FLEXURAL FATIGUE DESIGN CURVE FOR CONCRETE PAVEMENT IN CONSIDERATION OF PROBABILISTIC PROPERTIES OF FATIGUE PHENOMENA

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INTRODUCTION

In the mechanistic design method for concrete pavement, the flexural fatigue curve has a great influence when analyzing the fatigue life of the pavement. The flexural fatigue curve is obtained from the results of fatigue test of concrete. However, generally, the flexural fatigue strength of concrete shows a fair amount of scatter as its specific property, and accordingly, the reliability of concrete pavements are influenced considerably by this scatter. Therefore, in order to design concrete pavement rationally, it is necessary to determine the flexural fatigue design curve in consideration of the probabilistic properties of fatigue phenomena of concrete.

In this study, the probabilistic properties of flexural fatigue failure are obtained from test data. And, applying Monte Carlo simulation technique to them, analyses are made of the influence of scatter in the fatigue phenomena of concrete on the service life of concrete pavement and an investigation is made on the flexural fatigue curve for concrete pavement design.

PROBABILISTIC PROPERTIES OF FLEXURAL FATIGUE

The authors have previously reported on experiments conducted on the factors which the fatigue properties of pavement concrete are affected, such as stress ratio (the ratio of minimum to maximum stress of cyclic stress), the maximum size of coarse aggregate, and the difference in static flexural strength [1][2]. In these reports, it was clear by statistical analyses that these factors are ignored in the design of concrete pavements for practical purposes. Therefore, all data (173 specimens) obtained from these experiments were used for the investigation of fatigue curve. The numbers of specimens for each stress level S (= maximum stress / static strength) were as given in Table 1.

Table 1 Number of Specimens

Stress Level	Number of Specimens
0.90	46
0.85	36
0.80	49
0.80	42

In these fatigue tests, fatigue failures are scattered over broad ranges for the individual stress levels. This scatter can be considered to be a specific property of the fatigue phenomena, and a statistical study is required in preparation of a fatigue curve. That is, the probability distribution function for each stress level must be determined considering the test results of each stress level as a single population.

The failure probability of specimens belonging to the individual populations, according to the method of ordered statistics, is given by the following equation:

$$P_{fi} = \frac{i}{n + 1} \tag{1}$$

where,

P_{fi}: failure probability of i-th specimen at each stress level arrayed in order of cycles to failure

n: number of specimens at each stress level

When the run out specimens which did not fail are included, the failure probability is given by the following equation[3]:

$$P_{fi} = \frac{i}{n + 2} \tag{2}$$

The n in Eq.(2) includes the number of run out specimens.

Failure probability at each stress level using Eqs.(1) and (2), are given in Table 2. The arrows in the table indicate run out specimens. When these results are plotted on a log-normal probability paper as shown in Fig. 1, the data for the individual stress levels are arrayed on roughly a straight line. Consequently, the data

