

Quantifying Components of Permanent and Cyclic Warping in Concrete Pavements

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Abstract

Drying shrinkage causes both permanent and transient differential volume change because a portion of drying shrinkage is reversible. The reversible portion of drying shrinkage causes moisture warping, which varies seasonally with changes in ambient relative humidity. Differential drying shrinkage is caused by the irreversible portion of drying shrinkage and is permanent. Moisture warping and differential drying shrinkage in jointed plain concrete pavements can lead to cracking and premature fatigue failure if not properly accounted for in design. This study investigated how the ambient relative humidity in various locations around the country affects the amount of expected moisture warping and differential drying shrinkage in typical jointed plain concrete pavements. The goal of this research was to produce design aids for the prediction of moisture warping and differential drying shrinkage for standard pavement sections in different areas of the United States.

Background

A concrete slab subjected to a change in either temperature or moisture content will experience a change in volume. Increases in temperature or moisture are associated with expansion of the concrete, while decreases in either of these factors will result in contraction. If the concrete is restrained, these volumetric changes are limited, and stresses are induced. This restraint can come from tie bars, dowel bars, aggregate interlock, slab-base friction, and the slab's own self-weight. In a concrete pavement, these volumetric changes do not occur uniformly, but instead are generally more prevalent on the surface of the pavement, which is more directly exposed to the environment. The bottom of the pavement is more protected from fluctuations in temperature and moisture by the insulation of the concrete layer, and therefore tends to experience less change in volume. Due to the lack of uniformity in the volume change of the concrete through the depth of the slab, there is differential expansion or contraction through the thickness of the slab leading to slab bending.

When the top of the slab is warmer or wetter than the bottom (a positive gradient), the top of the slab expands, inducing a concave downward deflected shape. In some cases, this deflection is extreme enough to lift the middle of the slab off of the underlying layers, creating a gap under the middle of the slab (Shoukry 2000). Alternately, if the top of the slab is cooler or dryer than the bottom (a negative gradient), the slab will assume a concave upwards shape, sometimes lifting the corners off of the underlying layers (Hveem 1951). If no temperature or moisture gradient exists, the slab, in theory, should be perfectly flat and in complete contact with the underlying layer.

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Differential volume change in concrete slabs is due to the combined effects of temperature, moisture, and hydration. These effects manifest themselves as temperature curling, moisture warping, differential drying shrinkage, construction curl, and creep, all of which act to induce or reduce deformations. These differential volume changes can be quantified by an equivalent temperature difference required to deform a theoretically flat slab to the same shape as the actual slab. If a slab is not free to deform, stresses will be induced. These stresses can cause early age cracking or shorten the fatigue life of the pavement.

Equivalent Temperature Difference. The deformations induced by differential volume change are quantified by the equivalent linear temperature difference that would be required to deform an identical, but theoretically flat slab to the same shape as the curled/warped slab. This equivalent temperature difference is also called the total effective linear temperature difference or ΔT_{equiv} . The equivalent temperature difference is often divided into constituent components, which are considered separately in analysis, but often cannot be determined individually in field measurements. The components account for the effects of temperature and moisture gradients, construction curl, irreversible differential drying shrinkage, and creep (Rao and Roesler 2005). While an attempt can be made to separate the behavior of the slab into these components, this is a difficult task as all of these effects overlap and influence each other.

Temperature Effects. The top of the slab is directly exposed to the environment, and consequently, it experiences more frequent changes in temperature than the bottom of the slab. This causes a temperature gradient to develop through the thickness of the slab. During the day, the slab is often warmer on the surface, and cooler on the bottom, while at night, the reverse is true, leading to a slab profile which is curled downwards during the day and upwards at night. Many studies have found that the temperature gradient of a slab does not vary linearly with depth, but is instead non-linear (Teller and Sutherland 1935; Armaghani, Larsen et al. 1987; Poblete, Salsilli et al. 1988; Choubane and Tia 1995; Beckemeyer, Khazanovich et al. 2002). Whether linear or non-linear, this cyclical temperature gradient through the slab depth adds considerable bending stresses to a JPCP through its self-weight restraint.

Moisture Effects. Another factor impacting environmental stresses in JPCPs is caused by moisture within the concrete slab. Just as with temperature gradients, these moisture gradients also induce slab bending, and can lead to premature fatigue failure if not accounted for properly. While interrelated to concrete-specific material properties, these moisture effects can be divided into two categories: differential drying shrinkage and moisture warping. Both of these effects are caused by a drying shrinkage gradient through the thickness of the pavement.

