

A REFINED MODEL OF DOWELED JOINTS FOR CONCRETE PAVEMENT USING FEM ANALYSIS

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Mechanical behavior of concrete slabs near joints should be considered in the thickness design of concrete pavements. Thus, it is necessary to develop joint models for FEM analysis which represent the mechanism of load transfer across joint. In this study, the mechanism of joint is divided into two functions; (a) aggregate interlocking and (b) dowel action and bending action of dowel bar. Then, a refined joint model was developed, which includes these two functions. The numerical calculations were conducted using the model and the results were compared with the experiment data.

Transverse joints are provided in concrete pavements to control cracks due to volume changes of concrete slabs. However, they cause discontinuity in concrete slabs, and are structurally weak. Accordingly, in order to strengthen transverse joints, dowels are incorporated. The use of joints, even if with dowels, may reduce load carrying capacity of the pavements. Furthermore, considering the fact that concrete slabs are subjected to large number of repetitions of traffic loading at limited portions of transverse joints, the joint edge is under very severe stress condition.

In many cases in Japan, concrete pavements are failed by cracking at the transverse joint edges. These cracks are often caused by the fracture of dowels and so loss of load transfer. Thus, the mechanical behavior of the doweled joint should be considered in the design of concrete pavements.

For the analysis of concrete pavement, FEM(Finite Element Method) based on the thin plate theory is available. In order to develop the FEM of concrete pavements, a refined joint model is considered in this study.

The mechanism of load transfer at a joint is composed of aggregate interlocking

action and dowel action, so the joint model for the FEM should take these two functions into consideration. Huang, et al.(1) employed a method in which concrete slabs connected by a joint were deflected such that displacements of concrete slabs corresponded to a specified moment or deflection transfer ratio across the joint. This method needed iterative calculation, so the method was complicated and time-consuming. Next, a shear spring model was used to represent load transfer with dowels(2). Fukute, et al.(3) expanded this model to the aggregate interlocking function. On the other hand, Tabatabaie, et al.(4) used the bar element to represent the dowel function. However the bar element can not be used in the case where crack width is so narrow that the length of the bar element becomes too small and thus its rigidity becomes too high. In this study, a refined joint model is developed to represent various load transfer conditions.

Mechanism of load transfer at joints

Mechanism of load transfer at a joint is divided to two parts:

(a)Aggregate interlocking action

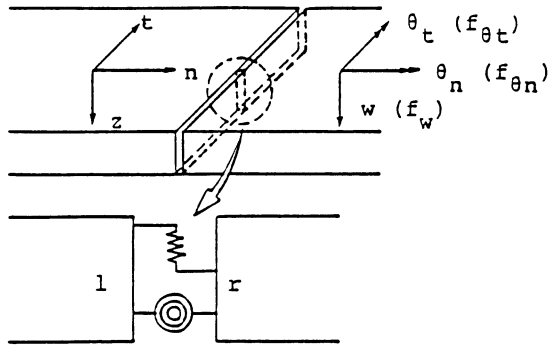
(b)Dowel action

The function (a) is the case for dummy joints and narrow cracks. Since its effect depends on the width of joint or crack opening, the load transfer mechanism varies with seasonal movement of concrete slab. The function (b) depends on interaction of a dowel and its surrounding concrete.

Formulation of joint element

The most simple model of load transfer across a joint is a spring model. Referring to Fig.1, this can be expressed by the following equation;

Figure 1. Load transfer model of a joint.



$$\{f_l - f_r\} = [T] \{d_l - d_r\} \quad (1)$$

in which

$$\{f\} = \langle f_w \ f_{\theta n} \ f_{\theta t} \rangle^T, \quad \{d\} = \langle w \ \theta_n \ \theta_t \rangle^T \quad (2)$$

T indicates transposition of matrix. Subscripts r and l indicate the right and left sides of the joint respectively. [T] is the elastic matrix representing the efficiency of load transfer at a point of the joint.

At this point, the following expression is derived by the virtual work theory;

$$\begin{Bmatrix} f_l \\ f_r \end{Bmatrix} = \begin{bmatrix} T & -T \\ -T & T \end{bmatrix} \cdot \begin{Bmatrix} d_l \\ d_r \end{Bmatrix} \quad (3)$$

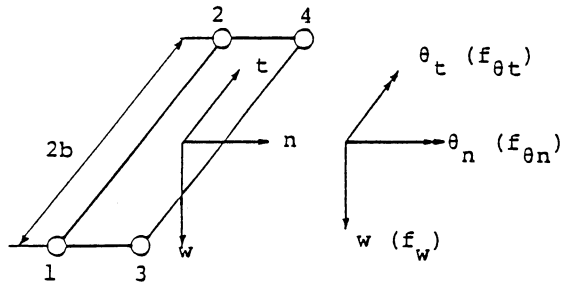
Generalizing Eq.(3), we have

$$\begin{Bmatrix} f_l \\ f_r \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{Bmatrix} d_l \\ d_r \end{Bmatrix} \quad (4)$$

Eq.(4) is the general stiffness equation of load transfer at a point. The components of [T_{ij}] depend on the type of load transfer.

A rectangular element which has 4 nodes and 12 degrees of freedom as shown in Fig.2 is employed in this study for a joint element. Such elements are placed along the joint of concrete pavement. Deflection and

Figure 2. Thin rectangular joint element.



rotations of the joint element are assumed to be expressed by the following equations (5);

$$\begin{aligned} w_l &= \alpha_1 + \alpha_2 t + \alpha_3 t^2 + \alpha_4 t^3 \\ \theta_{n,l} &= -\alpha_2 - 2\alpha_3 t - 3\alpha_4 t^2 \\ \theta_{t,l} &= \alpha_5 + 2\alpha_6 t \\ w_r &= \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 t^3 \\ \theta_{n,r} &= -\beta_2 - 2\beta_3 t - 3\beta_4 t^2 \\ \theta_{t,r} &= \beta_5 + 2\beta_6 t \end{aligned} \quad (5)$$

where t is the coordinate along the joint and $\alpha_1 \sim \alpha_6, \beta_1 \sim \beta_6$ are unknown constants.

