles 1 and 2

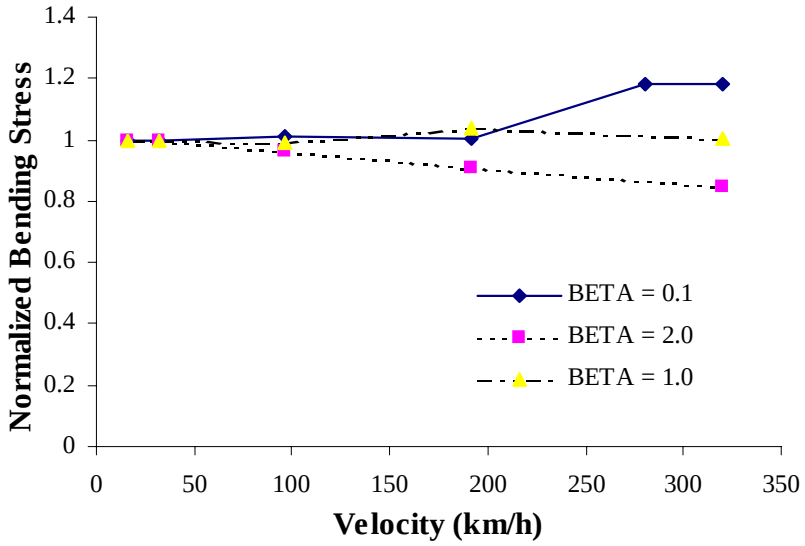

Figure 5. Effects of Radiation Damping and Speed on Peak Bending Stress

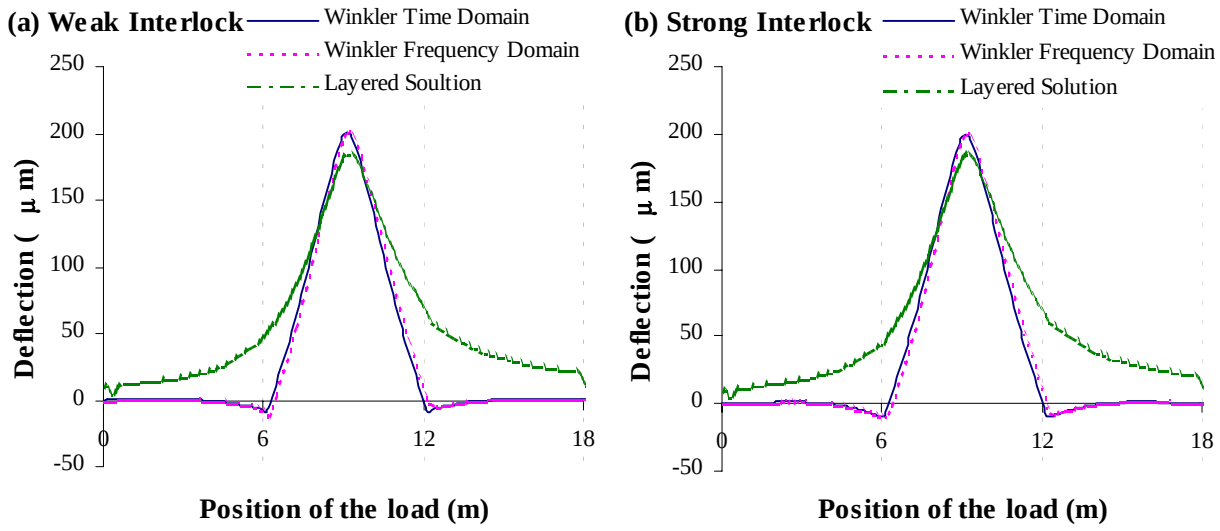

Figure 6. Interior Deflection Predictions Using Different Foundation Models Profile 1

