
BACKCALCULATION OF MATERIAL PROPERTIES FOR CONCRETE PAVEMENT

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ABSTRACT

This paper describes the development of a closed-form backcalculation procedure that yields a unique set of solutions for the layer moduli of a concrete pavement. The procedure is applicable to analyzing concrete pavements consisting of a concrete slab supported on an elastic foundation. For concrete pavement construction that involves a subbase layer, the backcalculation procedure computes the combined effective modulus of the subgrade and the subbase layer. The procedure makes use of surface deflection measurements obtained from falling weight deflectometer tests or any other nondestructive surface deflection measuring equipment. The algorithms use theoretical relationship between surface deflections and applied loads for rigid slabs supported on either a Winkler foundation or an elastic foundation. Numerical integration is performed to compute pavement surface deflections under loads in the forward calculation phase of the backcalculation analysis. Numerical examples are given to illustrate the versatility of the algorithm in backcalculation analysis, such as assessing the reliability of backcalculated layer moduli and detecting erroneous deflection measurements from faulty sensors.

INTRODUCTION

Backcalculation of layer moduli of pavements based on non-destructive surface deflection measurements has been widely accepted as a cost-effective way of pavement evaluation at both network and project levels. Most backcalculation procedures adopt trial-and-error techniques to search for a set of pavement layer moduli that produces a surface deflection basin closely matching the measured basin (1-3). A major setback of this approach is the non-uniqueness of solutions, which means that the backcalculated moduli may not be the true values.

The availability of the closed-form backcalculation algorithm ILLI-BACK by Ioannides et al (4) resolves to a large extent the theoretical issue of non-unique solutions for the case of concrete pavements. ILLI-BACK employs the concept of dimensional analysis and the unique relationship between the shape of deflection basin and the radius of relative stiffness of the pavement slab. This concept of ILLI-BACK is adopted by AASHTO (American Association of State Highway and Transportation Officials) for concrete pavement evaluation analysis in its 1993 Guide for Design of Pavement Structures (5).

This paper describes an alternative closed-form backcalculation algorithm for concrete pavements. Known as NUS-BACK, this algorithm presents a more general approach to the backcalculation problem and is more flexible in its applications. It also offers an effective means to assess the

reliability of the input data of deflection measurements, and to detect the possible presence of erroneous input measurements.

DEVELOPMENT OF A CLOSED-FORM BACKCALCULATION ALGORITHM

Theoretical Basis

For a concrete slab of known thickness resting on an elastic foundation, its surface deflections under a given load are a function of the following material properties: (a) elastic modulus and Poisson ratio of the concrete slab, and (b) modulus of subgrade reaction in the case of a Winkler foundation, or elastic modulus and Poisson ratio of the subgrade material in the case of an elastic foundation. The Poisson ratio of concrete materials and that of a given type of subgrade material vary within a relatively narrow range respectively. The choice of different values of the Poisson ratios has negligible effects on pavement response (including surface deflections) under loads. It is therefore common practice to assume these Poisson ratios to be known by assigning typical values to them.

In a standard backcalculation problem of a slab-on-elastic-foundation pavement system, the known and unknown pavement parameters are as follows:

- Known parameters:
slab thickness, h_c ; slab Poisson ratio, μ_c ; subgrade Poisson ratio, μ_s
- Unknown parameters:
elastic modulus of slab, E_c ; and
modulus of subgrade reaction, k (for Winkler foundation), or
elastic modulus of subgrade, E_s (for elastic foundation)

The problem thus becomes one with two unknowns -- E_c and k for concrete slab on Winkler foundation, or E_c and E_s for concrete slab on elastic foundation. In either case, the two unknowns can be determined if two independent pavement response equations could be set up. This is achieved by considering the pavement deflection equations as shown in Eqs. 1(a) and 1(b), or Eqs. 2(a) and 2(b):