According to the formulation procedure of FEM, the following relation between the displacement vector of a given point in the element and those of 4 nodal points is obtained;

$$\begin{Bmatrix} d_l \\ d_r \end{Bmatrix} = \begin{bmatrix} [B] [C]^{-1} & [0] \\ [0] & [B] [C]^{-1} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} \quad (6)$$

or

$$\{d_r\} = [B(t)] \{d_*\} \quad (7)$$

where

$$\{d_r\} = \langle d_l \ d_r \rangle^T$$

$$\{d_*\} = \langle d_1 \ d_2 \ d_3 \ d_4 \rangle^T$$

$$[B(t)] = \begin{bmatrix} [B] [C]^{-1} & [0] \\ [0] & [B] [C]^{-1} \end{bmatrix}$$

$$[B] = \begin{bmatrix} 1 & t & t^2 & t^3 & 0 & 0 \\ 0 & 1 & 2t & 3t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2t \end{bmatrix}$$

$$[C]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -3/4b^2 & 1/b & 0 & 3/4b^2 & 1/2b & 0 \\ 1/4b^3 & -1/4b^2 & 0 & -1/4b^3 & -1/4b^2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1/4b & 0 & 0 & 1/4b \end{bmatrix} \quad (8)$$

Applying the virtual work theory to the element, the stiffness matrix of the element can be obtained as follows;

$$\{f_*\} = [J_*] \{d_*\} \quad (9)$$

As stated above, the [J_e] can be divided into two parts: the aggregate interlocking action [J_{eI}] and the dowel action [J_{eS}] as follows;

$$[J_*] = [J_{*I}] + [J_{*S}] \quad (10)$$

where

$$[J_{*I}] = \int [B(t)]^T [T_I] [B(t)] dt \quad (11a)$$

Table 1. Input data for computation to obtain the relation between κ and e_{ff}

Item (unit)	Value
Young's modulus of concrete (kg/cm^2)	300,000
Poisson ratio of concrete	0.2
Thickness of concrete slab (cm)	25 and 30
Modulus of subbase reaction (kg/cm^3)	10, 30 and 50
Modulus of interlocking k (kg/cm^3)	between 10 and 10,000
load (t)	10 (Joint edge loading)
Dowel bar	None

Note: $1\text{kg/cm}^2=0.098\text{MPa}$, $1\text{kg/cm}^3=9.8\text{MPa/m}$

$$[J_{es}] = [B(t_s)]^T [T_s] [B(t_s)] \quad (11b)$$

Since the aggregate interlocking action is assumed to distribute over the joint face of the concrete slab, the integration over the element in Eq.(11a) is needed. On the other hand, load transfer of the dowel is done only at dowel position t_s , and thus integration over the element is unnecessary. We would like to name the joint element based on Eq.(11a) as the interlocking element, and also that based on Eq.(11b) as the dowel element.

Interlocking element

Since load transferred by the aggregate interlocking is shear force, $[T_I]$ in Eq.(11a) can be expressed as follows;

$$[T_I] = \begin{bmatrix} \kappa h & 0 & 0 & -\kappa h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\kappa h & 0 & 0 & \kappa h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

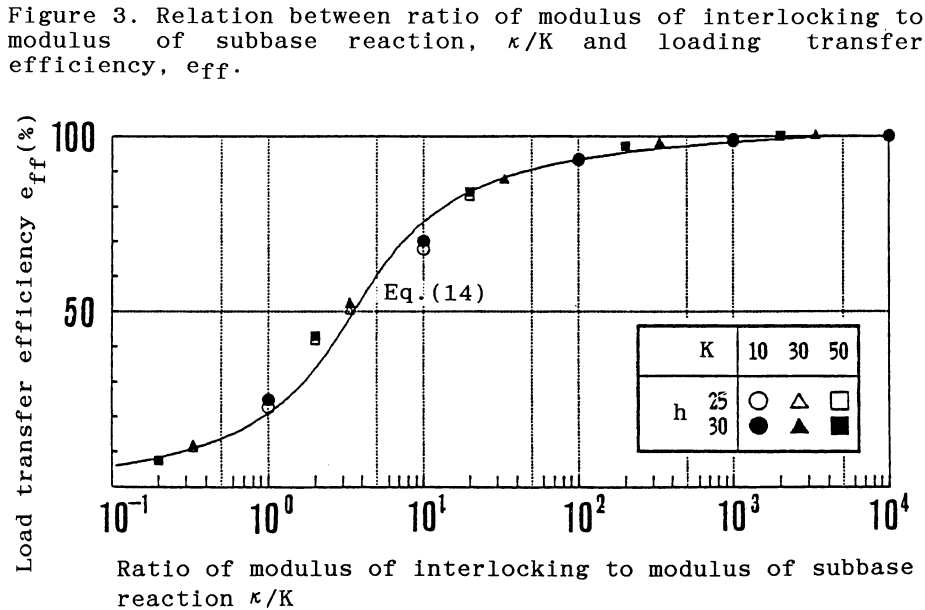
where h is thickness of the concrete slab. κ is the modulus of interlocking which means shear force per unit area transferred by the aggregate interlocking at the joint face.