Drying shrinkage is due to the loss of water from the concrete matrix to the ambient environment; particularly from the calcium silicate hydrate (C-S-H) gel pores and smaller capillary pores in the concrete's void system. Water loss is generally due to the evaporation of water from the concrete when the ambient relative humidity (RH) is less than the moisture content of the concrete, thereby creating a moisture gradient within the slab. Janssen (1987) found that the majority of moisture loss in concrete pavements occurs in the top two inches of the pavement, while the remaining thickness of the slab is at least 80% saturated, regardless of ambient conditions, and

the bottom of the slab is generally fully saturated. Within the top few inches, the moisture gradient is non-linear. Since most of the moisture movement in the slab occurs within the top few inches, the vast majority of drying shrinkage also occurs in this region, leading to differential volume changes in the slab (Wells, Phillips et al. 2006), which, in turn, causes a warped shape. Unlike temperature gradients, which generally follow a fairly set pattern of diurnal variation, moisture gradients are quite variable, depending on the ambient relative humidity and rain events. For this reason, moisture gradients are generally considered on a monthly basis (NCHRP 2003).

The shrinkage associated with these drying mechanisms may or may not be reversible. The deformations caused by the reversible portion of drying shrinkage are said to cause moisture warping. Differential drying shrinkage is attributed to the permanent portion of drying shrinkage. The amount of shrinkage which is reversible depends on the properties of the concrete, and the degree and duration of wetting. This phenomenon has been noted by some researchers (L'Hermite 1947; L'Hermite, Chefdeville et al. 1949; Shacklock and Keene 1957; Helmuth and Turk 1967; Granger, Torrenti et al. 1994), but it is still not well known or understood, and no prediction equation for the amount of shrinkage which will be reversible currently exists. It is generally assumed that half of all shrinkage is reversible, though this has been shown experimentally not to be the case (L'Hermite 1960; Lederle 2011). For a typical concrete paving mix undergoing minimal moisture curing, it has been found that only 30% of shrinkage is reversible upon rewetting, while the other 70% is permanent (Lederle 2011).

Proper estimation of the amount of moisture warping in a pavement is important because the equivalent temperature difference due to moisture warping is often of the same magnitude as that due to temperature curling (Hveem and Bailey 1957). It has been found that neglecting the effects of moisture warping results in an unconservative slab design. (Rao, Barenberg et al. 2001).

Another factor contributing to shrinkage in concrete is due to autogenous shrinkage, which is the self-desiccating effect of the hydration process due to the withdrawal of water from the capillary pores in the paste matrix. This typically occurs in the first 35 days after casting (ACI 2008). Due to its uniformity, autogenous shrinkage does not contribute to bending deformation in the slab.

Construction Curl. Construction curl is due to the presence of a temperature gradient at the time of initial set of the concrete. When a slab is cast with a temperature gradient, it cannot deform as it usually would because the concrete is plastic. Instead, it sets as a flat slab. If the temperature gradient through the cross section of the slab at the time of initial set differs from that at the time of casting, the slab will react by deforming. Therefore, if the slab is of uniform temperature it will behave as though there is a temperature gradient present of the same magnitude as the gradient that was present during the final set of the concrete. Similarly; the slab will only be flat and undeformed when it encounters a temperature gradient that is the same as it was at the time the slab was cast (disregarding all other differential volume change factors) (Yu and Khazanovich 2001). Since most slabs are cast during the day (in the presence of a positive temperature gradient), at a zero temperature gradient condition, the slabs behave as though the positive gradient has been removed (i.e. as though a negative gradient has been applied). This positive temperature gradient at the time of set is exacerbated by heat produced internally in the concrete as part of the chemical reactions associated with hydration.

Creep. Creep is defined as deformation due to sustained loading; in the case of a concrete slab, the load is the weight of the slab itself. Curl, warp, shrinkage and construction curl all induce deformations, which are countered by the self-weight of the slab. Regardless of how the other components cause the slab to deform, creep will always work against those deformations, which is why it is subtracted from all of the other components of the equivalent temperature difference. Studies on the influence of creep on the other factors of differential volume change have found that creep counteracts shrinkage (Altoubat and Lange 2001), curling (Teller and Sutherland 1935; Rao, Barenberg et al. 2001; Sondag and Snyder 2003), and warping (Teller and Sutherland 1935; Bissonnette, Attiogbe et al. 2007). Reduced deformations due to creep also cause stress relaxation, which can increase the strain capacity of the concrete (Altoubat and Lange 2001). One study found that if creep was neglected in estimating the total deformations induced by curl, warp, shrinkage and built-in curl, then edge displacements would be almost double those expected if creep were considered. Likewise, stresses computed without considering the stress relaxation effects of creep were three times higher than when creep was included in calculations (Lee, Lange et al. 2011). These creep effects are more prevalent for early-age deformations (construction curl) than for slower moving, long-term differential volumetric changes (warping and differential drying shrinkage) as the ability for concrete to creep is reduced as the concrete further hydrates.

