Rigid Pavement Backcalculation Using Differential Evolution

Kasthurirangan Gopalakrishnan[1] and Halil Ceylan[2]

Abstract

The backcalculation of pavement layer moduli from Falling Weight Deflectometer (FWD) measured surface deflections is a challenging task. It can also be formulated as a global optimization problem with the objective of finding the optimal pavement layer moduli values that minimize the error between measured and computed surface deflections. Over the years, several backcalculation methodologies have been developed including the use of soft computing techniques such as Neural Networks (NNs), Genetic Algorithms (GAs), etc. In this paper, Differential Evolution (DE), a stochastic parallel direct search evolution strategy optimization method is integrated with rapid surrogate mapping of Finite Element (FE) solutions through Neural Networks (NNs) in developing an automated rigid pavement backcalculation toolbox.

Introduction

Non-Destructive Testing (NDT) of in-service pavements using Falling Weight Deflectometer (FWD) equipment is carried out to measure the deflection response of the pavement structure to applied dynamic load that simulates a moving wheel. FWD testing is often preferred over destructive testing methods because it is faster than destructive test methods and does not entail the removal of pavement materials through coring. The deflection data collected with FWD equipment (see Figure 1) can provide both qualitative and quantitative data about the strength of a pavement at the time of testing. The raw deflection data directly beneath the load plate sensor provides an indication of the strength of the entire pavement structure. Likewise, the raw deflection data from the outermost sensor provides an indication of subgrade strength (FAA 2004). Many studies have addressed the interpretation of FWD pavement deflection measurements as a tool to characterize pavement-subgrade systems (Bush and Baladi 1989, Tayabji and Lukanen 2000).

The measured deflections from FWD can be correlated to in-situ material stiffness of each layer in the pavement structure through a procedure known as *backcalculation* or *inverse analysis*. Determination of in-situ material stiffness is essential in assessing the structural condition of exiting pavement for estimation of pavement remaining life and in determining the thickness of new overlay.

[1] Research Assistant Professor, Iowa State University, Ames, IA 50011, USA, Email: rangan@iastate.edu (corresponding author)

[2] Associate Professor, Iowa State University, Ames, IA 50011, USA, Email: hceylan@iastate.edu



Figure 1. Falling Weight Deflectometer (FWD)

The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. The most common approach in current commercial backcalculation software requires inputting initial seed modulus which is an assumed layer modulus for an iterative process. Thus, the reliability of the final optimized solution is dependent upon the initial seed modulus.

Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons. Researchers working in this area continue to explore advanced hybrid approaches to pavement moduli backcalculation with the main aim of facilitating speed of convergence, robustness, and computational efficiency (Gopalakrishnan et al. 2010).

Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms (Muttil and Liong 2004). Differential Evolution (DE) is one such popular stochastic parallel direct search evolutionary optimization strategy.

It is acknowledged that there are already closed-form solutions available for determining concrete slab modulus and the subgrade k value from nondestructive FWD testing. A major motivation for proposing the hybrid DE-based pavement inverse analysis approach is that it represents state-of-the-art backcalculation approach and yet the solutions are obtained rapidly in real-time or quasi real-time. Since it is a global stochastic optimization technique, it is almost always guaranteed to provide near-global solutions. This is especially useful in backcalculating concrete pavement layer moduli using field FWD data for which closed-form solutions may not be ideally suited. Field FWD measurements have inherent uncertainties, imprecision, and missing information which are handled best by soft computing techniques like DE. Another motivation was to introduce this popular evolutionary

optimization strategy, which has seen successful applications in diverse areas of science and engineering, to the pavement engineering community.

This paper proposes a hybrid DE-based heuristic optimization algorithm for analysis of rigid pavement non-destructive test data and backcalculation of concrete pavement layer moduli. The hybrid approach presented in this paper represents one of the most recent developments in backcalculating the mechanical properties of concrete pavement systems. This innovative approach takes advantage of the combined efficiency and accuracy achieved by integrating advanced pavement numerical modeling schemes, computational intelligence based surrogate mapping techniques, and heuristics based global optimization strategies, and yet provides a user-friendly pavement evaluation toolbox for the pavement engineer to use on a real-time basis for accurate infrastructure evaluation.