As mentioned earlier, κ depends on the width of joint opening, so it is practical to relate κ to the width of joint opening. Since it is almost impossible to derive the relation theoretically, we obtained the relation from numerical calculations. In the calculations, an efficiency of load transfer is defined as

$$e_{ff} = s_1/s_2 \times 100(\%) \quad (13)$$

where s_1 and s_2 are the maximum deflections of loaded slab and unloaded slab respectively. e_{ff} was calculated by varying κ , K and h as summarized in Table 1.

According to Colley, et al.(6), e_{ff} is affected by the modulus of subbase reaction, K , and thus the relation between e_{ff} , κ and K should be established. Fig.3 shows the relation between e_{ff} and κ/K , which was found to be expressed by the following equation;



Note: h =slab thickness(cm),
 K =modulus of subbase reaction(kg/cm^3),
 $1\text{kg/cm}^2=0.098\text{MPa}$, $1\text{kg/cm}^3=9.8\text{MPa/m}$

$$\log_{10} \kappa / K = 0.55 \frac{e_{ff}^2 - 183e_{ff} + 3383}{e_{ff}^2 - 100e_{ff} - 749} \quad (14)$$

On the other hand, another study (7) reported that there is the following linear relation between e_{ff} and width of joint opening;

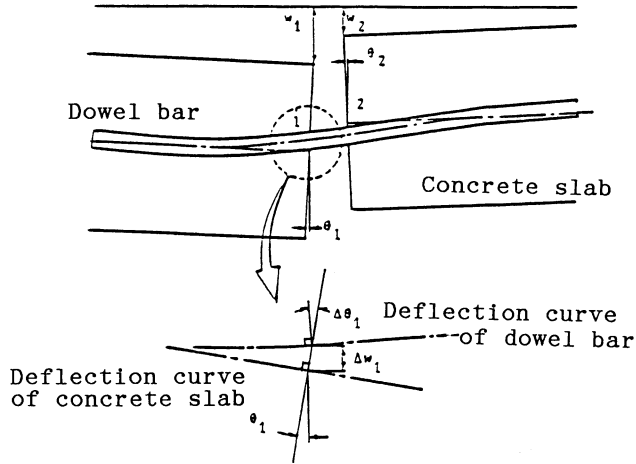
$$e_{ff} = 100 - 25xB \quad (15)$$

where B is the width of joint opening in mm. Hence, from Eqs. (14) and (15), we can obtain the relation between κ and B .

Dowel element

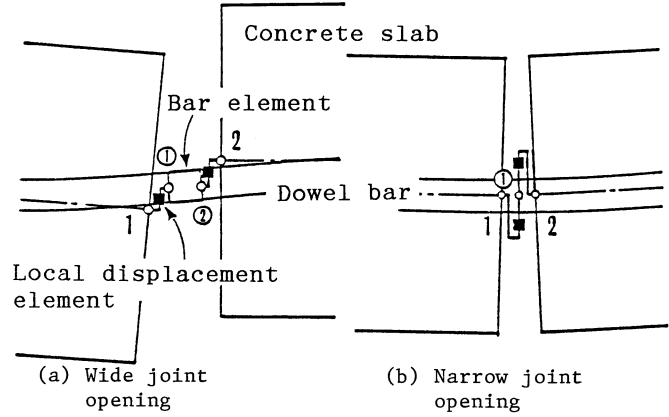
To consider the dowel function in the finite element model, Tabatabaie, et al. (4) used the bar element. However, in the case of very narrow width of joint opening, its rigidity becomes unrealistically large. This is caused by the assumption that the displacements at both end nodes of the bar element are the same as those of the slab element. In fact, there are small differences in those displacements as illustrated in Fig. 4. These differences result from local displacement of the surrounding concrete and can be evaluated by the theory of a bar supported by an elastic foundation.

Figure 4. Differences in displacements between dowel and concrete slab.



Considering inner nodes ① and ② which separate the nodes attached to the bar element from those of the slab elements, additional elements representing the differences in displacements between the slab and the bar are incorporated between these nodes, shown as Fig. 5. We call them the local displacement elements.

Figure 5. The refined dowel element.



The differences in displacement between the nodes 1 and ① can be expressed as (8);

$$\begin{Bmatrix} \Delta w \\ \Delta \theta t \end{Bmatrix} = \frac{1}{2\beta^2 EI A_3} \cdot \begin{bmatrix} (SC - sc) / \beta \\ (S^2 + s^2) \\ (S^2 + s^2) \\ 2\beta(SC + sc) \end{bmatrix} \cdot \begin{Bmatrix} \Delta f w \\ \Delta f \theta t \end{Bmatrix} \quad (16a)$$

In similar way, for the nodes 2 and ②,

$$\begin{Bmatrix} \Delta w \\ \Delta \theta t \end{Bmatrix} = \frac{1}{2\beta^2 EI A_3} \cdot \begin{bmatrix} (SC - sc) / \beta \\ -(S^2 + s^2) \\ -(S^2 + s^2) \\ 2\beta(SC + sc) \end{bmatrix} \cdot \begin{Bmatrix} \Delta f w \\ \Delta f \theta t \end{Bmatrix} \quad (16b)$$

where $\beta = \sqrt[4]{k\phi / 4EI}$, $A_3 = (S^2 - s^2)$
 $S = \sinh \beta l$, $C = \cosh \beta l$, $s = \sin \beta l$, $c = \cos \beta l$ (17)

EI : flexure rigidity of dowel,
 l : length of embedded dowel in concrete,
 k : modulus of dowel support,
 ϕ : diameter of dowel,
 and Δ indicates the difference in the nodal displacements and nodal forces.