Case A: Winkler foundation

$$D(R_i) = \frac{P}{k\pi a^2} \cdot F_k(R_i, a, l_k) \quad (1a)$$

$$D(R_j) = \frac{P}{k\pi a^2} \cdot F_k(R_j, a, l_k) \quad i \neq j \quad (1b)$$

Case B: Elastic foundation

$$D(R_i) = \frac{P(1 - \mu_s)}{E_s l_E} \cdot F_E(R_i, a, l_E) \quad (2a)$$

$$D(R_j) = \frac{P(1 - \mu_s)}{E_s l_E} \cdot F_E(R_j, a, l_E) \quad i \neq j \quad (2b)$$

where D = measured surface deflection, R_i and R_j = horizontal distances of points i and j respectively from the center of loading, P = magnitude of applied load, a = radius of loaded area, F and F = deflection factors which are infinite integral functions of the three variables in the parentheses (6), and l_k and l_E are the radii of relative stiffness of the slab given by

$$l_k = \left(\frac{E_c h_c}{12(1 - \mu_c^2)k} \right)^{1/4} \quad (3)$$

$$l_E = \left(\frac{E_c h_c^3 (1 - \mu_s^2)}{6(1 - \mu_c^2)E_s} \right)^{1/3} \quad (4)$$

where all variables are as defined earlier.

For any pair of measured deflections $D(R_i)$ and $D(R_j)$, there is a unique solution of lk and k for a slab on a Winkler foundation, and lE and E_s for a slab on an elastic foundation.

Determination of Unknown Pavement Parameters

The parameters lk and k (for Winkler foundation) can be determined by solving the simultaneous equations Eqs. 1(a) and 1(b). Similarly, lE and E_s (for elastic foundation) can be obtained by solving Eqs. 2(a) and 2(b). Solving of the simultaneous equations requires the determination of deflection factors F_k or F_E that involves computation of infinite integrals of the product of Bessels functions of parameters R_i (or R_j), a and lk (or lE). In this study, numerical integration has been adopted and was found that truncated errors of less than 0.0001 could be achieved in the computation of deflection factors F_k and F_E . The magnitude of the truncated errors are negligible as the normal values of the deflection factors vary from 0.01 to 0.40 (7). Once lk and k (or lE and E_s) values are determined, the concrete slab elastic modulus E_c can be determined from either Eq. 3 or Eq. 4. The flow chart in Fig. 1 summarizes the computational procedure of the backcalculation analysis.

Backcalculation Computer Program

It has been established from the preceding sections that only two surface deflection measurements are needed to solve for the two unknown parameters. A micro-computer backcalculation program NUS-BACK has been developed using the described algorithm based on deflection measurements input at any two points.

Practically all falling weight deflectometers (FWD) available commercially record surface deflections at more than 2 points, with 7-point deflection measurements being the most common today. NUS-BACK allows any two deflection measurements to be selected for backcalculation computation. It also allows more than one pair of the deflection measurements to be considered, and gives the individual backcalculated results as well as the mean backcalculated values.

RELIABILITY ASSESSMENT OF DEFLECTION MEASUREMENTS

For a falling weight deflectometer (FWD) that measures deflections at N number of points during a falling weight test, a total of $N(N-1)/2$ sets of solutions can be obtained using NUS-BACK. That is, when 7 deflection sensors are used, 21 different pairs of deflection readings can be used to generate 21 sets of solutions. NUS-BACK can process and give 21 solutions and the mean solutions in 4 seconds on AT486 PC with a clock speed of 66 MHz.

In an ideal situation where all conditions are perfect as assumed by the theoretical model, and all sensors provide exact measurements of surface deflections, the $N(N-1)/2$ sets of backcalculated lk and k (or lE and E_s) values would be identical. This provides a useful means to assess the relative reliability of FWD deflection data.

Effect of Random Errors in Deflection Readings

Errors in deflection measurements during a FWD test are unavoidable. A typical measuring error of 2% $\pm$ 2 mm is stated in most supplier specifications of commercial FWD. This section illustrates using NUS-BACK how such random measurement errors would affect the backcalculated pavement parameters.

Consider the example illustrated in Fig. 2. For easy illustration, assume that surface deflections are measured at the following 4 distances away from the center of loading: 0 mm, 305 mm, 610 mm and 915 mm. The exact deflections at these four points can be computed to be $D_1 = 0.086$ mm, $D_2 = 0.080$ mm, $D_3 = 0.068$ mm and $D_4 = 0.056$ mm respectively for a Winkler foundation, and $D_1 = 0.077$ mm, $D_2 = 0.072$ mm, $D_3 = 0.063$ mm and $D_4 = 0.053$ mm respectively for an elastic foundation. Table 1 randomly generates 30 sets of deflection readings with random errors of $\pm 2\%$ for each reading.