Differential Evolution (DE)

Differential Evolution (DE) is a simple and efficient heuristic for global optimization over continuous spaces. Storn and Price (1995) proposed DE as a new heuristic approach that has the ability to handle non-differentiable, nonlinear, and multimodal cost functions. It has the ability to lend itself very well to parallel computation, has few control variables to steer the minimization, and has good convergence properties. It has been demonstrated through several benchmark optimization problems that DE converges faster and with more certainty than many other acclaimed global optimization methods.

Owing to its simplicity and advantages over other optimization methods, DE has been successfully applied in different fields for solving complex, nonlinear, non-differentiable, and non-convex optimization problems such as in reservoir system optimization (Reddy and Kumar 2007), distribution network reconfiguration problems (Chiou et al. 2004), electrical power distribution (Chang and Chang 2000), efficient parameter estimation in bio-filter modeling (Babu and Angira 2006), structural system identification (Tang et al. 2008), optimal design of gas transmission network (Babu et al. 2003), asphalt pavement inverse analysis (Gopalakrishnan and Khaitan 2010), etc.

A brief description of the DE algorithm is as follows. A fixed number of vectors, within a population of potential solutions in an n-dimensional search space, are randomly initialized (*initialization*), and then evolved over time to explore the search apace to locate the objective function minima. At each iteration, new vectors are generated by the combination of vectors randomly chosen from the current population (*mutation*). The produced vectors are then mixed with a predetermined target vector (*recombination*) and a trial vector is produced. For the next generation, the trial vector is accepted if, and only if, it yields a reduction in the cost function value (*selection*).

DE is considered to have some attractive characteristics compared to Genetic Algorithms (GAs) and other heuristic algorithm such as the Particle Swarm Optimization (PSO). For instance, DE uses a simple differential operator and one-to-one competition scheme compared to the complicated generic search operators employed in GA. In GA, previous knowledge of the problem is destroyed once the population changes and in the case of PSO, a secondary archive is needed, whereas in DE, the memory of good solutions is retained and it has constructive cooperation between individuals in the population (Tang et al. 2008).

The population size (NP), scaling factor (F), and crossover control constant (CR) are the three important control variables in the DE algorithm. A reasonable choice for NP is 5-10 times that of dimensionality (D) of the problem, but NP must be

at least 4 to ensure that DE will have enough mutually different vectors with which to work. In general, F and CR affect the convergence speed and robustness of the search process. A value of 0.5 is usually a good initial choice for F while values smaller than 0.4 or greater than 1.0 are only occasionally effective. Similarly, a value of 0.1 is a good first choice for CR, although values of 0.9 or 1.0 may be tried out first to see if a quick solution is possible. The optimal values of F and CR depend both on the characteristics of objective function and the population size and thus, the selection of optimal values for F and CR is application-specific. For the current study, optimal values of NP, F, and CR were chosen as 10, 0.85, and 1.0, respectively, based on preliminary parametric sensitivity analysis results. The DE module in the proposed approach was adapted from the MATLAB® version of the DE code presented by Price et al. (2005).

FE-NN-DE Hybrid Approach to Rigid Pavement Backcalculation

The elastic modulus of the slab, E, and modulus of subgrade reaction, k, are the two most important backcalculated concrete pavement properties. Over the years, researchers have developed many different methodologies for backcalculation of concrete pavement properties from FWD measurements, including the AREA method for rigid pavements (Ioannides et al. 1989, Ioannides 1990, Barenberg and Petros 1991), ILLI-BACK (Ioannides 1994), graphical solution using ILLI-SLAB (Foxworthy and Darter 1989), use of regression analysis to solve AREA method for rigid pavements (Hall 1992, Hall et al. 1996), use of best fit algorithm to find radius of relative stiffness (l) (Hall et al. 1996, Smith et al. 1996), among others.

Based on a backcalculation study of concrete pavement properties using 277 deflection basins obtained from the Denver International Airport (DIA), Rufino et al. (2002) studied the effect of slab modeling (number of layers, interface condition, and model type) as well as effect of different methodologies and sensor configurations on backcalculated pavement properties. It was found that backcalculated slab modulus of elasticity is lower on average when the pavement layers on top of subgrade are bonded versus unbonded interface. Higher backcalculated k-values are obtained when the slab is modeled as plate compared to modeling the slab as elastic layer. Modeling of the slab and base as elastic layers seem to yield more reasonable backcalculated results since the interface bonding condition can be reflected both in the backcalculated slab elastic modulus and subgrade k-value.