Solving Eqs. (16a) and (16b) for the displacements, we have

$$\begin{Bmatrix} \Delta f w \\ \Delta f \theta t \end{Bmatrix} = \frac{2\beta^2 EI (S^2 - s^2)}{S^2 C^2 - 2S^2 c^2 - s^2 C^2 - S^4 - s^4} \cdot \begin{bmatrix} 2\beta(SC + sc) & -(S^2 + s^2) \\ -(S^2 + s^2) & (SC - sc) / \beta \end{bmatrix} \cdot \begin{Bmatrix} \Delta w \\ \Delta \theta \end{Bmatrix} \quad \text{for node 1} \quad (18a)$$

$$\begin{Bmatrix} \Delta f w \\ \Delta f \theta t \end{Bmatrix} = \frac{2\beta^2 EI (S^2 - s^2)}{S^2 C^2 - 2S^2 c^2 - s^2 C^2 - S^4 - s^4} \cdot \begin{bmatrix} 2\beta(SC + sc) & S^2 + s^2 \\ S^2 + s^2 & (SC - sc) / \beta \end{bmatrix} \cdot \begin{Bmatrix} \Delta w \\ \Delta \theta \end{Bmatrix} \quad \text{for node 2} \quad (18b)$$

or

$$\{\Delta f\} = [M_1] \{\Delta d\} \quad \text{for node 1} \quad (19a)$$

$$\{\Delta f\} = [M_2] \{\Delta d\} \quad \text{for node 2} \quad (19b)$$

where

$$\begin{aligned} \{\Delta f\} &= \langle \Delta f_w \quad \Delta f_{\theta t} \rangle, \quad \{\Delta d\} = \langle \Delta w \quad \Delta \theta t \rangle \\ [M_1] &= \frac{2\beta^3 EI (S^2 - s^2)}{S^2 C^2 - 2S^2 c^2 - s^2 C^2 - S^4 - s^4} \\ &\quad \cdot \begin{bmatrix} 2\beta (SC + sc) & -S^2 + s^2 \\ -S^2 + s^2 & (SC - sc)/\beta \end{bmatrix} \\ [M_2] &= \frac{2\beta^3 EI (S^2 - s^2)}{S^2 C^2 - 2S^2 c^2 - s^2 C^2 - S^4 - s^4} \\ &\quad \cdot \begin{bmatrix} 2\beta (SC + sc) & S^2 + s^2 \\ S^2 + s^2 & (SC - sc)/\beta \end{bmatrix} \end{aligned} \quad (20)$$

Based on Eqs. (19a) and (19b), the stiffness matrix of the local displacement element is derived from the virtual work principle as follows;

$$\begin{Bmatrix} f_1 \\ f_0 \end{Bmatrix} = \begin{bmatrix} M_1 & -M_1 \\ -M_1 & M_1 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_0 \end{Bmatrix} \quad \text{for node 1} \quad (21a)$$

and

$$\begin{Bmatrix} f_2 \\ f_0 \end{Bmatrix} = \begin{bmatrix} M_2 & -M_2 \\ -M_2 & M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_2 \\ d_0 \end{Bmatrix} \quad \text{for node 2} \quad (21b)$$

The stiffness equation of the bar element is

$$\begin{Bmatrix} f_0 \\ f_0 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \cdot \begin{Bmatrix} d_0 \\ d_0 \end{Bmatrix} \quad (22)$$

where

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \frac{EI}{B} \begin{bmatrix} 12/B^2 & -6/B & -12/B^2 & -6/B \\ & 4 & 6/B & 2 \\ & & 12/B^2 & 6/B \\ \text{sym} & & & 4 \end{bmatrix} \quad (23)$$

and B indicates the width of joint opening. Eqs. (21a), (21b) and (22) are superimposed to formulate the stiffness equation of the dowel as follows;

$$\begin{Bmatrix} f_1 \\ f_0 \\ f_0 \\ f_2 \end{Bmatrix} = \begin{bmatrix} M_1 & -M_1 & 0 & 0 \\ -M_1 & M_1 + K_{11} & K_{12} & 0 \\ 0 & K_{21} & M_2 + K_{22} & -M_2 \\ 0 & 0 & -M_2 & M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_0 \\ d_0 \\ d_2 \end{Bmatrix} \quad (24)$$

Since no external force is applied at the inner nodes, $f_1 = f_2 = 0$. Thus,

$$\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \begin{bmatrix} M_1 + K_{11} & K_{12} \\ K_{21} & M_2 + K_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_0 \\ d_0 \end{Bmatrix} \quad (25)$$

Then Eq. (24) becomes

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \cdot \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} - \begin{bmatrix} M_1 + K_{11} & K_{12} \\ K_{21} & M_2 + K_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \quad (26)$$

In the case of a small narrow width joint opening, the bar element is removed and the local displacement elements are directly connected at the inner node ① as shown Fig.5b. Then,

$$\begin{Bmatrix} f_1 \\ f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} M_1 & -M_1 & 0 \\ -M_1 & M_1 + M_2 & -M_2 \\ 0 & -M_2 & M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_1 \\ d_2 \end{Bmatrix} \quad (27)$$

Again, external force is not applied at the inner node, that is $f_0 = 0$, thus,

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \cdot \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} - \begin{bmatrix} M^{-1}M_1 & M^{-1}M_2 \\ M^{-1}M_1 & M^{-1}M_2 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \quad (28)$$

where

$$[M^{-1}] = \frac{1}{4\beta^2 EI A_3} \begin{bmatrix} (SC - sc)/\beta & 0 \\ 0 & 2\beta (SC + sc) \end{bmatrix} \quad (29)$$

Eqs. (26) and (28) are the stiffness equations for the dowel in wide and narrow width joint openings respectively. The limit value of the width of joint opening in which Eq. (27) can be used was found to be 1mm from the result of numerical calculations. In practice, if the width of joint opening is less than 1mm, Eq. (28) will be used, otherwise Eq. (26).

The 4x4 matrices in Eqs. (26) and (28) are expanded to 6x6 matrices of $[T_S]$ in Eq. (16a) by adding components corresponding to the terms, θ_n and $f_{\theta n}$. The element based on Eqs. (26) and (28) is named the refined dowel element (RDE).