Table 2 Probability of Failure of Specimens

i	S=0.90		S=0.85		S=0.80		S=0.70	
	C.F.	P_f	C.F.	P_f	C.F.	P_f	C.F.	P_f
1	150	0.021	315	0.027	740	0.020	2.3E4	0.022
2	264	0.043	451	0.054	1547	0.039	6.8E4	0.046
3	319	0.064	652	0.081	3124	0.059	1.1E5	0.068
4	471	0.085	811	0.108	3685	0.078	1.1E5	0.090
5	498	0.106	979	0.135	4204	0.098	2.1E5	0.114
6	591	0.128	989	0.162	5729	0.118	3.4E5	0.136
7	663	0.149	1093	0.189	5980	0.137	4.5E5	0.159
8	670	0.170	1274	0.216	6671	0.157	5.9E5	0.182
9	733	0.192	1346	0.243	7582	0.177	6.2E5	0.205
10	952	0.213	1372	0.270	9525	0.196	7.7E5	0.227
11	952	0.234	1647	0.297	1.0E4	0.216	8.5E5	0.250
12	1081	0.255	1665	0.324	1.0E4	0.235	9.0E5	0.273
13	1307	0.277	1759	0.351	1.0E4	0.254	9.4E5	0.296
14	1444	0.298	1966	0.378	1.1E4	0.275	1.1E6	0.318
15	1767	0.319	2511	0.405	1.6E4	0.294	1.2E6	0.341
16	1805	0.340	3060	0.432	2.3E4	0.313	1.4E6	0.364
17	1947	0.362	3257	0.460	2.3E4	0.333	1.5E6	0.386
18	1957	0.383	3640	0.487	2.6E4	0.353	1.7E6	0.409
19	2283	0.404	3997	0.514	3.2E4	0.373	2.1E6	0.432
20	2834	0.426	4193	0.541	3.2E4	0.392	2.6E6	0.455
21	2845	0.447	4367	0.568	3.2E4	0.412	2.0E6→	
22	3245	0.468	4837	0.595	3.7E4	0.431	2.0E6→	
23	3849	0.489	4897	0.622	3.9E4	0.451	2.0E6→	
24	3907	0.511	5455	0.649	4.2E4	0.471	2.0E6→	
25	4147	0.532	6232	0.676	4.5E4	0.490	2.0E6→	
26	4297	0.553	6666	0.703	4.6E4	0.510	2.0E6→	
27	4464	0.575	8897	0.730	5.1E4	0.529	2.0E6→	
28	4506	0.596	1.1E4	0.757	5.1E4	0.549	2.0E6→	
29	4801	0.617	1.1E4	0.784	6.6E4	0.569	2.0E6→	
30	5714	0.638	1.4E4	0.811	6.7E4	0.588	2.0E6→	
31	6359	0.660	1.8E4	0.838	7.5E4	0.608	2.0E6→	
32	8900	0.681	3.4E4	0.865	7.7E4	0.628	2.0E6→	
33	1.0E4	0.702	3.5E4	0.895	1.6E5	0.647	2.0E6→	
34	1.4E4	0.723	4.6E4	0.919	2.4E5	0.667	2.0E6→	
35	1.4E4	0.745	6.9E5	0.946	4.2E5	0.686	2.0E6→	
36	1.4E4	0.766	9.6E5	0.973	7.2E5	0.706	2.0E6→	
37	1.7E4	0.787			7.4E5	0.726	2.0E6→	
38	2.1E4	0.809			1.2E6	0.745	5.0E6→	
39	2.2E4	0.830			1.4E6	0.765	5.0E6→	
40	2.3E4	0.851			1.5E6	0.784	5.0E6→	
41	2.3E4	0.872			2.1E6	0.804	5.0E6→	
42	2.5E4	0.894			3.1E6	0.824	5.0E6→	
43	2.7E4	0.915			2.0E6→			
44	3.0E4	0.936			2.0E6→			
45	4.9E4	0.957			2.0E6→			
46	3.0E5	0.979			3.0E6→			
47					3.0E6→			
48					5.0E6→			
49					5.0E6→			

S:Stress Level C.F.:Cycles to Failure
P_f:Probability of Failure

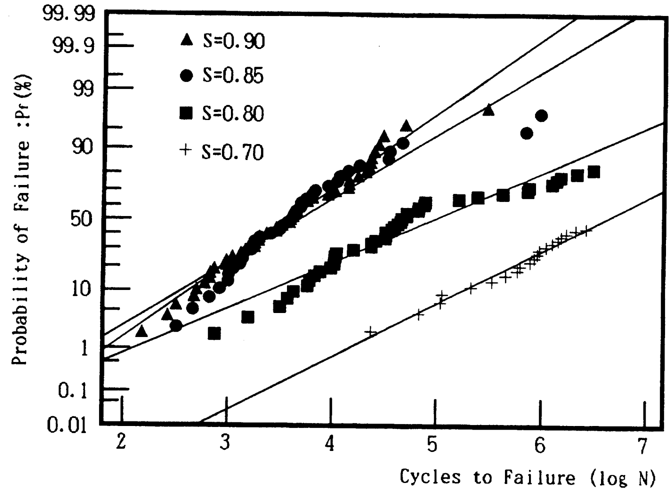

Fig.1 P_f-N Diagram for Stress Level

Table 3 Parameters of Log-Normal Distribution for Fatigue Strength

S	λ	ζ
0.90	8.231	1.6972
0.85	8.431	1.9955
0.80	11.401	2.7894
0.70	15.137	2.3832

for the individual stress levels were shown to respectively belong to a population of log-normal distribution. This was confirmed by the method of Kolmogorov-Smirnov of 5% significance level.