Over the past decade, the use of computational intelligence techniques in pavement systems modeling, analysis, and design, has become increasingly common. Ceylan (2002) employed NNs in the analysis of concrete pavement systems and developed NN-based design tools that incorporated the ISLAB 2000 (Tabatabaie and Barenberg 1978, Khazanovich 1994, Khazanovich et al. 2000) finite element solutions into routine practical design at several orders of magnitude faster than ISLAB 2000.

Khazanovich and Roesler (1997) developed a program called DIPLOBACK for backcalculation of moduli values of composite pavements based on NNs. NNs have also been applied along with dimensional analysis to backcalculate joint properties from FWD testing (Ioannides et al. 1996). The advantage of using NNs and dimensional analysis together is that they both reduce the database size necessary to accurately estimate pavement properties (Rufino et al. 2002). In the development of the new Mechanistic-Empirical Pavement Design Guide (MEPDG) for the American Association of State Highway and Transportation Officials (AASHTO), NNs were recognized as nontraditional, yet very powerful computing techniques and ANN

models were used in preparing the concrete pavement analysis package (Khazanovich et al. 2001). Ceylan et al. (2009) developed a suite of NN-based flexible, rigid, and composite pavement backcalculation models from comprehensive synthetic databases.

This paper discusses the implementation of the DE optimization approach for a slab-on-grade rigid pavement structure although it can be used for a variety of pavement geometry and types owing to its flexible and integrated modular systems approach. The objective (fitness) function or the cost function for the proposed DE optimization approach is the difference between measured FWD deflections and computed pavement surface deflections.

The proposed DE global optimization backcalculation approach is presented in Figure 2. This approach treats backcalculation as a global optimization problem where the cost function to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the DE algorithm as described previously. Thus, for every update of the population of moduli solutions in the DE search scheme, the forward pavement response model has to be invoked to compute the resulting surface deflections.

In this paper, the DE optimization technique is hybridized with NN surrogate forward pavement response model for rapid prediction of surface deflections using concrete pavement PCC moduli and modulus of subgrade reaction and thicknesses of PCC layers as inputs. This reduces the computational time of DE significantly considering the number of times the surface deflections need to be computed using different sets of pavement layer moduli during the optimization process.

The DE, in essence, finds the optimal values of the NN inputs (pavement layer moduli) iteratively such that the corresponding values of the network outputs (deflections) match the measured pavement surface deflections to minimize the differences between the measured and computed deflections. Although the error-minimization deflection-based objective function can be defined in a number of ways, a simple objective function (f) representing sum of the squared differences between measured (D_i) and computed (d_i) deflections as shown in Equation 1 was selected for this study (where n = 6):