Modulus of dowel support

The modulus of dowel support is a measure of the support of the surrounding

concrete as the dowel is deflected. In our study, the following linear relation is assumed;

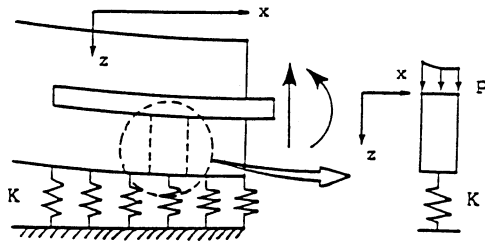
$$p = k \Delta w \quad (30)$$

in which p : vertical contact stress between dowel and concrete,
 k : modulus of dowel support,
 Δw : local deflection of concrete.

Yoder, et al.(9) suggested that its value is between 8,300 and 41,500kg/cm³, and Tabatabaie(4) summarized the values between 8,000 and 250,000kg/cm³ from the previous studies. Accordingly, the value is vague and can not be exactly determined.

In our study, it is assumed that surrounding concrete is an elastic solid as shown in Fig.6, and the modulus is related to the characteristics of the concrete slab.

Figure 6. Model for estimation of modulus of dowel.



The relation between the vertical stress and strain is

$$\sigma_z = (\lambda + 2G) \epsilon_z \quad (31)$$

where λ and G are constants of concrete elasticity. And the relation between the vertical strain and deflection is

$$\epsilon_z = \partial w(x, z) / \partial z \quad (32)$$

The vertical strain is assumed to be distributed in the concrete as follows;

$$\epsilon_z = A(x) \Phi(z) \quad (33)$$

Then, the deflection is

$$w(x, z) = \int \epsilon_z dz = A(x) \int \Phi(z) dz \quad (34)$$

And Eq.(31) becomes

$$\sigma_z = (\lambda + 2G) A(x) \Phi(z) \quad (35)$$

Substituting Eqs. (34) and (35) into Eq. (30), the modulus of dowel support is expressed as follows;

$$k = \frac{p}{\Delta w} = \frac{\sigma_z(x, 0)}{w(x, 0) - w(x, h)}$$

$$= \frac{(\lambda + 2G) A(x) \Phi(0)}{A(x) \int \Phi(z) dz} = \frac{(\lambda + 2G) \Phi(0)}{\int \Phi(z) dz} \quad (36)$$

Therefore, if $\Phi(z)$ is determined, the value of k can be estimated by Eq.(36). Fig.7 shows ϵ_z distribution measured by Teller, et al.(10) From this figure, it was found that the following function can approximate this strain distribution;

$$\Phi(z) = \frac{(1 - z')}{(z' + 1)^3} \quad (37)$$

where $z' = z/h'$ and $h' = (h - \phi)/2$. Substituting Eq.(37) into Eq.(36), we can obtain

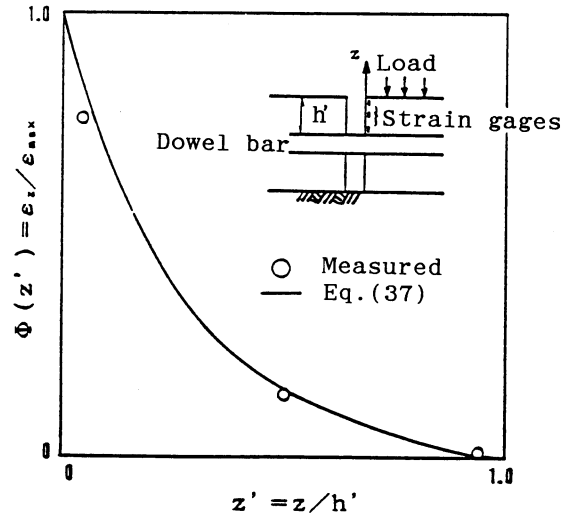
$$k = \frac{0.25 (\lambda + 2G)}{h'} \quad (38)$$

or

$$k = \frac{8.0 (1 - \mu) E}{(1 + \mu) (1 - 2\mu) (h - \phi)} \quad (39)$$

According to Eq.(39), assuming $h=25$ cm, $E=300,000$ kg/cm², $\mu=0.2$, $\phi=25$ mm, then $k=120,000$ kg/cm³.

Figure 7. Distribution of vertical strain in joint face (Teller et al.).



Verification

Laboratory experiment

The results of a model experiment were analyzed by the proposed FEM model. The values used in the analysis were based on the measured ones in the experiment as summarized in Table 2.

Table 2. Input data for FEM analysis of laboratory experiment.

Item (unit)	Value
Young's modulus of concrete (kg/cm ²)	240,000
Poisson ratio of concrete	0.22
Thickness of concrete slab (cm)	10
Modulus of subbase reaction (kg/cm ³)	10
load (t)	3 (Joint edge loading)
Diameter of loading plate (cm)	30
Diameter of dowel (mm)	11 and 23
Spacing of dowel (mm)	25
Young's modulus of dowel (kg/cm ²)	2,100,000

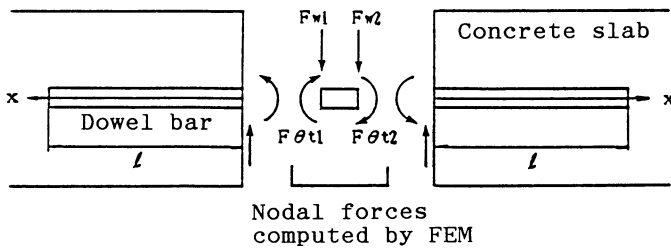
Note: 1kg/cm²=0.098MPa, 1kg/cm³=9.8MPa/m, 1t=9.8kN