NUS-BACK was applied to backcalculate the pavement parameters for each of the 30 sets of data in Table 1. Table 2 presents the statistical analysis of the backcalculation results. It shows that with random errors associated with the deflection readings of a typical FWD, the backcalculated parameters k and E_c (for Winkler foundation) and E_s and E_c (for elastic foundation) each has a 95% confidence interval that includes the actual parameter value given in Fig. 2. The coefficients of variation (i.e. standard deviation expressed as a percentage of the mean) of the four backcalculated parameters are correspondingly 7.56%, 11.16%, 4.87% and 13.96%. These results provide an indication of the reliability levels one could expect from pavement parameters backcalculation using a typical falling weight deflectometer.

Effect of Inaccurate Deflection Measurements

It is of interest to investigate how inaccurate measurements from faulty deflection sensors would affect the backcalculated values of pavement parameters. Examples are given using the pavement structure depicted in Fig. 2. For illustration purpose, the effect of having one inaccurate deflection reading at a time is analyzed. The following 4 magnitudes of measurement errors are considered: +10%, -10%, +20% and -20%.

The results of backcalculation analysis using NUS-BACK are plotted in Figs. 3 and 4 for the cases of Winkler and elastic foundation respectively.

The following observations can be made:

1. The backcalculated concrete slab modulus values E_c are most sensitive to errors in deflection measurements. With a $\pm 10\%$ error in one of the deflection measurements, the percent error in back-calculated E_c reaches up to 199% (Winkler foundation) or 335% (elastic foundation), compared with 36% for k and -19% for E_s . With a $\pm 20\%$ error, the corresponding maximum percent errors are 400% and 670% for E_c , 69% for k and -29% for E_s .
2. In terms of percent error in backcalculated results, k is found to be affected more by errors in individual deflection measurements than E_s . The maximum percent errors of backcalculated k are 36% and 69% for $\pm 10\%$ and $\pm 20\%$ errors respectively, while the corresponding errors for E_s are -19% and -29%. This is believed to be related to the fact that the Winkler foundation assumes no shear interaction in the foundation.
3. In general, too high a D_1 measurement leads to over-estimation of k and E_s values, and under-estimation of E_c values. When the D_1 measurement is too low, the effects are reversed. Errors in D_4 measurements have the opposite effects to those of D_1 . The effects of errors in D_2 and D_3 tend to follow the trend observed for D_4 measurements.

DETECTION OF ERRONEOUS DEFLECTION READINGS FROM FAULTY SENSORS

Examples in the preceding section have shown that even a single inaccurate or faulty deflection measurement would seriously affect the accuracy of the backcalculated pavement parameters. The presence of faulty sensors causes significant errors in the backcalculated pavement parameters by conventional backcalculation methods that rely on matching of measured and computed deflection basins. This problem can be overcome by the use of NUS-BACK developed in this study.

Two examples are presented in this section to demonstrate the use of NUS-BACK to detect the presence of faulty sensors. The pavement structure depicted in Fig. 2 is again considered. The first example considers the case of a Winkler foundation with a 10% error in the deflection measurement D_1 , while the second considers the case of an elastic foundation with the same margin of error in D_1 .

The procedure to follow in detecting an erroneous deflection measurement is to perform backcalculation analysis using NUS-BACK with two deflection measurements at a time. For a total of 4 deflection measurements, 6 sets of backcalculated solutions can be calculated. Table 3 presents the results of the backcalculation analysis. Since D_1 , D_3 and D_4 are exact measurements, the last 3 cases of the backcalculation analysis all give the true answers of $k = 57.30 \text{ MN/m}$ and $E_c = 31.00 \text{ GPa}$ for liquid foundation, and $E_s = 191.00 \text{ MPa}$ and $E_c = 31.00 \text{ GPa}$ for elastic foundation.

It is apparent from the results in Table 3 that whenever D_1 is entered in the backcalculation analysis, erroneous parameters are obtained. This clearly points to the conclusion that the deflection measurement D_1 is problematic, and should be discarded in the backcalculation analysis.