$$f = \sum_{i=1}^{n} (D_i - d_i)^2 \quad (1)$$

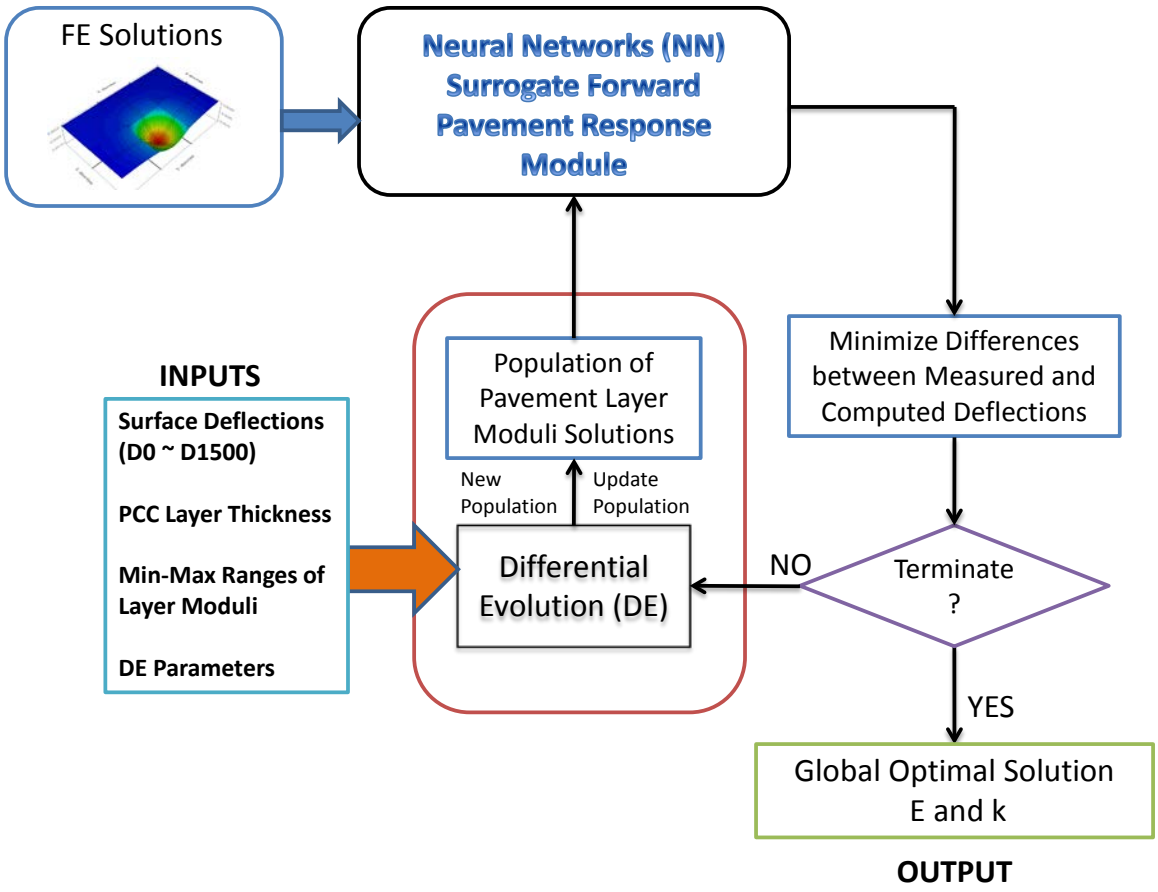


Figure 2. Finite Elements (FE) - Neural Networks (NN) - Differential Evolution (DE) Hybrid Global Optimization Backcalculation Approach

The FE-NN-DE hybrid optimization toolbox for rigid pavement backcalculation was implemented in MATLAB®. The input variables to the toolbox include six FWD measured surface deflections at 300-mm radial offsets starting from the center of the FWD loading plate (D0, D300, D600, D900, D1200, and D1500), PCC layer thickness, and the corresponding min-max ranges of pavement layer moduli. For ease of implementation and to avoid NN network saturation problems, all values were normalized in the range of 0.1 to 0.9.

A trained NN serves as a surrogate forward pavement response model that has learned the mapping between pavement layer moduli and resulting pavement surface deflections for a variety of case scenarios generated using the ISLAB 2000 pavement finite element program, as described in the next section.

NN Surrogate Forward Pavement Response Module

A Back Propagation (BP) type NN model was trained in this study with the results from the ISLAB 2000 finite element program to develop the surrogate forward pavement response model. BP NNs are very powerful and versatile networks that can be taught a mapping from one data space to another using example of the mapping to be learned. The term "back-propagation network" actually refers to a multi-layered; feed-forward neural network trained using an error back-propagation algorithm. The learning process performed by this algorithm is called "back-propagation learning" which is mainly an "error minimization technique" (Haykin 1999). Backpropagation

networks excel at data modelling with their superior function approximation capabilities (Haykin 1999).

A total of 41,106 data vectors generated by modeling slab-on-grade concrete pavement systems using ISLAB 2000 were used for NN training and testing. Concrete pavements analyzed in this study were represented by a six-slab assembly, each slab having dimensions of 6.1 m by 6.1 m (20 ft by 20 ft). The dense liquid model, proposed by Winkler (1867), was used to characterize the subgrade behavior. To maintain the same level of accuracy in the results from all analyses, a standard ISLAB 2000 finite element mesh was constructed for the slab. This mesh consisted of 10,004 elements with 10,209 nodes. The 40-kN (9,000-lb) FWD loading condition was simulated in ISLAB 2000.