In the FEM analysis, only nodal forces of joint elements are obtained. Therefore the strain distribution in a dowel should be estimated from the nodal forces by using the solution of the bar on the elastic foundation as shown in Fig.8. Then, the

moment distribution in the dowel can be calculated by the following equation;

$$M = -\frac{1}{(S^2 - s^2)} \left[\frac{F_w}{\beta} \{ S \cdot G_1(x, l-x) - s \cdot G_1(l-x, x) \} - F \theta t \{ S \cdot G_4(x, l-x) + S \cdot G_3(x, l-x) - s \cdot G_4(l-x, x) - s \cdot G_3(l-x, x) \} \right] \quad (40)$$

Figure 8. Model for calculation of moment in a dowel.



where $S = \sinh \beta l$, $s = \sin \beta l$

$$G_1(\xi, \eta) = \sin \beta \xi \cdot \sinh \beta \eta$$

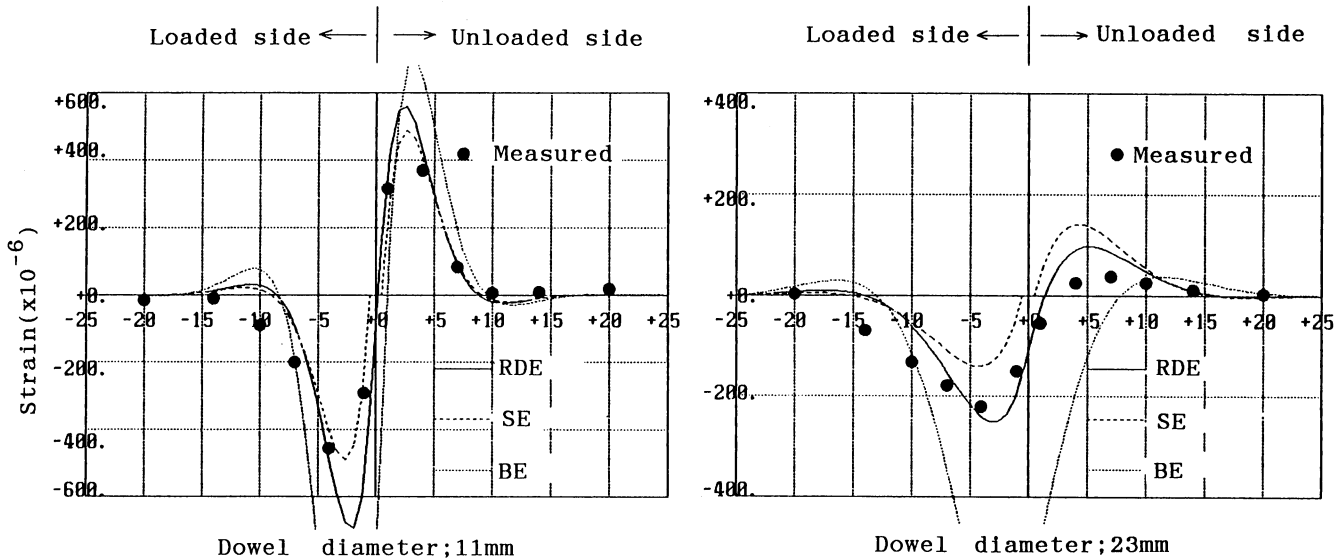
$$G_3(\xi, \eta) = \sin \beta \xi \cdot \cosh \beta \eta$$

$$G_4(\xi, \eta) = \cos \beta \xi \cdot \sinh \beta \eta$$

(41)

Fig.9 shows the strain distributions in the dowel comparing the FEM solutions with the experimental results. Three types of joint elements were used in the analysis; the spring element(SE), the bar

Figure 9. Comparison of strain distributions between FEM solutions and experiment results.



Note: RDE(Refined dowel element); SE(Spring element); BE(Bar element)

element(BE) and the refined dowel element(RDE). In the case of small diameter dowels, the results of the analysis with the SE and the RDE agree with that of the experiment. The calculated strains with the BE is much larger than the measured ones. In the case of large diameter dowels, on the other hand, the measured strain distributions are closer to those of the RDE and the BE. Hence, it could be concluded that the RDE is the most applicable to doweled joints for various dowel diameters.

Full-scale experiment(11)

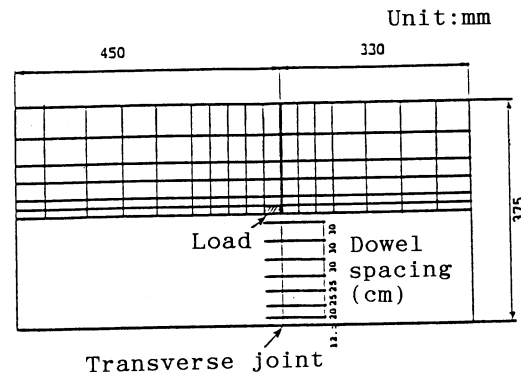
Some results of full-scale experiments conducted by Iwama at the Research Institute, Ministry of Construction were used to verify the joint model. The input data for the FEM analysis are summarized in Table 3. The discretization of the pavements, the dowel placements and the loading condition are illustrated in Fig.10.

Fig.11 shows comparisons of FEM solutions with results of experiments in the case of loading at the corner of a dummy joint without dowel. In this type of joint, load transfer is achieved by only aggregate interlocking, so interlocking elements were used in FEM. In computations, the width of joint opening was assumed to be 1mm. The strains computed by FEM agree with measured ones, especially in loaded slabs.