Table 3 gives the parameters λ and ζ of log-normal distributions at the individual stress levels indicated by natural logarithm values. From this, it is possible to determine the number of cycles N to fatigue failure for a certain failure probability P_f. Further, as is clear from Table 3, there is a tendency seen for scatter of data to become greater as the stress level becomes lower.

Since the probability distribution function of flexural fatigue at the individual stress levels have been made clear by the above, it is possible to determine the fatigue curve for a certain failure probability P_f which is the relationship of the stress level S and number of cycles N to fatigue failure. The fatigue curves for failure probabilities P_f = 10 to 90% are shown in Fig. 2. In general, a fatigue curve is expressed in the form of Eq.(3), with a and b being coefficients determined from experiments.

$$S = a - b * \log(N) \qquad (3)$$

where,
S: stress level
N: number of cycles to failure

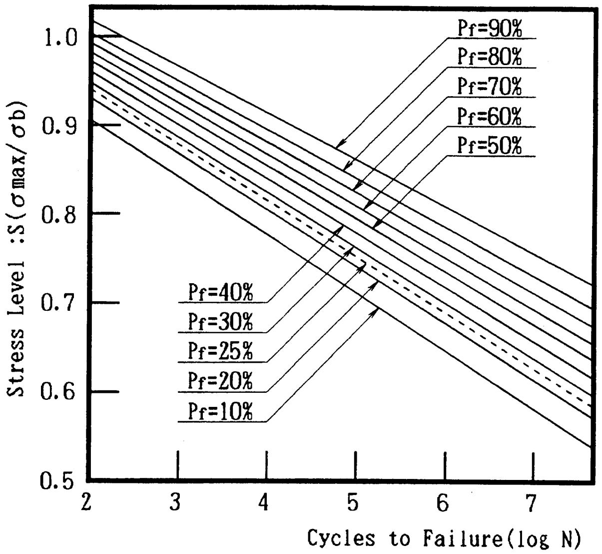

Fig.2 S-N Diagram for Pf

Table 4 Coefficents of Fatigue Strength Equation

P_f (%)	a	b
10	1.037	0.065
20	1.058	0.063
25	1.066	0.063
30	1.072	0.062
40	1.082	0.061
50	1.091	0.059

Of the fatigue curves shown for each failure probability in Fig. 2, a and b for each curve concerning failure probability P_f = 10 to 50% are as given in Table 4, where N ≧100.

INFLUENCE OF SCATTER OF FATIGUE PHENOMENA ON CONCRETE PAVEMENT

As described above, the results of fatigue tests show a fair amount of scatter. And it is considered that the service life of concrete pavements are affected by this scatter. Therefore, in this study, the influence of this scatter was examined by Monte Carlo simulation.

The flow chart of the simulation made in this study is given in Fig. 3. For fatigue analysis of the concrete slab to be the basis for this simulation, the design method of the Manual for Design and Construction of Concrete Pavements[4] was partially modified and used. With regard to this modified design method, the compatibility with performance of actual pavements has been ascertained in the past study by the authors[5].

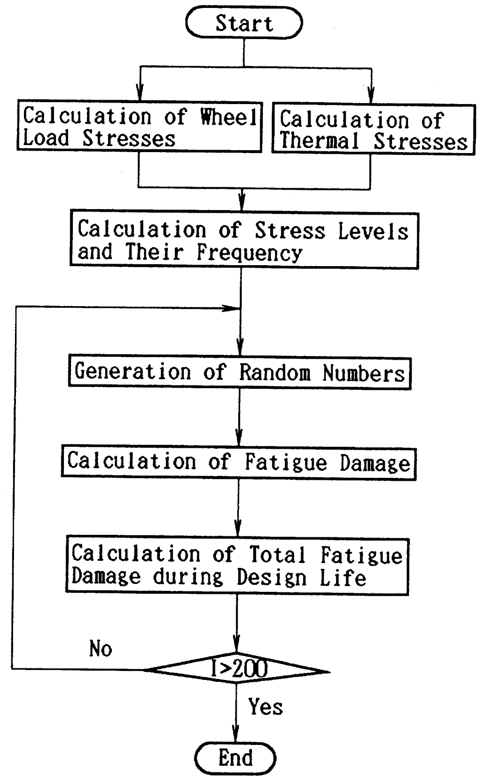

Fig. 3 Flow Chart of Simulation

Table 5 Transverse Distribution of Wheel Load Placement and Its Relative Frequency[6]