The ISLAB 2000 solutions database was generated by varying E, k and thickness of PCC (h_{pcc}) over a range of values representative of realistic variations in the field. The E ranged from 6.9 to 103.4 GPa (1,000 to 15,000 ksi); k ranged from 13.6 to 217 MPa/m (50 to 800 psi/in); and h_{pcc} ranged from 152 to 635 mm (6 to 25 in) considering that most design thicknesses would be in this range. Note that the US design approach for concrete pavements limits k to 0.16-0.20 N/mm^3. A Poisson's ratio (μ) of 0.15 was assumed for PCC. Thus a total of 41,106 ISLAB 2000 analyses (51 different values of E × 31 different values of k × 26 different values of h_{pcc}) were conducted to represent a complete factorial of all the input values.

A network with two hidden layers was exclusively chosen for the NN models trained in this study. Satisfactory results were obtained in the previous studies with these types of networks due to their ability to better facilitate the nonlinear functional mapping (Ceylan 2002, Ceylan et al. 2005).

In the BP NNs used in this study, the connection weights are initially selected at random. Inputs from the mapping examples are propagated forward through each layer of the network to emerge as outputs. The errors between those outputs and the correct answers are then propagated backwards through the network and the connection weights are individually adjusted to reduce the error. After many examples (training patterns) have been propagated through the network many times, the mapping function is learned with some specified error tolerance. This is called supervised learning because the network has to be shown the correct answers for it to learn (Haykin 1999; Ceylan et al. 2005).

The 3-60-60-6 architecture (3 inputs [E, k, and PCC layer thickness], 60 nodes in the first and second hidden layers, and 6 outputs [D0 ~ D1500], respectively) was chosen as the best architecture based on its lowest training and testing Mean Squared Errors (MSEs). The NN training and testing were conducted with the MATLAB® Neural Networks toolbox using the most commonly used Levenberg-Marquardt training algorithm to achieve efficient convergence. In the BP learning algorithm, the error energy used for monitoring the progress toward convergence is the generalized value of all errors that is calculated by the least-squares formulation and represented by a Mean Squared Error (MSE) as follows (Haykin 1999):

$$MSE = \frac{1}{MP}\sum_{1}^{P}\sum_{k=1}^{M}\left(d_k - y_k\right)^2 \tag{2}$$

Where y_k and d_k are actual and desired outputs, respectively, M is the number of neurons in the output layer and P represents the total number of training patterns.

Other performance measures such as the Root Mean Squared Error (RMSE), Average Absolute Error (AAE), etc. are also used.

The training and testing MSE progress curves for the 3-60-60-6 network for the output were in the same order of magnitude thus depicting proper training. Exceptional prediction performance of the NN surrogate response models were achieved validating their suitability for use in DE hybrid optimization approach.

Prediction Performance of Proposed Backcalculation Tool

Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed FE-NN-DE hybrid concrete pavement backcalculation tool. A total of about 150 datasets were independently selected from the comprehensive synthetic FE solutions database to assess the prediction performance. The performance of FE-NN-DE optimization approach in backcalculating concrete pavement layer moduli is reported in Figure 3. As shown in the plots, all 150 FE-NN-DE backcalculation predictions fell on the line of equality for the two pavement layer moduli (E and k) thus indicating a proper training and excellent performance of the proposed hybrid concrete pavement backcalculation model.

Quantitative assessments of the degree to how close the models could predict the actual outputs are used to provide an evaluation of the models' predictive performances. A multi-criteria assessment with various goodness-of-fit statistics was performed using the 150 test vectors as an independent dataset which was not used in training the models. The criteria that were employed for evaluation of models' predictive performances were the coefficient of correlation (*R*), the coefficient of determination (R^2) with reference to the line of equality, root-mean-square error (*RMSE*) between the actual and predicted values, and the average absolute error (*AAE*).