CONCLUSIONS

This paper describes the development of a closed-form backcalculation procedure NUS-BACK that computes the unique solutions for the layer moduli of a concrete pavement resting on either Winkler foundation or an elastic foundation. Surface deflection measurements from any non-destructive test equipment can be used for the backcalculation analysis. Theoretically, only two deflection measurements are required, and true layer modulus value will be obtained if all deflection measurements are exact.

Using the NUS-BACK computer program, it has been shown that with the typical random errors in deflection measurements of common falling weight deflectometers, the coefficients of variation of the backcalculated parameters were of the order of about 10%. The 95% confidence intervals of the backcalculated parameters contain the true values of the respective parameters.

Examples have been presented to show that compared with the backcalculated subgrade modulus values, the backcalculated estimates of the concrete slab modulus are much more sensitive to errors in the deflection measurements. It is illustrated in the paper that by considering the deflection measurements in pairs and examining the corresponding backcalculated results, it is possible to identify the presence of inaccurate deflection measurements from faulty sensors.

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TABLE 1 Deflection Measurements with Random System Errors of FWD

Data Set	Winkler Foundation				Elastic Foundation			
	D1	D2	D3	D4	D1	D2	D3	D4
1	0.087	0.080	0.071	0.058	0.077	0.070	0.063	0.054
2	0.088	0.081	0.070	0.057	0.077	0.069	0.063	0.053
3	0.085	0.081	0.069	0.058	0.075	0.071	0.061	0.054
4	0.086	0.079	0.071	0.055	0.075	0.069	0.063	0.051
5	0.087	0.081	0.070	0.057	0.076	0.071	0.063	0.053
6	0.084	0.079	0.067	0.058	0.074	0.069	0.060	0.054
7	0.085	0.081	0.070	0.055	0.075	0.071	0.063	0.051
8	0.086	0.082	0.071	0.057	0.076	0.070	0.063	0.051
9	0.086	0.080	0.071	0.058	0.076	0.070	0.063	0.054
10	0.084	0.079	0.069	0.056	0.074	0.069	0.062	0.052
11	0.082	0.079	0.068	0.057	0.075	0.071	0.061	0.053
12	0.084	0.081	0.069	0.055	0.074	0.070	0.062	0.051
13	0.085	0.082	0.069	0.057	0.075	0.072	0.062	0.053
14	0.084	0.079	0.070	0.055	0.074	0.069	0.063	0.051
15	0.087	0.081	0.067	0.055	0.077	0.070	0.060	0.051
16	0.086	0.079	0.068	0.058	0.076	0.069	0.063	0.054
17	0.085	0.081	0.069	0.056	0.075	0.071	0.062	0.052
18	0.088	0.081	0.070	0.056	0.077	0.071	0.063	0.052
19	0.087	0.083	0.067	0.057	0.077	0.073	0.060	0.053
20	0.086	0.082	0.069	0.057	0.075	0.072	0.063	0.053
21	0.085	0.082	0.071	0.058	0.074	0.072	0.064	0.054
22	0.085	0.079	0.069	0.057	0.075	0.070	0.062	0.053
23	0.088	0.078	0.069	0.055	0.078	0.069	0.062	0.051
24	0.087	0.081	0.070	0.057	0.078	0.070	0.063	0.053
25	0.086	0.083	0.067	0.058	0.075	0.073	0.060	0.054
26	0.085	0.083	0.070	0.056	0.075	0.073	0.063	0.052
27	0.085	0.081	0.070	0.056	0.075	0.071	0.063	0.052
28	0.087	0.082	0.068	0.058	0.076	0.072	0.061	0.054
29	0.088	0.078	0.069	0.055	0.077	0.070	0.062	0.051
30	0.086	0.083	0.071	0.055	0.075	0.073	0.064	0.051

Note: (1) All deflections are in mm.

(2) D1, D2, D3 and D4 are surface deflections measured at 0, 305, 610 and 915 mm from center of loading area.

TABLE 2 Effect of Random Errors in Deflection Measurements on Backcalculated Pavement Parameters

Winkler Foundation				Elastic Foundation			
k (MN/m ³)		Ec (GPa)		Es (MPa)		Ec (GPa)	
Mean	Std Dev.	Mean	Std Dev.	Mean	Std Dev.	Mean	Std Dev.
53.64	4.05	34.60	3.86	186.8	9.09	37.11	5.18
COV = 7.56%		COV = 11.16%		COV = 4.87%		COV = 13.96%	

Note: Std Dev. stands for standard deviation, and COV for coefficient of variation.