Differential Volume Change in the MEDPG. The MEDPG uses two terms to account for differential volume change when calculating stresses: equivalent temperature gradient (ETG) and built-in curl (BIC). The ETG term is used to quantify the effects of curl due to actual temperature gradients and warp due to moisture gradients, while the BIC term is the equivalent temperature gradient due to construction curl, the permanent component of warp due to differential drying shrinkage, and creep, collectively called built-in curl (NCHRP 2003). The relation of these terms is given in Figure 1.

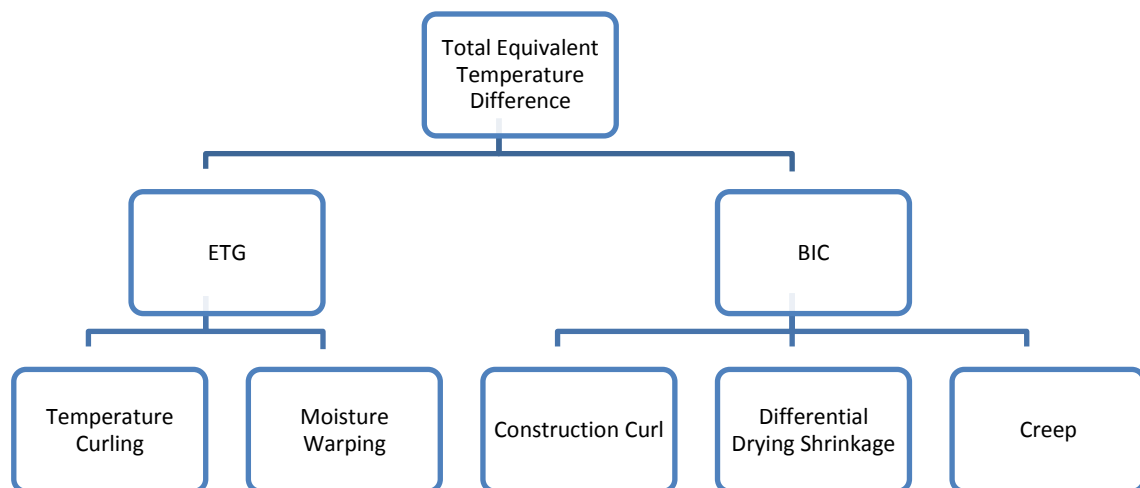


Figure 1: Accounting for equivalent temperature difference in the MEDPG.

To calculate the total ETG, the actual temperature gradient from climatic data is summed with the equivalent temperature gradient due to moisture warping. The

moisture warping is predicted based on an assumed linear shrinkage distribution through the top few inches of the pavement due to variations in ambient relative humidity, also taken from climatic data. This value is then adjusted to account for the age of the slab. Temperature curling is computed hourly, while moisture warping is computed monthly (NCHRP 2003). It has been found that the moisture warping model currently used in the MEPDG predicts unrealistic values for the equivalent temperature difference due to moisture gradients (Lederle and Hiller 2011).

Accounting for Moisture Effects

Failure to properly account for the effects of differential volume change in the design of jointed plain concrete pavements (JPCP) can lead to early age cracking and premature fatigue failure. The goal of this research was to use newly proposed models for the prediction of moisture warping and differential drying shrinkage to produce design aids. These design aids will help an engineer determine quickly and easily how much moisture warping and differential drying shrinkage a standard pavement will experience in different regions of the country. These design aids use contour lines to show the variation in warping or differential drying shrinkage for a pavement of a given thickness and water to cement ratio throughout the United States. Additionally, by comparing design aids for different pavement conditions, it can be seen how changing certain parameters, such as pavement thickness, will affect the expected moisture warping and differential drying shrinkage in various geographic regions. Though temperature curl, construction curl, and creep also play an important role in determining the warped shape of the pavement, their prediction is such that design aids cannot be generated to show regional variations. Therefore they were considered to be outside the scope of this project.

Moisture Warping and Differential Drying Shrinkage Models. To predict the expected amount of moisture warping and differential drying shrinkage, models developed by Lederle and Hiller (2011) were used, see Equation 1 and Equation 2 respectively. These models predict the expected equivalent linear temperature difference based on an assumed shrinkage distribution through the shrinkage zone (h_s) of the slab which follows the curve of a quarter of an ellipse, as shown in Figure 2. This elliptical shrinkage distribution improves upon the rectangular shrinkage model (Eisenmann and Leykauf 1990) and the triangular model (NCHRP 2003) developed by previous researchers. It should be noted that these models are not intended to predict early age behavior, but rather behavior during the service life of the pavement. These models use ambient relative humidity as the predictor of the concrete's internal relative humidity distribution, and ultimately, warping and differential drying shrinkage. Both models depend on a reversible shrinkage factor to determine the amount of drying shrinkage which causes warping versus differential drying shrinkage. The amount of reversible shrinkage expected for concrete typically used in paving applications is 30% (Lederle 2011). Currently, in the MEPDG, the model used to compute moisture warping yields improbable values (Yu et al. 2002), while there is no model to predict differential drying shrinkage (Lederle and Hiller 2011).