Coefficient of Correlation (R)

$$R = \frac{\sum_{i=1}^{n}(y_i^t - \overline{y^t})(y_i^p - \overline{y^p})}{\sqrt{\sum_{i=1}^{n}(y_i^t - \overline{y^t})^2}\sqrt{\sum_{i=1}^{n}(y_i^p - \overline{y^p})^2}} \quad (3)$$

Coefficient of Determination (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i^t - y_i^p)^2}{\sqrt{\sum_{i=1}^{n}(y_i^t - \overline{y^t})^2}} \quad (4)$$

Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i^t - y_i^p)^2}{n}} \quad (5)$$

Average Absolute Error (AAE)

$$AAE = \frac{\sum_{i=1}^{n}\left|\frac{y_i^t - y_i^p}{y_i^t} \times 100\right|}{n} \quad (6)$$

Where y_i^t and y_i^p are the target and predicted modulus values, respectively, $\overline{y_i^t}$ and $\overline{y_i^p}$ are the mean of the target and predicted modulus values corresponding to *n* patterns. y_x^p is the number of predicted modulus values (out of n total predicted) for which the absolute relative error less *x*% from the model.

The *R* and R^2 are a measure of correlation between the predicted and the measured values and therefore, determines accuracy of the prediction model (higher *R* and R^2 equates to higher accuracy). The *RMSE* and the *AAE* indicate the relative improvement in accuracy and thus a smaller value is indicative of better accuracy. A summary of performance metrics for the developed FE-NN-DE rigid pavement moduli prediction is shown in Table 1.

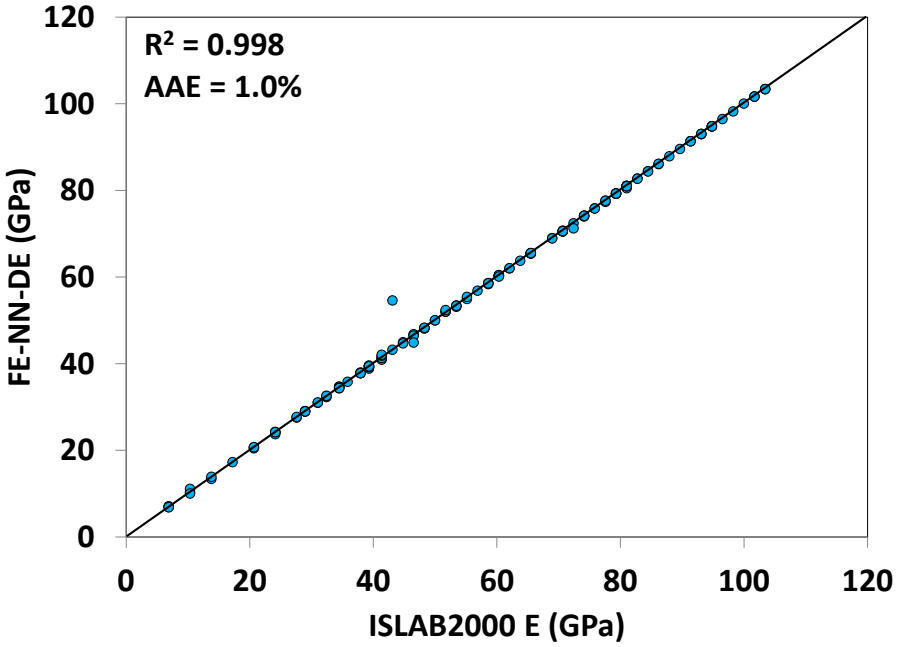
The DE search process in locating the global minima (normalized pavement layer moduli) is illustrated for one hypothetical solution (among the 150 independent test sets) in Figure 4 along with the contour lines of the 2-D and 3-D peaks function. At iteration 1, a population of 10 solution vectors is randomly initialized within the bounds of the search space (0.1 to 0.9). Through successive iterations, the solution vectors are evolved to find optimal solutions through the mutation, crossover, and selecting operation procedures of DE strategy. The corresponding cost of the system (Mean Squared Error) is also plotted against number of iterations wherein the cost decreases with successive iterations as the DE algorithm finds new child generations with lower cost. It is clearly seen that DE is able to locate the global minima in less than 15 iterations (less than a second).