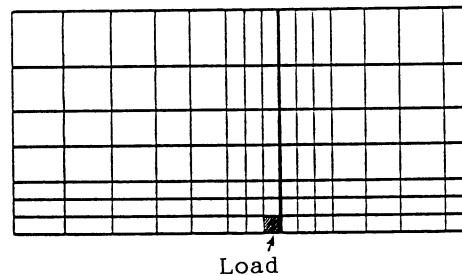
Figs.12 and 13 show the same comparisons in the case of loading at the edge and corner of a dummy joint with dowels respectively. In these cases, load transfer is achieved by aggregate interlocking and dowels, thus RDE as well as interlocking elements were used in FEM. The width of joint opening assumed in computations is also 1mm. Again, good agreements of FEM solutions with the measured data are recognized.

Fig.14 shows relations between the joint efficiency and the width of joint opening in dummy joints with and without dowels. The relations expressed by the

Figure 10. Discretization of pavements and dowel placement.



(a) Joint edge loading



(b) Joint corner loading

straight lines in the figure are reported in Ref.(7). As well known, joint efficiency of doweled joint little depends on the width of joint opening. The dotted line illustrates this tendency. FEM solutions agree with these lines. In this figure, joint efficiency based on flexure stress of the concrete slab computed by FEM is also presented. Apparently, such efficiency is much smaller than that based on deflection.

Conclusions

In this paper, for the analysis of concrete pavements by FEM, the refined

Table 3. Input data for FEM analysis of Iwama's experiment.

Item (unit)	Value
Young's modulus of concrete (kg/cm^2)	340,000
Poisson ratio of concrete	0.27
Thickness of concrete slab (cm)	19.9 and 24.9
Modulus of subbase reaction (kg/cm^3)	3.5
load (t)	5 and 8
Diameter of loading plate (cm)	30
Diameter of dowel (mm)	20 and 25
Spacing of dowel (mm)	40

Note: $1\text{kg/cm}^2=0.098\text{MPa}$, $1\text{kg/cm}^3=9.8\text{MPa/m}$, $1\text{t}=9.8\text{kN}$

Figure 11. Comparisons of FEM solutions with results of experiments loaded at the corner of dummy joint without dowel.

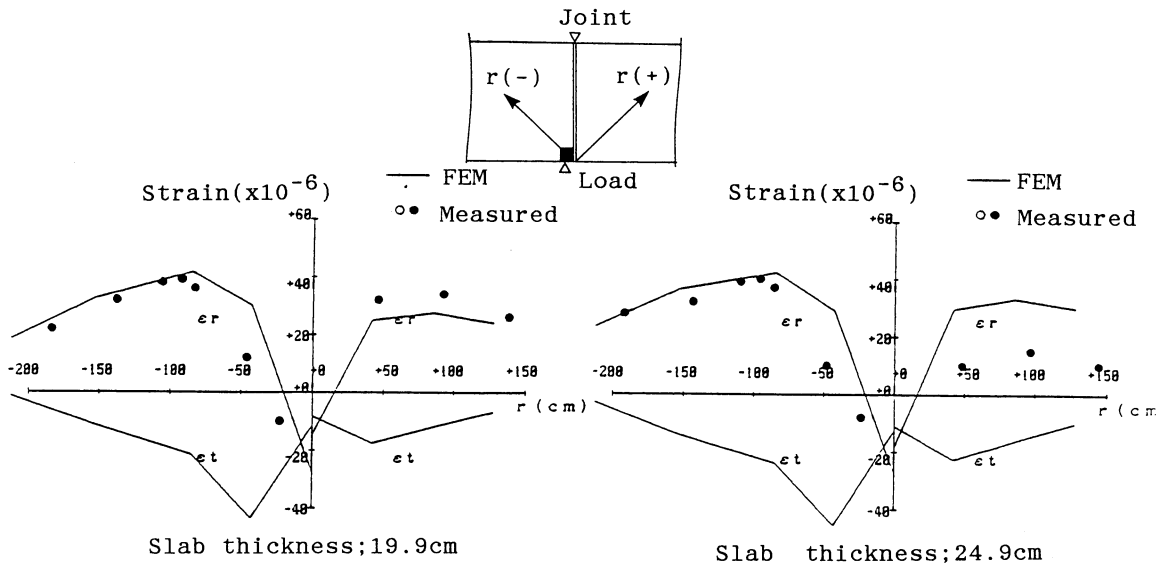


Figure 12. Comparisons of FEM solutions with results of experiments loaded at the edge of dummy joint with dowel.