TABLE 3 Backcalculation Analysis with an Erroneous Deflection Measurement

Input Deflections	Winkler Foundation		Elastic Foundation	
	k (MN/m ³)	Ec (GPa)	Es (MPa)	Ec (GPa)
D1 and D2	150.02	9.25	304.68	8.72
D1 and D3	81.84	17.44	219.62	17.65
D1 and D4	65.82	21.95	201.28	20.80
D2 and D3	57.30	31.00	191.00	31.00
D2 and D4	57.30	31.00	191.00	31.00
D3 and D4	57.30	31.00	191.00	31.00

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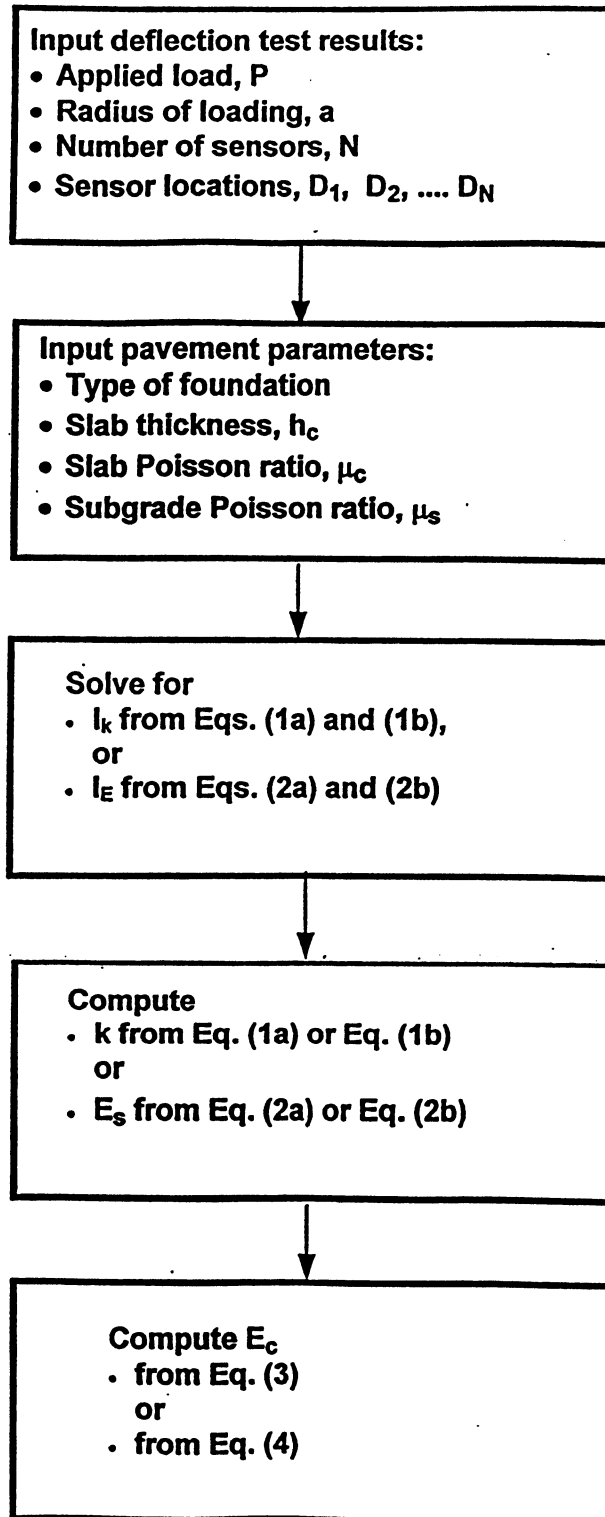
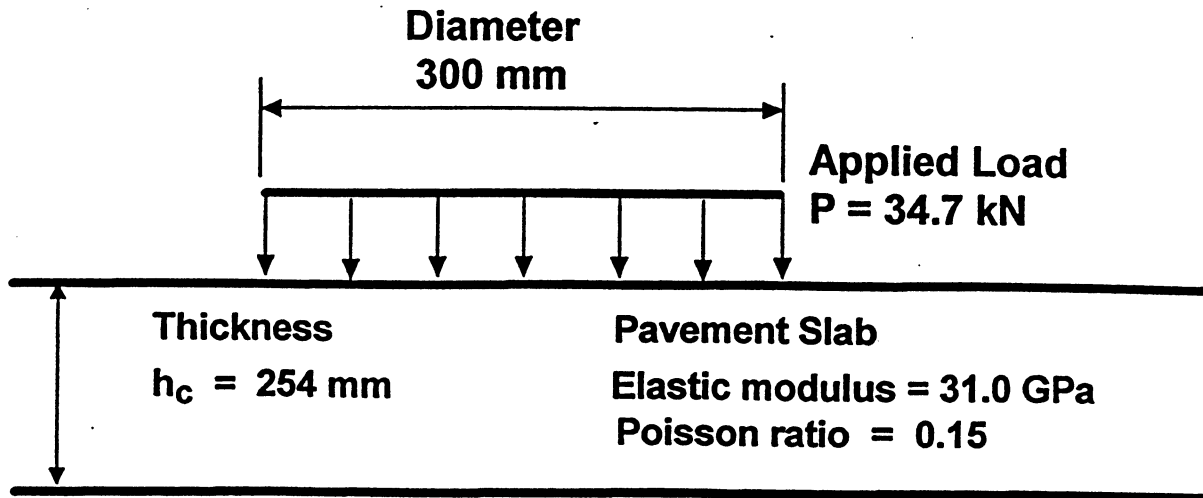