Placement of Wheel Load* (cm)	15	45	75	Over 75
Relative Frequency (%)	51	43	5	1

*Distance from the Center of the Most Frequent Passing Position of Wheel Loads

The principal modification points are as follows:

(a) The fatigue characteristics of concrete given as the result of this study are used.
(b) The transverse joint edge of the concrete slab is made the basis of stress analysis.
(c) For lateral position of wheel loads and its frequency, Table 5 giving the results of survey by the authors[6] is used.
(d) For distribution of wheel loads, Table 6 [7] giving the results of measurements by the Ministry of Construction is used.

Table 6 Distribution of Wheel Loads [7]

Traffic Volume Level	M.N.T.V.	M.N.C.V.	Number of Wheel Loads											
			0-1 t	1-2 t	2-3 t	3-4 t	4-5 t	5-6 t	6-7 t	7-8 t	8-9 t	9-10 t	10-12 t	12-14 t
B-traffic	4986	749	8371	810	420	229	119	79	45	27	15	6.7	4.3	0.70
C-traffic	8930	1889	13934	1855	1210	620	335	208	118	59	30	13.0	8.5	1.81
D-traffic	12727	4696	16610	3611	3091	1978	1005	623	378	176	86	36.0	19.0	2.04

M.N.T.V. : Mean Number of Total Vehicles(V/day/direction)
M.N.C.V. : Mean Number of Commercial Vehicles(V/day/direction)

In the simulation, the combined stresses of wheel load stresses and thermal warping stresses are first obtained, and the stress levels and their frequency are calculated from this combined stresses. Next, random numbers corresponding to the frequency are developed. These random numbers were obtained applying the central limiting theory to uniformly random numbers according to the multiplicative congruential method, and were produced according to log-normal distribution having the parameters λ and ζ given in Table 3. The allowable number of cycles to fatigue failure are obtained by the respective random numbers, and by applying Miner's rule to them, the fatigue damage of the concrete slab is calculated. This simulation was made multiplying the number of wheel load passages in Table 6 by 365 for the traffic during one year. And the fatigue damage in design life of 20 years was determined by multiplying the fatigue damage obtained in this simulation by 20. Furthermore, the service life was calculated from the fatigue damage obtained. The number of simulations that were tried was set at 200 times referring to the study on this type of simulation made by the authors[8].

An example is presented, the concrete pavement and the traffic conditions considered were as follows:

Flexural strength of concrete:
σ_b = 45kgf/cm²

Elastic modulus of concrete:
E = 350,000kgf/cm²

Poisson's ratio of concrete:
μ = 0.25

Coefficient of thermal expantion of concrete:
α = 0.00001/°C

Modulus of subbase reaction:
K_{75}= 8kgf/cm³

Concrete slab thickness:
H = 25cm

Traffic volume level:
D-traffic level shown in Table 6

Thermal condition:
moderate climate

Fig. 4 is a histogram indicating the scatter in service life obtained as a result of the simulation for the above calculation condition. Next, this histogram was approximated by the beta-distribution function as indicated in the figure. Here, the area obtained by integrating the zone of service life ≧ 20 yrs of this curve is the reliability of the concrete pavement against failure caused by scatter in fatigue strength of concrete. If this reliability concerning fatigue is represented by P_R, then P_R = 29.4% in this example.