$$ETG_{warp} = \frac{\phi R \omega \varepsilon_u h_s [-3h(-4 + \pi) - 20h_s + 6\pi h_s](1 - \mu)}{2h^2 \alpha} \quad \text{Equation 1}$$

$$\frac{ETG_{DDS}}{(1-\phi)\mathcal{R}\omega\varepsilon_u h_s[-3h(-4+\pi)-20h_s+6\pi h_s](1-\mu)} = \frac{2h^2\alpha}{2h^2\alpha} \quad \text{Equation 2}$$

Where:

ω is a factor to account for the amount of total shrinkage due to drying shrinkage based on the water to cement ratio (w/c) of the concrete, see Equation 3

$$\omega = \begin{cases} 3.6308(w/c) - 0.5675, & w/c \leq 0.36 \\ 0.5463(w/c) + 0.4901, & w/c > 0.36 \end{cases} \quad \text{Equation 3}$$

Where:

$\mathcal{R}$ is the relative humidity adjustment factor based on the monthly average ambient relative humidity (RH) as a percent, see Equation 4.

$$\mathcal{R} = 1 - (RH/100)^3 \quad \text{Equation 4}$$

Where:

h is the depth of the pavement, in inches

h_s is the depth of the shrinkage zone, in inches

ε_u is the ultimate shrinkage of the concrete, in $\mu\epsilon$

μ is poison's ratio of the concrete

ϕ is reversible shrinkage factor, recommended to be 0.3 (Lederle

2011)

α is the coefficient of thermal expansion of the concrete, in

inch/inch/ $^{\circ}\text{F}$

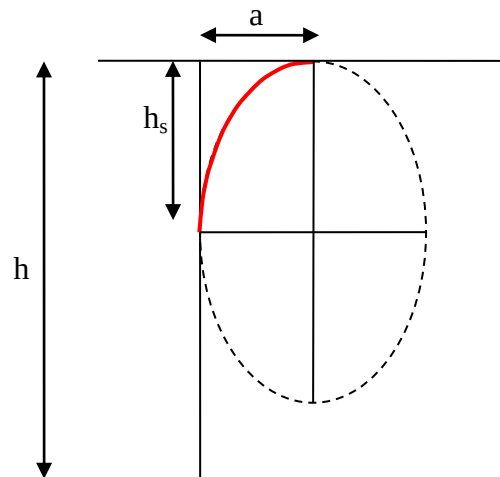


Figure 2: Non-linear shrinkage gradient modeled as a quarter of an ellipse where h is the pavement thickness, h_s is the depth of the shrinkage zone, and a is the amount of drying shrinkage at the top of the slab, equal to $\phi\mathcal{R}\omega\varepsilon_u$ for moisture warping and $(1-\phi)\mathcal{R}\omega\varepsilon_u$ for differential drying shrinkage.

Relative Humidity Data. Data for the ambient relative humidity for each month of the year was obtained from the National Oceanic and Aeronautics Administration (NOAA) climatic database for 256 sites scattered around the United States. At each

location, the mean value of average morning (AM) and afternoon (PM) relative humidity in each month was given based on historic data. Depending on the location, the historical climatic record stretched back between 6 and 96 years (NOAA 2008). To obtain an average daily value of relative humidity from morning and afternoon values, a weighted average was used, as given in Equation 5 (Daly and Taylor 2001).

$$\text{Daily average RH} = 0.65 * RH_{AM} + 0.3 * RH_{PM} \quad \text{Equation 5}$$

Where :

RH_{AM} is the average morning relative humidity

RH_{PM} is the average afternoon relative humidity

The annual average of the average daily relative humidity was used as an input in the proposed models to compute the expected equivalent temperature in a pavement of given thickness and water to cement ratio. Calculations were also made on a monthly basis, as in the MEPDG, but there was insufficient variation in most locations to justify monthly maps. For each pavement case, two maps were created, one for moisture warping and one for differential drying shrinkage. For the case of $\phi = 0.5$, the map is applicable to both moisture warping and differential drying shrinkage half of the overall drying shrinkage is attributed to each.

Correct determination of the depth of shrinkage zone is critical to accurately predicting warping and differential drying shrinkage. Given that the actual depth of the shrinkage zone varies by location and is not the conventionally assumed two inches in all areas, design aids were created as well for various values of h_s , water to cement ratio, ϕ and ϵ_{su} .

Design Aids for the Predication of Moisture Warping and Differential Drying Shrinkage. Design aids for the prediction of the amount of expected moisture warping and differential drying shrinkage in different regions of the United States were generated using Equation 1 and Equation 2 respectively and are shown in Figure 3 through Figure 9. These design aids were generated for many common JPCPs, not all of which are shown here due to space constraints, and are represented as an equivalent temperature difference (in °F) to account for the bending moment created by the moisture gradient in the slab.