Conclusions

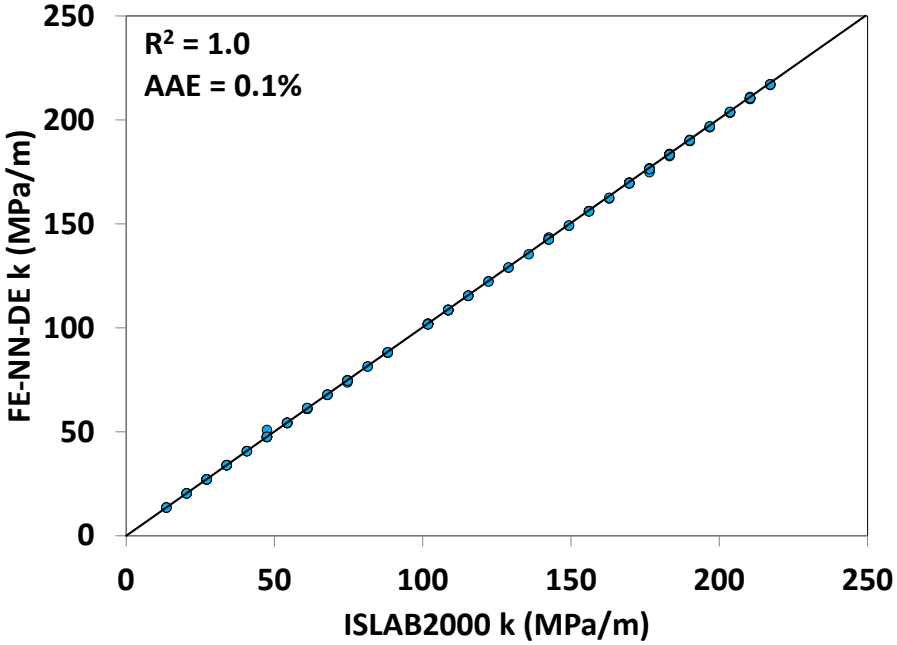
The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons. This paper focused on the development of a hybrid evolutionary global optimization algorithm using Differential Evolution (DE) stochastic search optimization technique. In this approach, the backcalculation problem is treated as a global optimization problem where the cost function to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the DE algorithm. Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed FE-NN-DE hybrid concrete pavement backcalculation tool. The results demonstrated the excellent performance of the developed backcalculation tool. Future studies will focus on further validation of the proposed backcalculation tool using field FWD data.

Table 1. Summary of FE-NN-DE backcalculation model performance metrics

Performance Measure	Backcalculated PCC Modulus, E	Backcaculated Subgrade Modulus of Reaction, k
R	0.9993	1.0000
R^2	0.9985	1.0000
RMSE	0.9666 GPa	0.3593 MPa/m
AAE	1%	0.1%

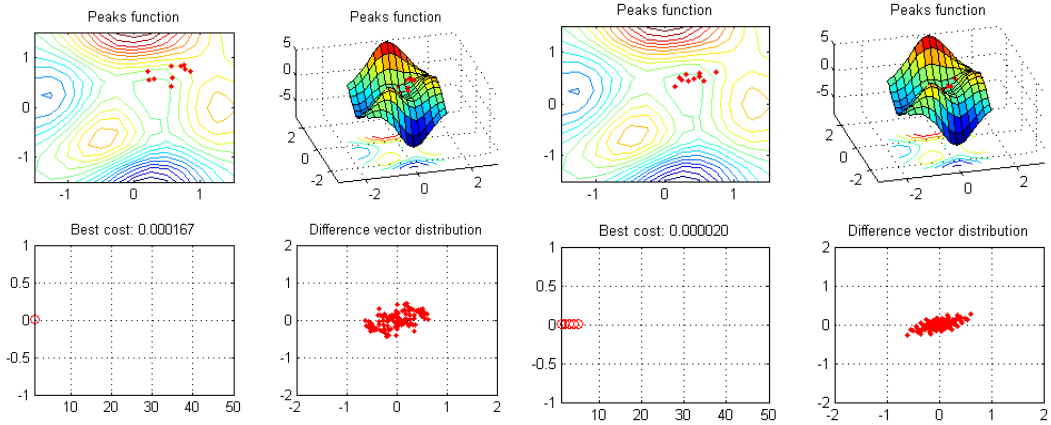


(a)