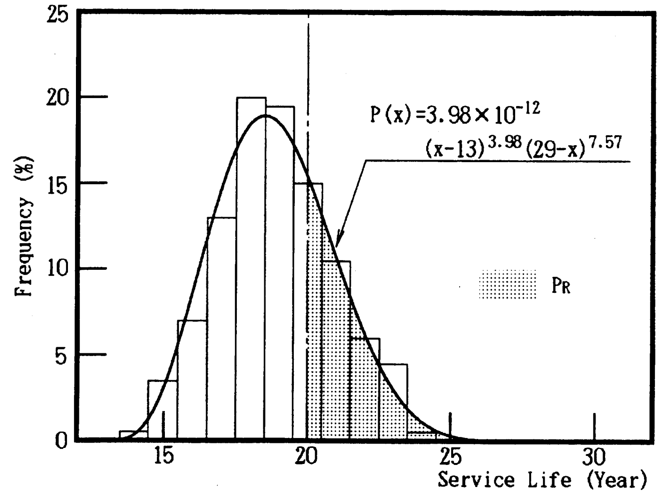

Fig.4 Service Life Distribution

RELATIONSHIP BETWEEN FAILURE PROBABILITY OF FLEXURAL FATIGUE CURVE AND RELIABILITY OF CONCRETE PAVEMENT

In the mechanistic design method of concrete pavements, the flexural fatigue design curve is selected in order to give the design results on the conservative side. In case of the design method of the Manual in Japan[4], the envelope of the experimental values of fatigue failure is selected for the flexural fatigue design curve[9]. And in case of PCA design method, a small probability of fatigue failure is selected[10]. But in such a method, the relationship between failure probability from the flexural fatigue curve and the fatigue damage of a concrete slab is not made distinct, and hence, the reliability of a

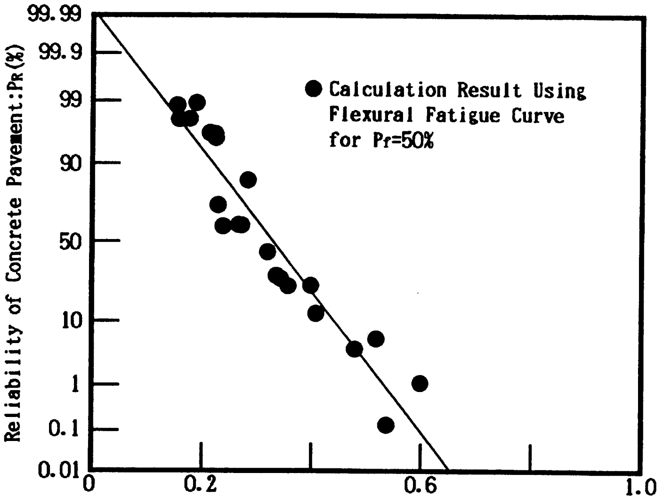

Fig.5 Relationship between Fatigue Damage and Reliability in Case of H=25cm, D-traffic