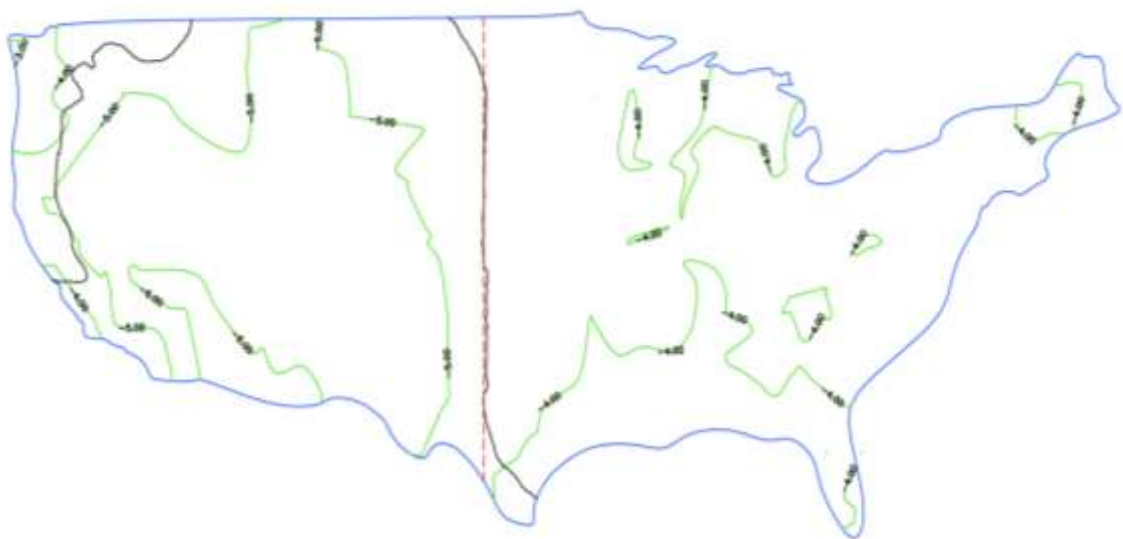


Figure 3: Moisture warping for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 3\text{in}$, $\phi = 0.3$ and $\epsilon_{su} = 600\mu\epsilon$.

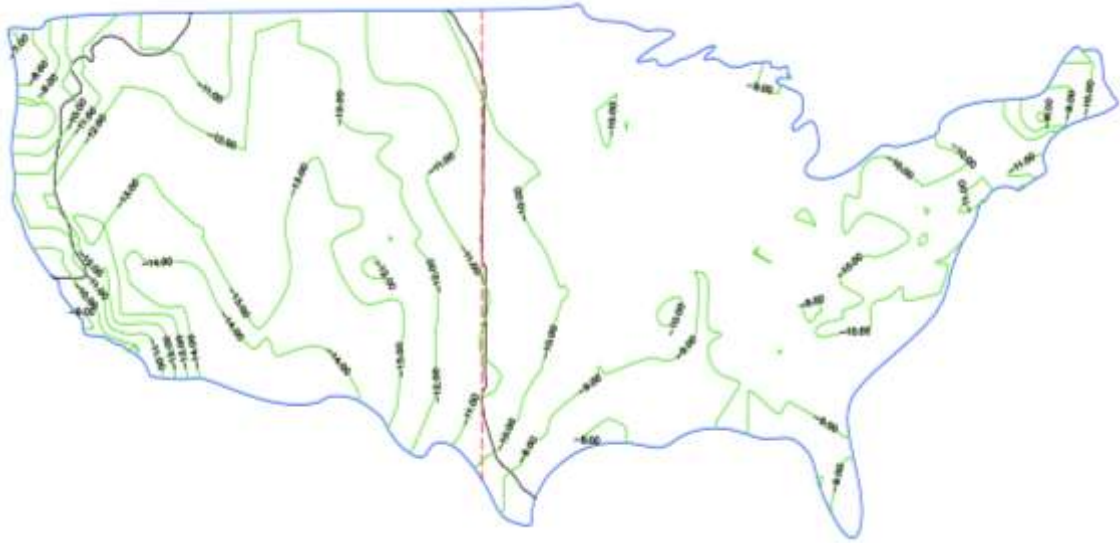


Figure 4: Differential drying shrinkage for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 3\text{in}$, $1-\phi = 0.7$ and $\epsilon_{su} = 600\mu\epsilon$.