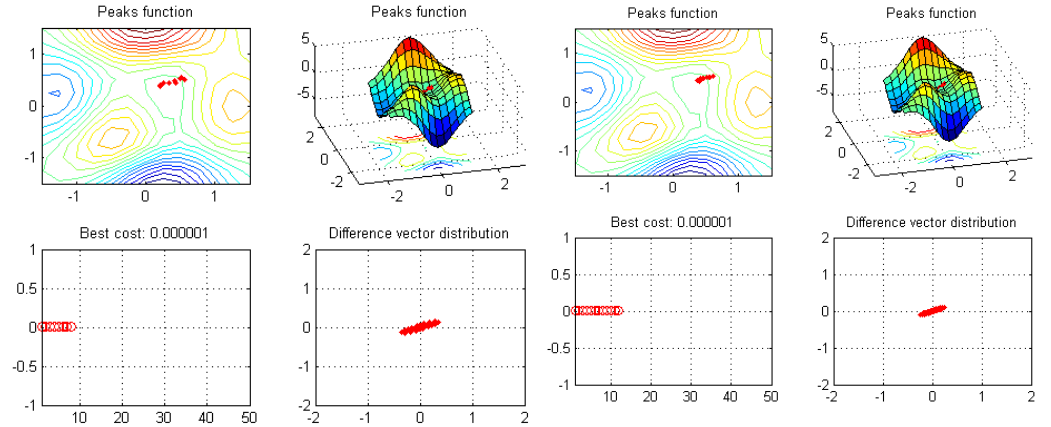


(b)

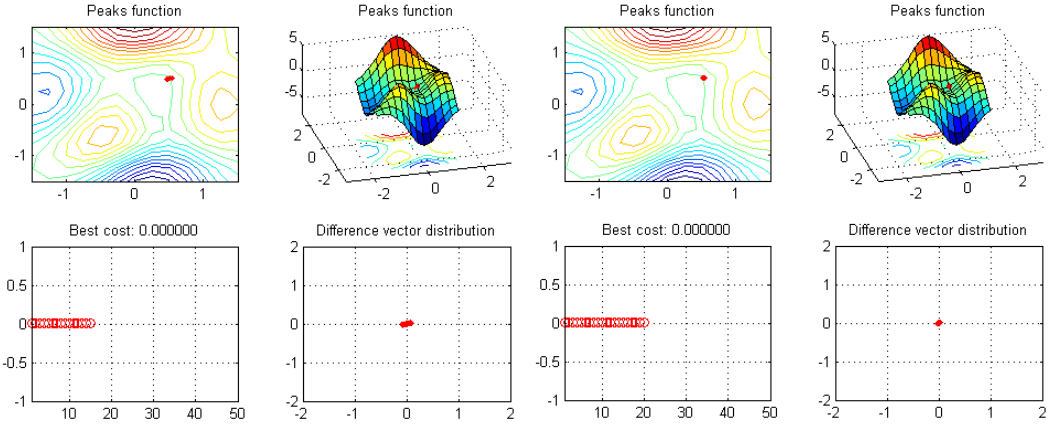
Figure 3. Prediction performance of FE-NN-DE backcalculation tool with hypothetical data: (a) E; (b) k



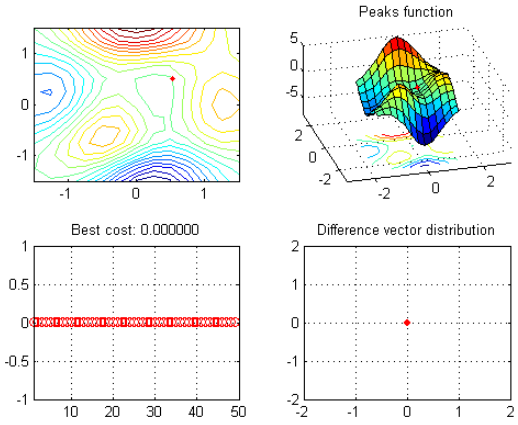
(a) Iteration = 1 (b) Iteration = 5



(c) Iteration = 8 (d) Iteration = 12



(e) Iteration = 15 (f) Iteration = 20



(g) Iteration = 49

Figure 4. Differential Evolution (DE) of pavement layer moduli solutions: actual solution, E = 0.52857 (58.6 GPa); k = 0.5 (115 MPa/m)

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