Fig. 1 Computational Procedure of the Proposed Closed-Form Backcalculation Analysis

**Winkler Foundation**

Modulus of Subgrade reaction $k = 57.3 \text{ MN/m}^3$
or

Solid Foundation

Elastic modulus $E_s = 191.0 \text{ MPa}$

Poisson ratio $\mu_s = 0.45$

Fig. 2 Two-Layer Structure of Concrete Slab supported on Elastic Foundation

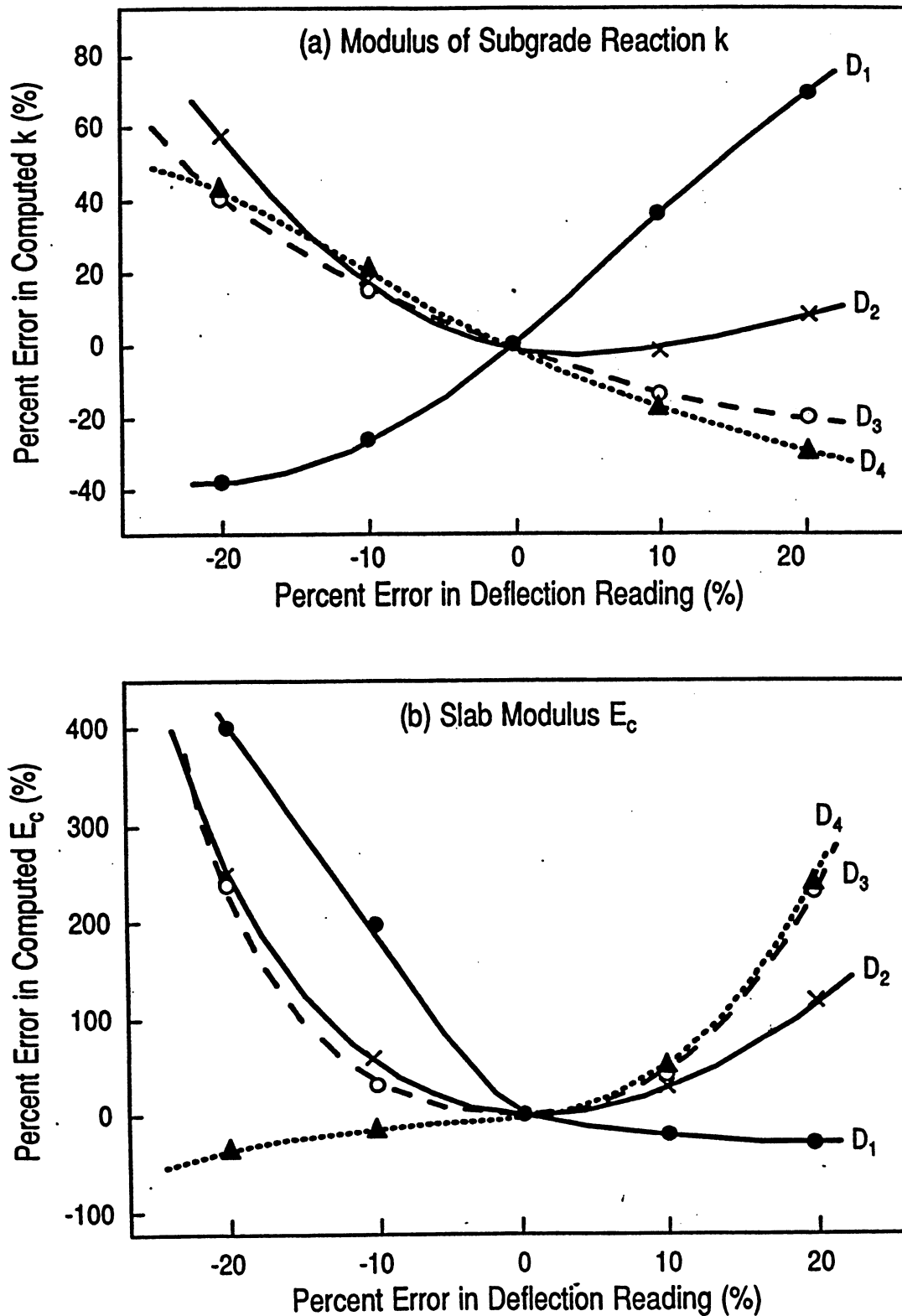


Fig. 3 Effect of Inaccurate Deflection Measurements on Backcalculation Analysis of Concrete Pavement resting on Winkler Foundation