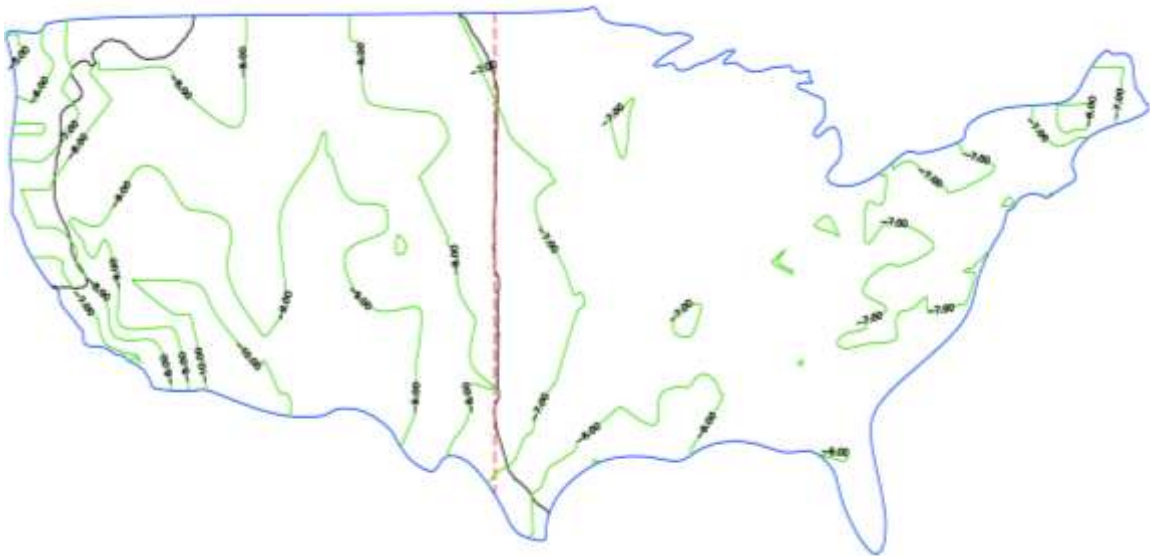


Figure 5: Moisture warping for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 2\text{in}$, $\phi = 0.3$ and $\epsilon_{su} = 600\mu\epsilon$.

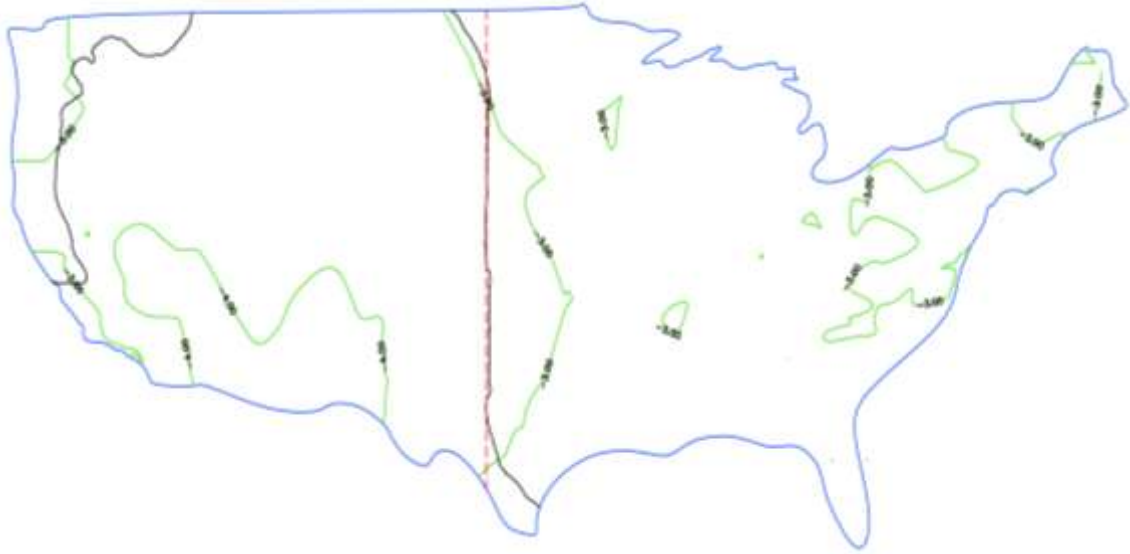


Figure 6: Differential drying shrinkage for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 2\text{in}$, $1-\phi = 0.7$ and $\epsilon_{su} = 600\mu\epsilon$.