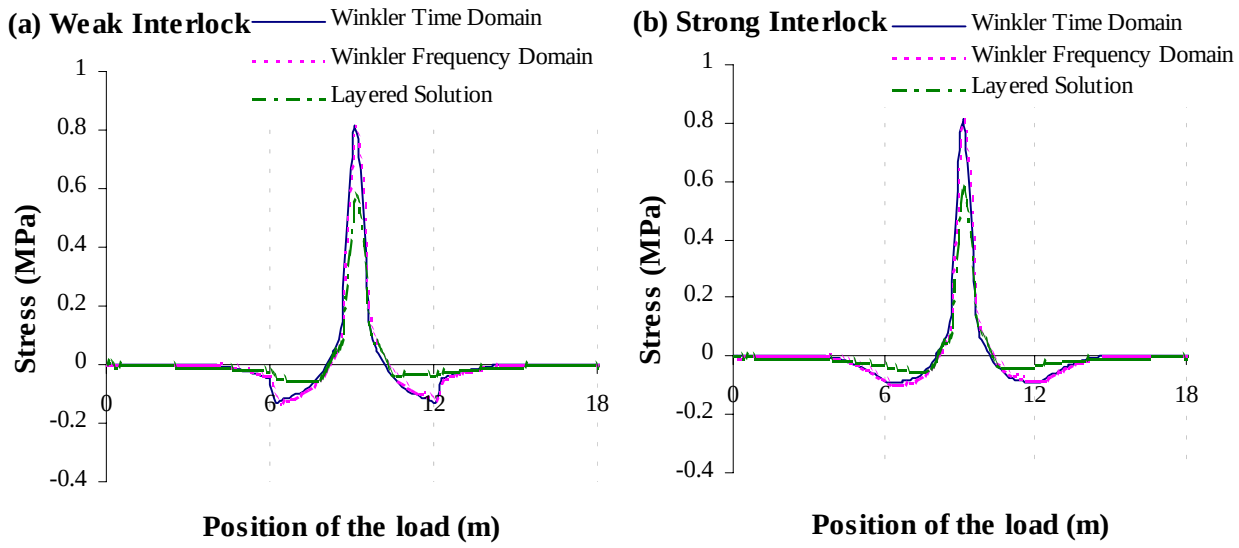

Figure 7. Interior Stress Predictions Using Different Foundation Models Profile 1

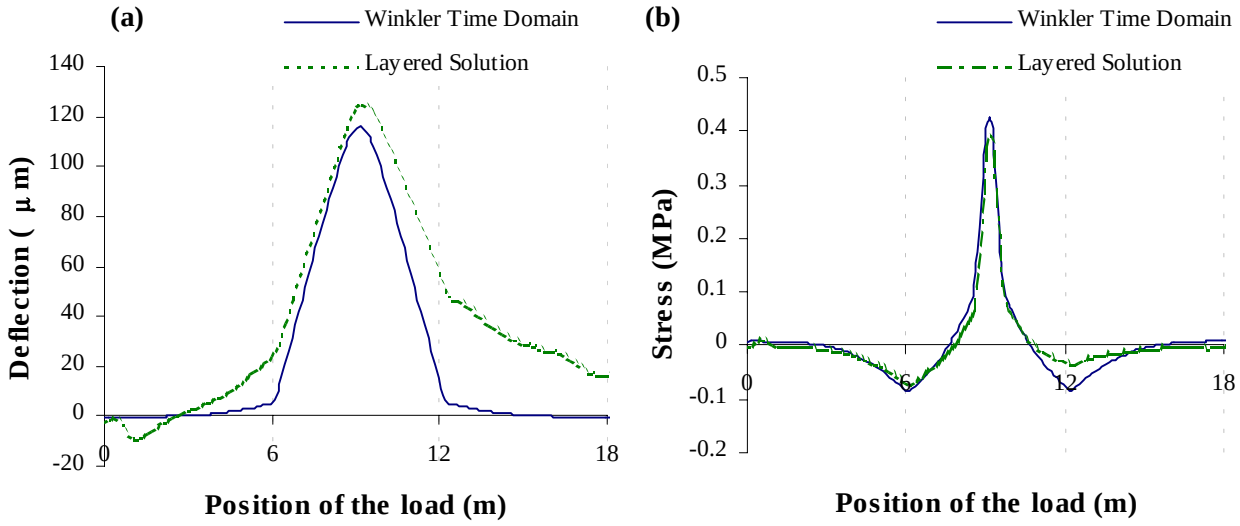

Figure 8. Interior Deflection and Stress Predictions Using Different Foundation Models - Profile 2