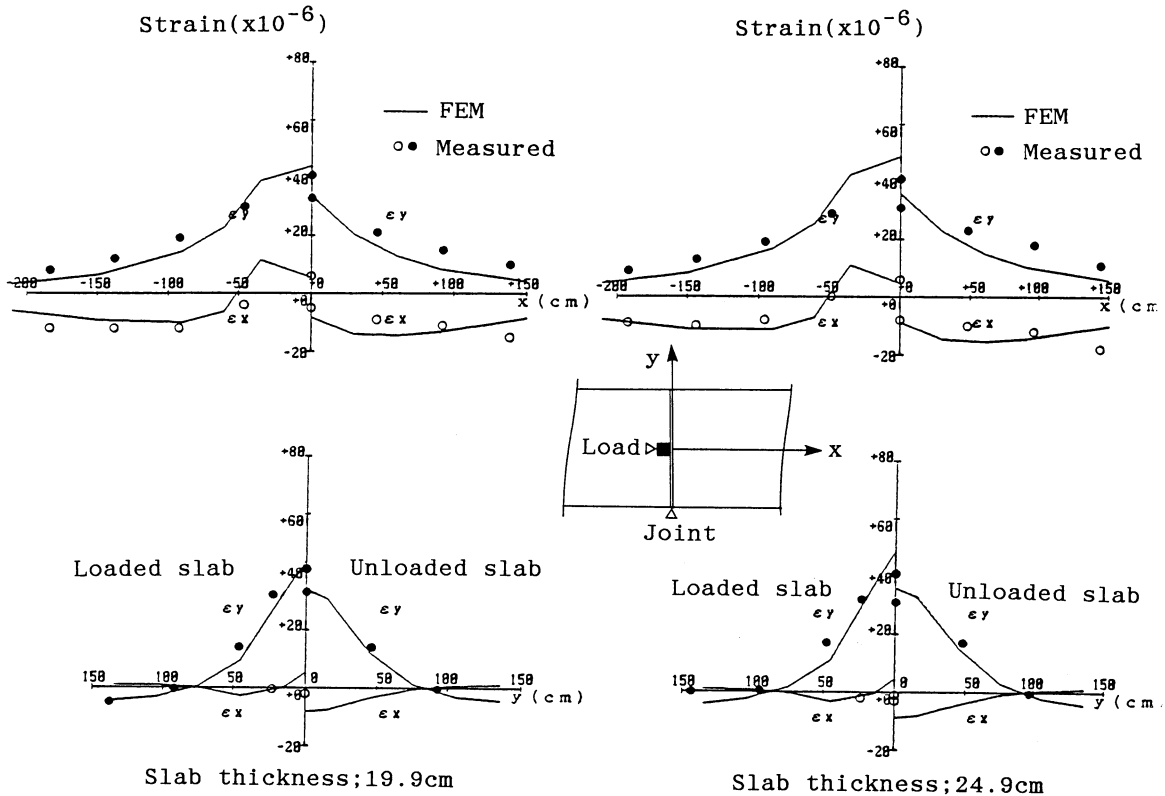


Figure 13. Comparisons of FEM solutions with results of experiments loaded at the corner of dummy joint with dowel.

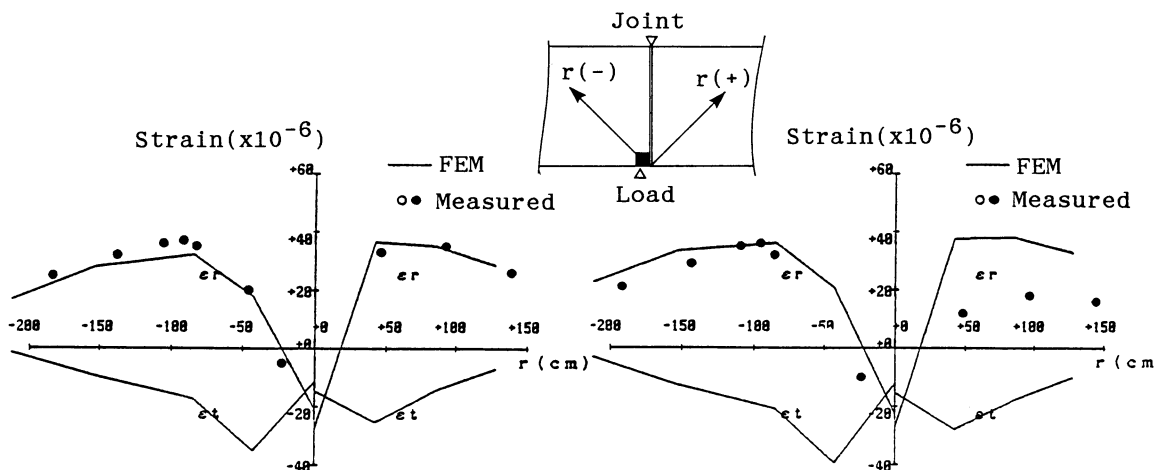
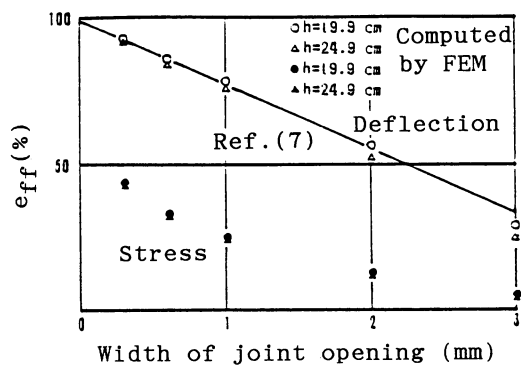
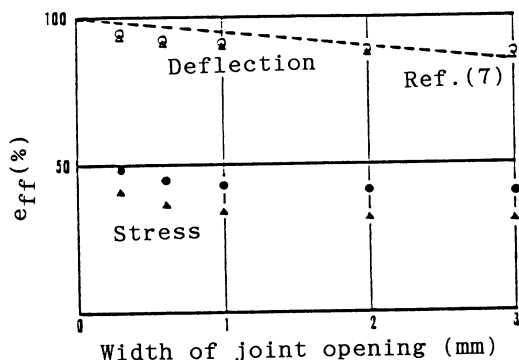


Figure 14. Relations between joint efficiency and width of joint opening.



(a) Non-doweled joint



(b) Doweled joint

Note: $eff = s_2/s_1 \times 100$,
 s_1 = maximum value in loaded slab
 s_2 = maximum value in unloaded slab
 h = thickness of concrete slab

joint element is formulated. Load transfer across a joint is considered by two functions: aggregate interlocking action and dowel action. The interlocking element and dowel element are derived for respective functions by using the standard formulation procedure of FEM. Especially, the dowel element is refined by combining the bar element and the local displacement element.

In the interlocking element, the modulus of aggregate interlocking, k is introduced to express load transfer efficiency by the aggregate interlocking. In the dowel element, the modulus of dowel support is also related to mechanical properties of concrete slabs.

From the comparison between FEM solutions and the results of laboratory and full-scale experiments, it is observed that the refined joint element could express load transfer of the joint and reasonably predict behaviors of not only concrete slabs but also dowels.

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