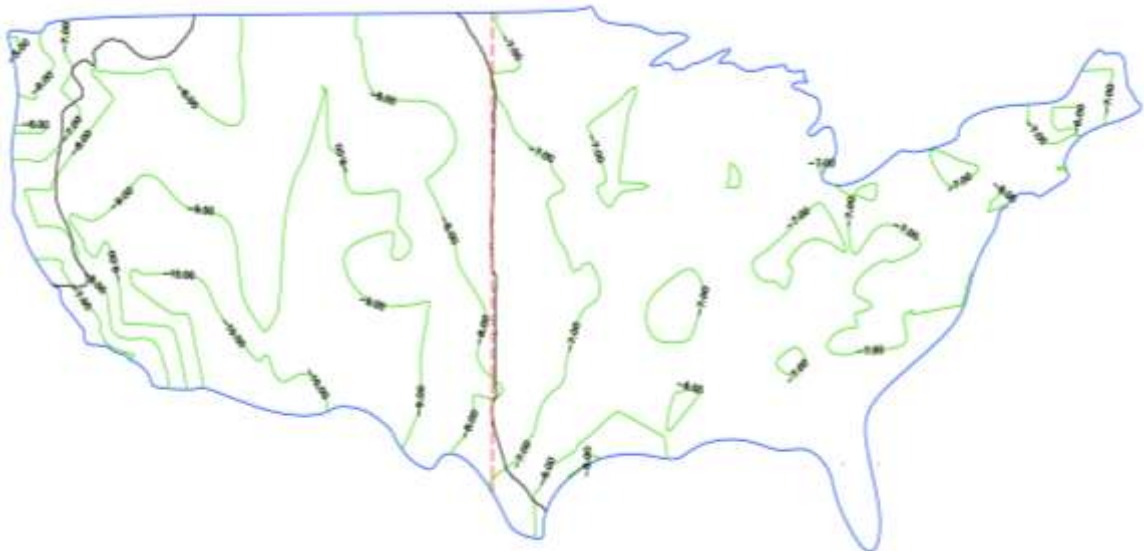


Figure 7: Moisture warping for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 3\text{in}$, $\phi = 0.7$ and $\epsilon_{su} = 1200\mu\epsilon$.

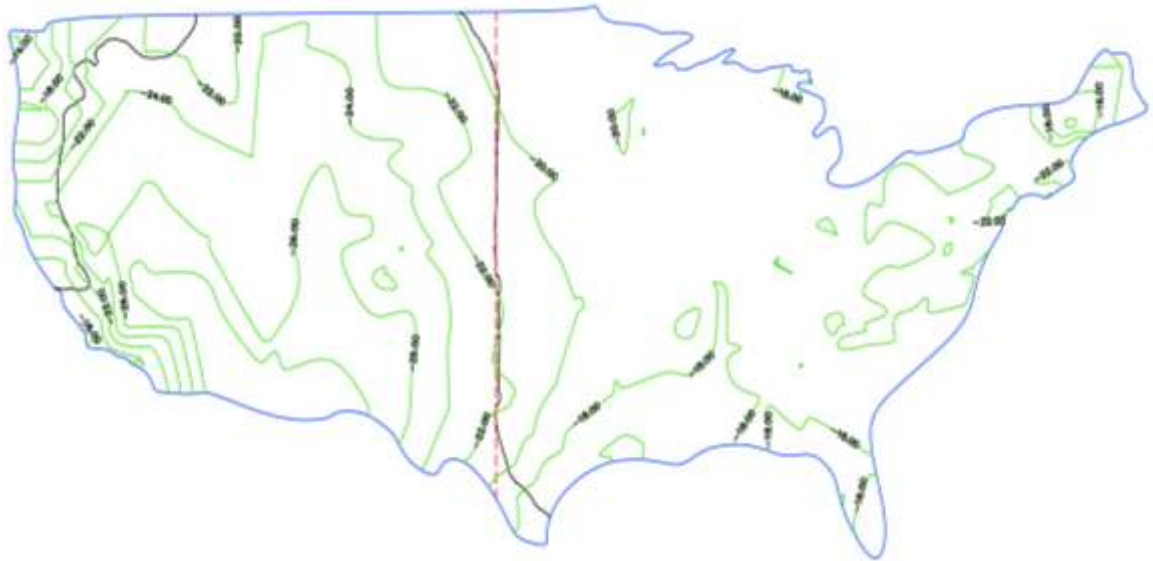


Figure 8: Differential drying shrinkage for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 3\text{in}$, $1-\phi = 0.7$ and $\epsilon_{su} = 1200\mu\epsilon$.