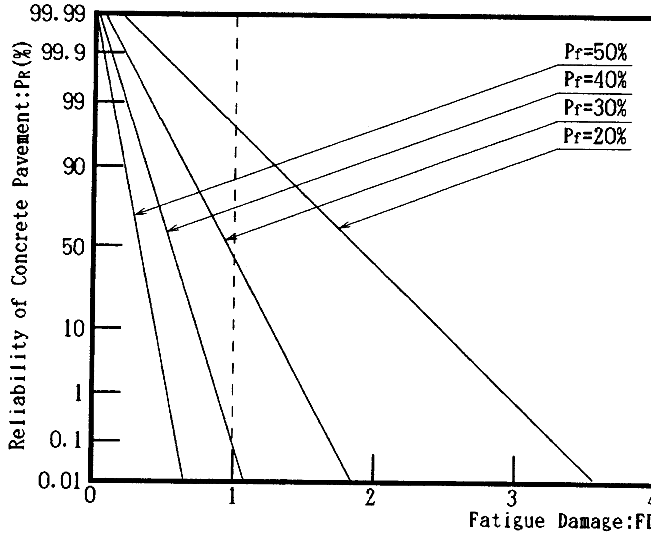

Fig.6 PR-Pf-FD Relationship in Case of H=25cm, D-traffic

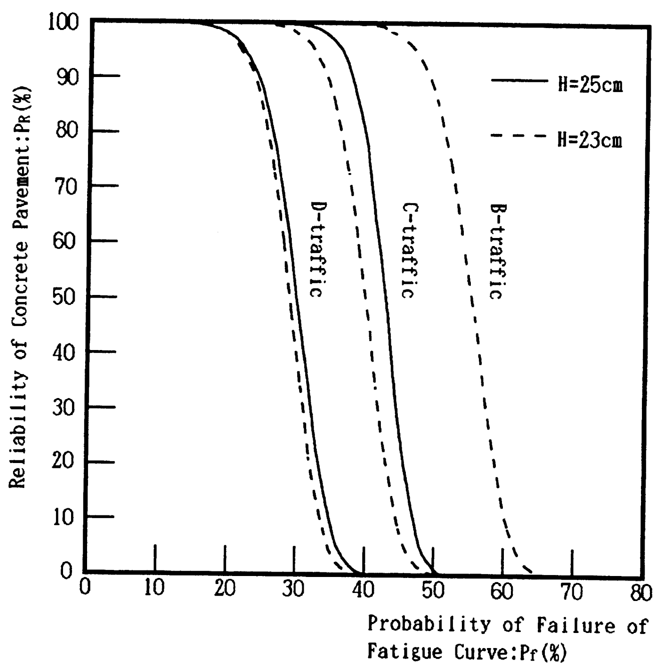

Fig.7 Relationship between PR and Pf

design is indistinct. Therefore, in this study, the relationship between the failure probability from the flexural fatigue curve and the reliability of concrete pavements was examined.

As mentioned above, the reliability concerning fatigue P_R for a certain case of concrete pavement was obtained by Monte Carlo simulation. On the other hand, when analysis is performed deterministically using the flexural fatigue curve of a certain probability of failure P_f, the fatigue damage FD in that case can be obtained. For example, using the flexural fatigue curve for $P_f = 50\%$, the relationship of P_R-FD in case of varying modulus of subbase reaction, elastic modulus and Poisson's ratio of concrete will be as shown in Fig. 5.

According to Fig. 5, that the fatigue damage FD obtained by the deterministic fatigue analysis, and the reliability P_R obtained by simulation have a roughly linear relationship can be seen. Further, this relationship changes to the form shown in Fig. 6 by changing the failure probability P_f of the fatigue curve used in the deterministic analysis. Therefore, on obtaining from this P_R-P_f-FD relationship the relationship of P_R and P_f at fatigue damage FD = 1.0 to be the basis in designing, the result will be the solid line(D-traffic) in Fig. 7. That is, in case of designing using the flexural fatigue curve of failure probability $P_f = 25\%$, the reliability P_R in the design life of 20 years will be approximately 85%.

In the present study, the foregoing analysis was performed for slab thickness of H = 23 and 25cm, and traffic volume level of B, C, and D, and the P_R-P_f-FD relationship for design conditions of even broader scope was determined. The pavement structure conditions made the objects of calculations were selected covering a wide range, but since in the fatigue analysis, the results are the same in case the numbers of cycles for each stress level are the same, when the pavement structure conditions for which calculations were made are arranged in order, they will be as shown in Tables 7 and 8. Further, in case of slab thickness 28cm and over, according to the modified design method used in the present analysis, failure due to fatigue of the concrete slab will not occur during the disign life of 20 years, so that this was excluded as an object of the study.

On comparison of the cases of C-traffic and D-traffic in Fig. 7, when the traffic conditions are the same, the relationship of P_f and P_R may be said to indicate a roughly similar trend irrespective of slab thickness. In effect, it may