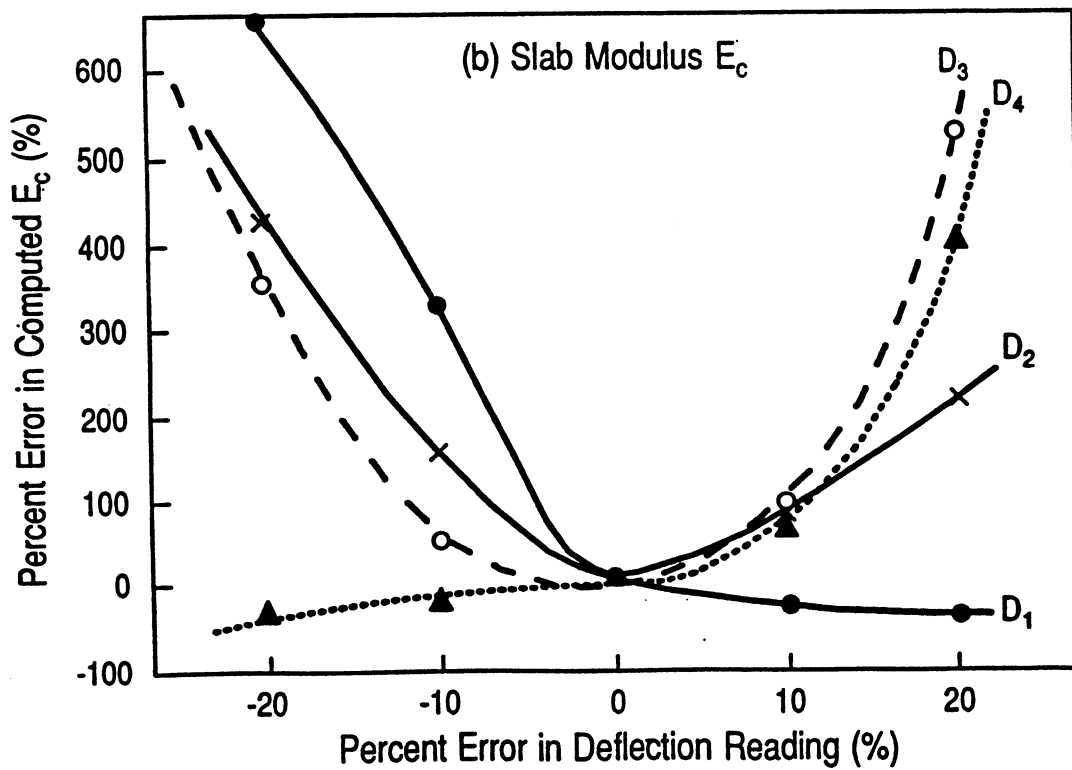
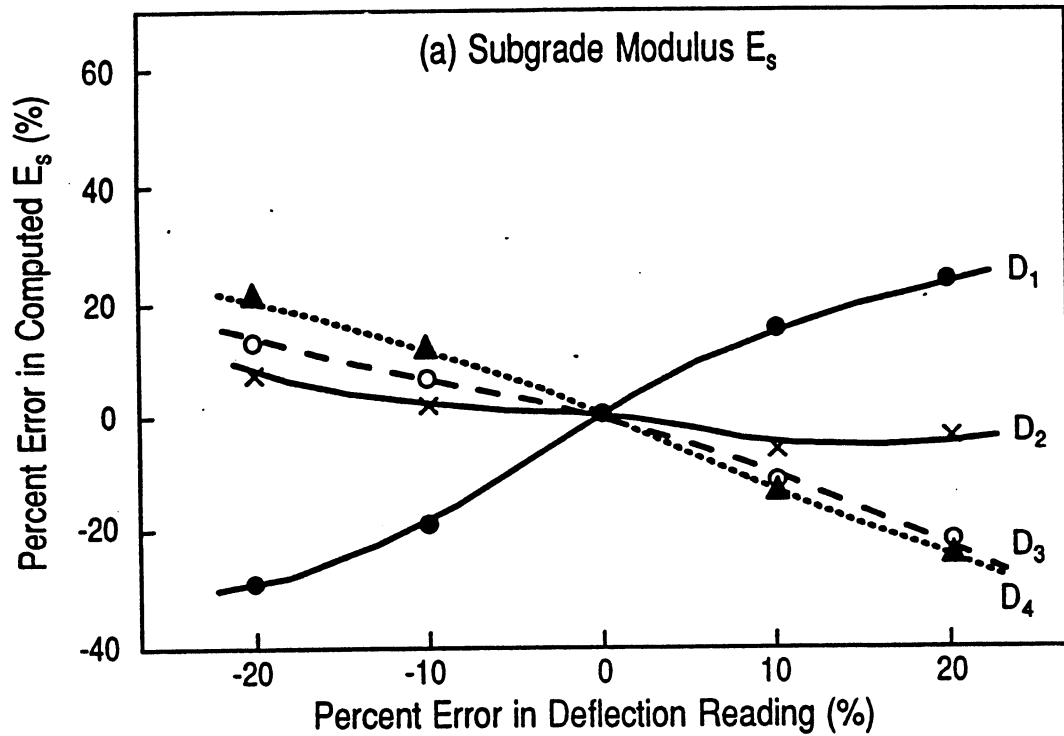


Fig. 4 Effect of Inaccurate Deflection Measurements on Backcalculation Analysis of Concrete Pavement resting on Elastic Foundation

REFERENCES

1. Lytton, R.L. Backcalculation of Pavement Layer Properties. Special Technical Publication STP 1026, ASTM, Philadelphia, Pa., 1994, pp. 7-40.
2. Bush, A. J. and D. R. Alexander. Pavement Evaluation using Deflection Basin Measurements and Layered Theory. Transportation Research Record 1022, TRB, Washington D.C., 1985, pp. 16-28.
3. Uzan, J. Advanced Backcalculation Techniques. Special Technical Publication STP 1198, ASTM, Philadelphia, Pa., 1994, pp. 3-37.
4. Ioannides, A. M., E. J. Barenberg and J. A. Lary. Interpretation of Falling Weight Deflectometer Results using Principles of Dimensional Analysis. Proc. 4th Int. Conf. on Concrete Pavement Design and Rehabilitation, Purdue Univ., West Lafayette, IN, 1989, pp. 231-247.
5. AASHTO. AASHTO Guide for Design of Pavement Structures. American Association of State Highway and Transportation Officials, Washington D.C., 1993.
6. Panc, V. Theories of Elastic Plates. Noordhoff International Publishing, Leyden, Czechoslovakia, 1975.
7. Li, S. Evaluation of Concrete Pavement Properties using Non-Destructive Measurements of Deflections. PhD Dissertation, National University of Singapore, 1997.

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