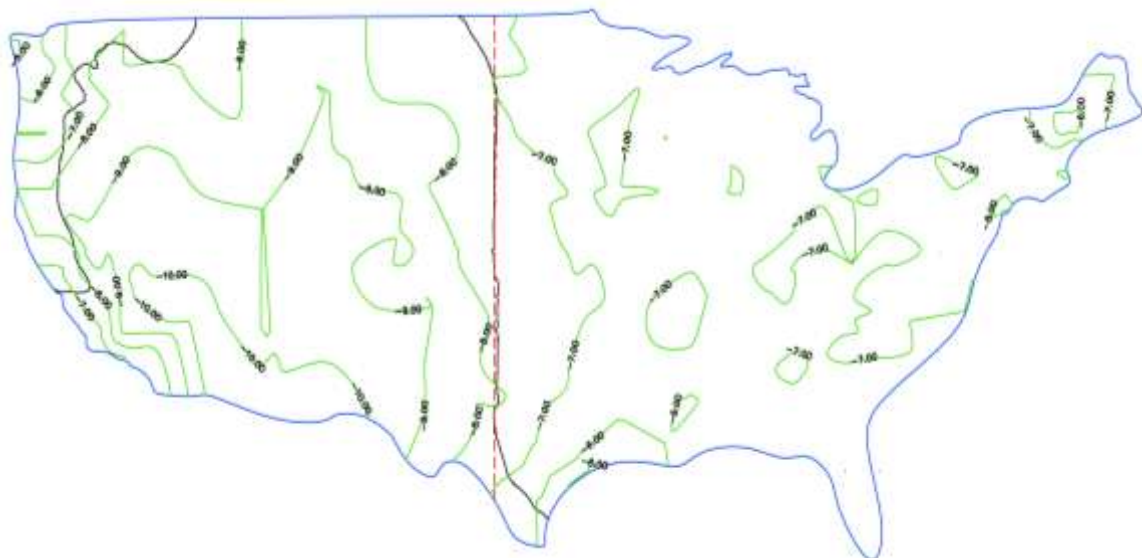


Figure 9: Moisture warping and differential drying shrinkage for a pavement with $w/c = 0.4$, $h = 10\text{in}$, $h_s = 3\text{in}$, $\phi = 1-\phi = 0.5$ and $\epsilon_{su} = 500\mu\epsilon$.

Analysis

From the design aids generated in Figure 3-9, it can be seen that the majority of the variation in the amount of predicted warping and differential drying shrinkage occurs on the west coast, in the southwest, and in New England region. This is because those locations have areas of very different climates close together due to the effects of topography and moisture from the oceans. The highest level for both moisture warping and differential drying shrinkage occurs in the southwest due to its dry ambient relative humidity. Conversely, New England and the northwest have less warping and differential drying shrinkage, as they are relatively wetter regions. The largest amount of moisture warping and differential drying shrinkage generally occurs in the arid region of the United States. This region was defined in 1879 by geologist

John Wesley Powell as areas which receive less than an average annual rainfall of 20 inches, as shown in

Figure 10, and is delineated on the design aids as well. Powell (1879) noted that the eastern boundary of this arid region corresponds well with the 100th meridian, which is also shown on the design aids. This indicates that special attention should be given to warping and differential drying shrinkage issues in this arid region as the associated shift in the equivalent temperature gradients from these factors can radically change the timing and location of fatigue crack development in JPCPs (Hiller and Roesler 2008).



Figure 10: Map defining John Wesley Powell’s “Arid Region” in the western United States (Powell 1890).

Changing the input parameters in the warping and differential drying shrinkage models greatly affects both the value of equivalent temperature difference as well as the spacing of the contour lines. Increasing the water-to-cement ratio causes more warping and differential drying shrinkage overall, and leads to more tightly spaced contours since this would increase the proportion of drying shrinking expected in comparison to the total shrinkage found using ASTM C157 testing

(ASTM 2008). Thinner pavements have more warping and differential drying shrinkage, as well as closer spaced contour lines since the drying shrinkage zone would occupy a larger portion of the slab depth, thereby causing more eccentricity and bending of the slab. Pavements with a larger amount of reversible shrinkage will have more warping but less differential drying shrinkage. A concrete mix with a higher value of total shrinkage will experience increased moisture warping and differential drying shrinkage, and more closely spaced contours.

Changing the depth of the shrinkage zone had the most drastic effect on both the magnitude of the warping and differential drying shrinkage as well as its spatial distribution. A map with a depth of the shrinkage zone equal to one inch will have almost no variation as the entire country will show an equivalent temperature gradient of -2 to -3°F . In contrast, a map with a depth of the shrinkage zone equal to three inches shows significant variation in the expected equivalent temperature distribution around the country. A deeper shrinkage zone would result in even higher values of both differential drying shrinkage and moisture warping, as both quantities increase with the square of the depth of the shrinkage zone. Additionally, the equivalent temperature differences values are much larger. This is especially important as in the MEPDG, h_s is one input which is often left at the default setting of two inches, despite evidence that this is often not the case (Kim and Lee 1999; Qin 2011).

Conclusions

Thermal and moisture differences through the depth of a concrete slab can result in cyclical curling, as well as both reversible and permanent warping deformations. These deformations can radically affect JPCP fatigue performance as the slab undergoes lift-off and subsequent stress development due to its own self-weight. Based on new differential drying shrinkage and warping models, design aids for the determination of the equivalent temperature difference throughout the United States were generated.

These design aids are intended for use by engineers to predict the amounts of moisture warping and differential drying shrinkage expected in a JPCP since the current MEPDG design procedure does not account for moisture warping on an appropriate scale and neglects differential drying shrinkage completely. Though implementing a new model in the MEPDG will take time, these design aids can be used immediately. To account for the expected amount of warping and/or differential drying shrinkage, the equivalent temperature difference from the design aid can be directly added to the amount of built-in curl anticipated. This will neglect any time-dependent effects of warping. However, including the overall deformed shape and any associated damage is an improvement upon the current values predicted by the MEPDG.

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