Table 7 Calculation Conditions for Slab Thickness of H=23cm

B-traffic					C-traffic					D-traffic				
K_{75}	μ	E	P_R	FD	K_{75}	μ	E	P_R	FD	K_{75}	μ	E	P_R	FD
5	0.20	300000	95.518	0.741	8	0.20	300000	99.726	0.250	9	0.18	300000	99.039	0.164
8	0.25	330000	99.134	0.761	9	0.18	320000	99.608	0.267	10	0.18	310000	97.046	0.163
6	0.20	320000	98.408	0.800	10	0.20	330000	99.109	0.346	10	0.20	310000	94.848	0.213
8	0.20	350000	97.661	0.841	9	0.20	320000	99.619	0.354	10	0.18	320000	92.703	0.246
7	0.25	320000	98.981	0.885	8	0.20	310000	99.264	0.381	9	0.20	300000	92.405	0.215
7	0.20	340000	97.666	0.969	10	0.18	340000	98.922	0.399	9	0.18	310000	78.002	0.248
9	0.25	350000	92.955	1.066	9	0.18	330000	99.476	0.406	8	0.18	300000	73.222	0.263
5	0.20	310000	33.621	1.130	8	0.18	320000	68.775	0.436	10	0.20	320000	67.951	0.322
8	0.25	340000	26.955	1.160	7	0.20	300000	29.268	0.438	9	0.20	310000	64.570	0.326
6	0.20	330000	15.843	1.217	7	0.18	310000	29.310	0.500	10	0.18	330000	46.189	0.373
7	0.25	330000	13.877	1.351	10	0.20	340000	69.147	0.527	8	0.18	310000	10.820	0.400
7	0.20	350000	32.141	1.472	9	0.20	330000	67.113	0.539	8	0.20	300000	3.608	0.347
5	0.25	300000	3.548	1.614	8	0.20	320000	24.098	0.581	7	0.18	300000	2.765	0.452
5	0.20	320000	1.778	1.723	9	0.18	340000	51.795	0.617	9	0.18	320000	1.867	0.377
8	0.25	350000	5.203	1.767	6	0.18	300000	18.363	0.624	10	0.18	340000	1.367	0.564
6	0.20	340000	2.877	1.852	8	0.18	330000	20.323	0.663	9	0.18	330000	1.317	0.570
					7	0.20	310000	20.343	0.669	9	0.20	320000	1.027	0.496
					7	0.18	320000	7.779	0.761	10	0.20	330000	0.751	0.489
					9	0.20	340000	14.049	0.819	8	0.20	310000	0.329	0.528
					6	0.20	300000	1.046	0.840					
					8	0.20	330000	5.463	0.884					

be comprehended that the reliability P_R does not depend on the structure of the concrete pavement, but is determined by the failure probability P_f of the flexural fatigue curve selected and traffic conditions. Furthermore, it may be seen that, according to the same figure, in order to obtain the same level of reliability P_R, the failure probability P_f of the flexural fatigue design curve must be smaller the heavier the traffic.

CONCLUSIONS

This paper reports the results of examinations concerning the effect of scatter in the flexural fatigue phenomena of concrete on fatigue failure of concrete slabs. The main conclusions are as follows:

It was shown by a statistical study on the results of flexural fatigue tests of concrete that the fatigue life for the individual stress levels were shown to respectively belong to a population of log-normal distribution.

By means of the Monte Carlo simulation which took account of the probabilistic properties of fatigue phenomena, the reliability of concrete pavement against fatigue failure was obtained. Further, by applying this simulation to varying conditions of concrete pavements, the relationship of this reliablity and failure probability of fatigue curve was obtained. By using the flexural design fatigue curve selected based on this relationship, it will be possible to assure the reliability of the concrete pavement concerning flexural fatigue phenomena of concrete.

Table 8 Calculation Conditions for Slab Thickness of H=25cm

C-traffic					D-traffic				
K_{75}	μ	E	P_R	FD	K_{75}	μ	E	P_R	FD
5	0.25	320000	99.487	0.375	10	0.25	350000	98.637	0.154
7	0.25	350000	99.403	0.380	7	0.25	320000	97.709	0.158
5	0.20	340000	99.834	0.430	5	0.20	310000	97.776	0.176
5	0.25	330000	88.549	0.567	7	0.20	340000	98.799	0.188
5	0.20	350000	34.479	0.647	6	0.20	330000	96.191	0.213
5	0.25	340000	21.915	0.857	9	0.25	350000	96.104	0.222
					8	0.25	340000	95.487	0.223
					5	0.25	300000	72.786	0.228
					7	0.25	330000	60.519	0.237
					5	0.20	320000	61.485	0.265
					6	0.25	320000	61.135	0.271
					7	0.20	350000	84.309	0.282
					6	0.20	340000	43.873	0.319
					8	0.25	350000	29.422	0.336
					5	0.25	310000	27.777	0.344
					7	0.25	340000	24.103	0.357
					5	0.20	330000	24.119	0.399
					6	0.25	330000	12.323	0.409
					6	0.20	350000	0.479	0.479
					5	0.25	320000	5.870	0.518
					7	0.25	350000	0.157	0.537
					5	0.20	340000	1.151	0.599

It was learned that this relationship, within the scope of pavement structure conditions considered in this study, is not related to the structural characteristics of the pavement, but is affected only by traffic conditions.

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