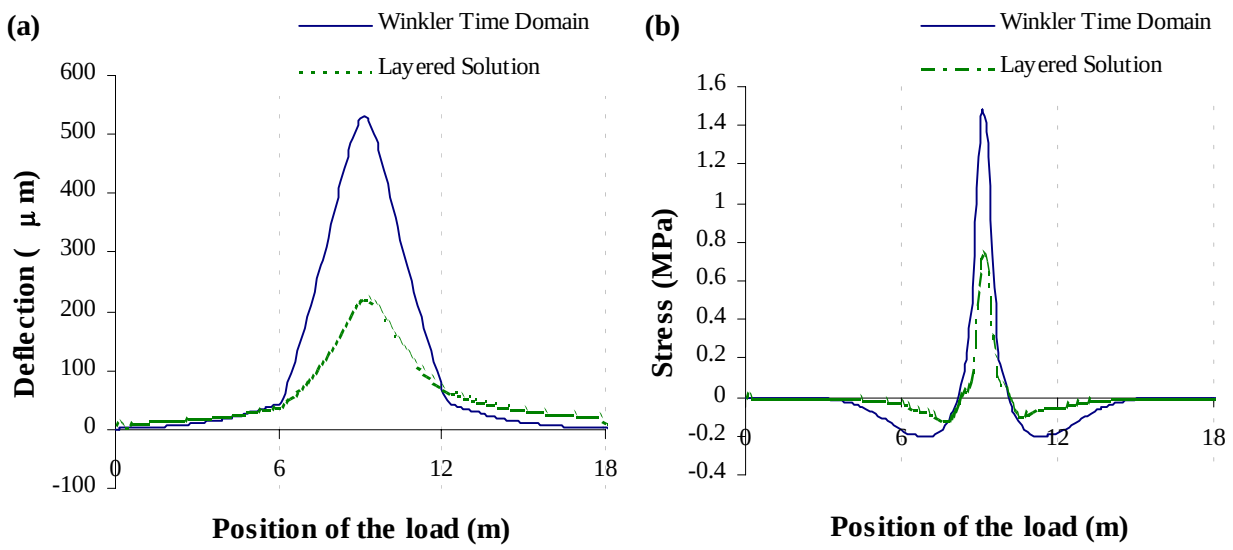

Figure 9. Edge Deflection and Stress Predictions Using Different Foundation Models - Profile 1

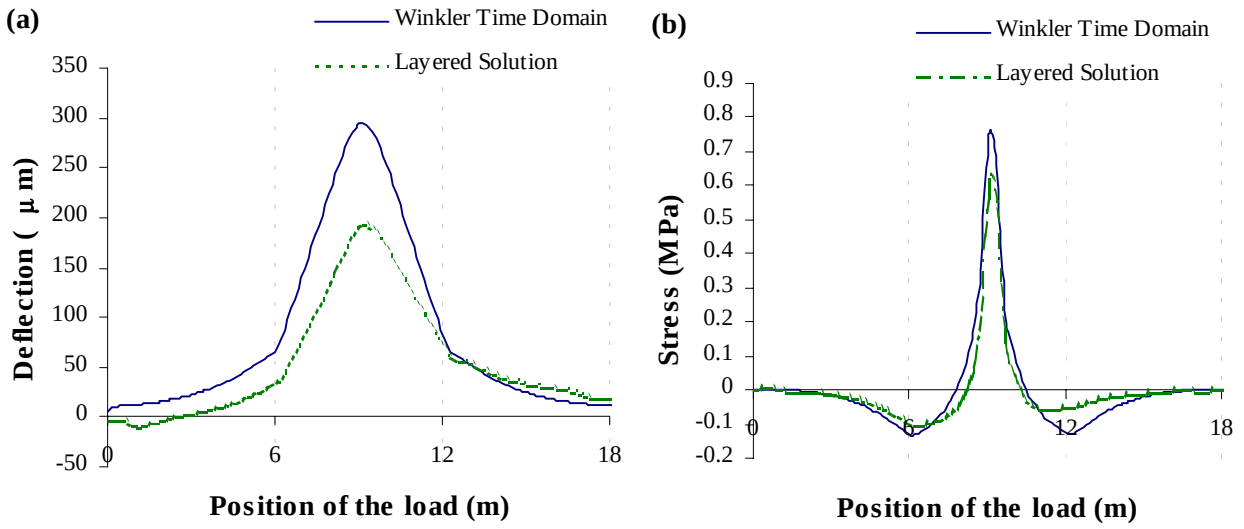

Figure 10. Edge Deflection and Stress Predictions Using Different Foundation Models - Profile 2

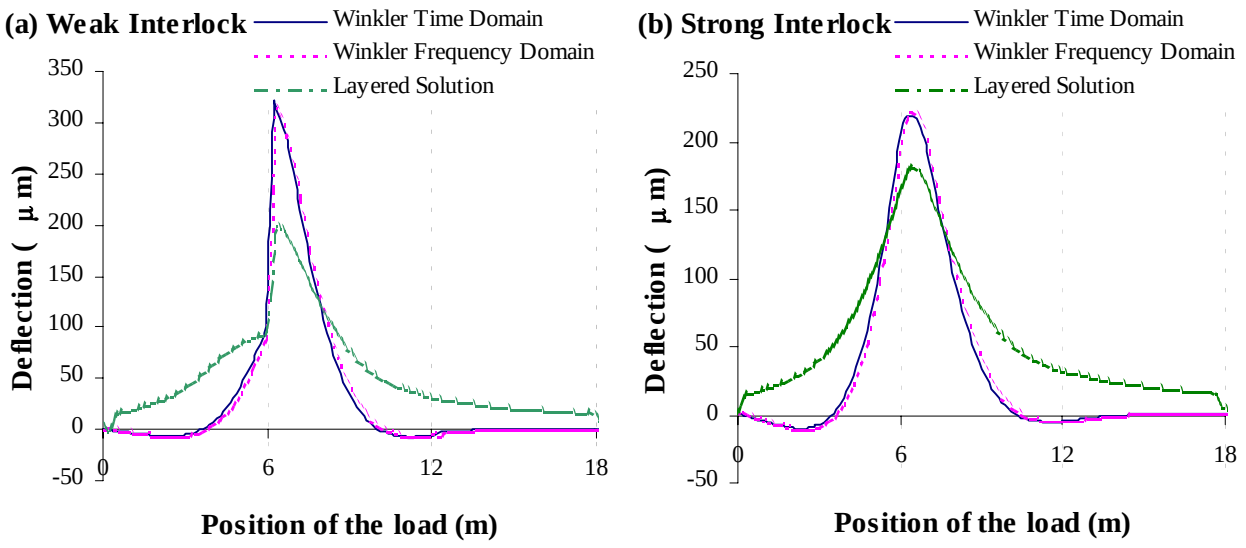

Figure 11. Transverse Joint Deflection Predictions Using Different Foundation Models - Profile 1

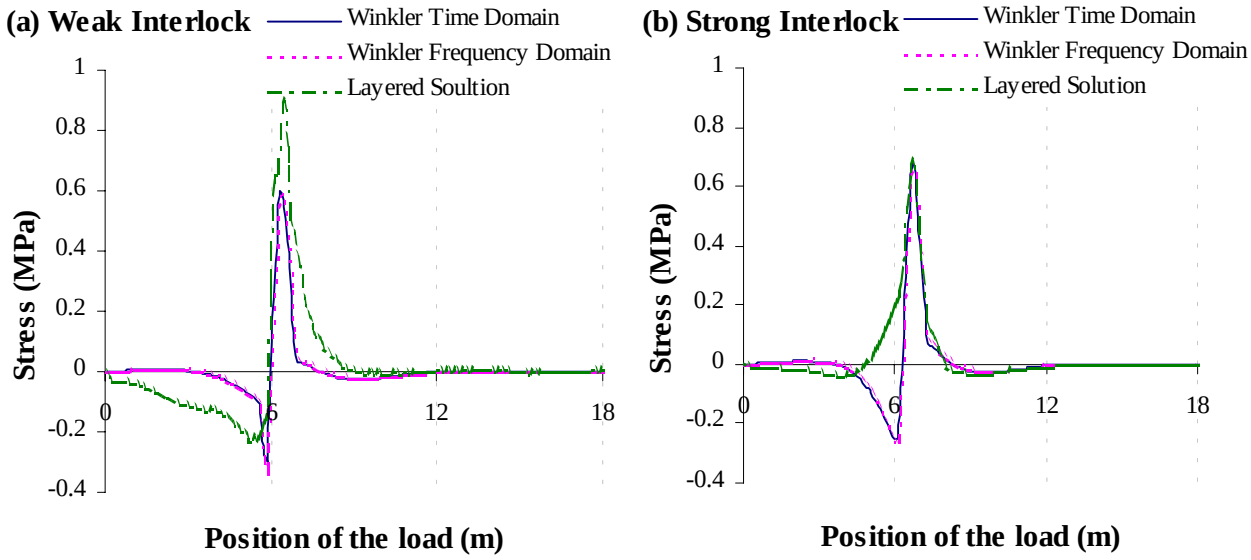

Figure 12. Transverse Joint Stress Predictions Using Different Foundation Models - Profile 1