

DEVELOPMENT OF NEW DESIGN METHOD FOR CONTROL OF CRACKING IN CONTINUOUSLY REINFORCED CONCRETE PAVEMENT

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This paper contains a detailed description on the crack controlling method for Continuously Reinforced Concrete Pavement (CRCP). A longitudinal crack originated from the transverse cracks is considered as a failure criteria in CRCP. As the possibility of this cracking depends on the load transfer efficiency at the transverse crack, the crack control, namely controlling both crack width and crack spacing, is thought as a key stone in designing CRCP. This study consists of the following steps. Firstly new formulas for calculating stress in the reinforcement have been derived. By using those, crack spacing and crack width can be estimated with accuracy. And then, full scale test pavements have been constructed and the behaviour of the pavements has been monitored continuously from the construction. Finally, the validity of those formulas has been verified by various data obtained in the test pavement. By using the new formulas, it is confirmed that the position of the reinforcement adopted in this test pavement is supported to be theoretically reasonable.

Introduction

Concrete pavements are considered suitable for airport pavement; for airport pavement has to carry repetitive heavy aircraft loadings. In particular, Continuously Reinforced Concrete Pavement (CRCP) not only provides comfortable riding quality but also lessens maintenance and/or rehabilitation works since it can eliminate transverse joints.

Needless to say, the stress in the concrete slab is caused by aircraft loadings. In addition to this, the stress will occur if the movement of the concrete slab due to temperature change and drying shrinkage is restrained by the subgrade. Although the maximum stress occurs along joints in the case of jointed concrete pavements, the maximum stress occurs along transverse cracks in CRCP; that is, a longitudinal crack originated from the transverse cracks is considered as a failure criteria in CRCP. The longitudinal cracks will be caused unless an efficiency of load transfer at the transverse cracks is sufficient. As it is necessary for keeping good load transfer to close the crack tightly, the crack control, namely controlling both crack width and

crack spacing, is thought as a key stone in designing CRCP.

Continuous reinforcing steel bars are embedded longitudinally in the slab to close the crack tightly. In the current design method of CRCP in Japan, Vetter's equation has been used. (1) This equation has been originally developed to calculate the amount of reinforcements when they are installed in the mid-depth of the concrete slab. However, in most cases, the reinforcements have been embedded in the upper portion. This means that the basic assumption in Vetter's equation is not fulfilled.

The authors have developed a new design method for CRCP of which principle is to determine the amount and position of reinforcements enough to close the transverse cracks tightly. The new formula can calculate crack width through the slab thickness and stress in the reinforcing steel bars regardless the position of the reinforcement in the concrete slab. In these equations, characteristics of bond between the reinforcement and the concrete as well as mechanical properties of materials such as creep, drying shrinkage and thermal expansion of concrete besides elastic properties of concrete and steel bars are taken into consideration. By using this formula, crack spacings and crack width in the future can also be estimated. These kinds of information are very important since stress in the steel bars varies with the crack spacings, which are, needless to say, related with crack width in CRCP.

Crack controlling procedure is considered as essential in the design method for CRCP. Therefore, this is mainly discussed in this paper.

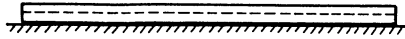
Development of Equations for Controlling Cracks in CRCP

Mechanism of Cracking in CRCP

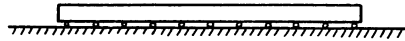
Stress and strains vary with respect of length of CRCP as shown in Figure 1, since its deformation due to drying shrinkage and temperature change (drop) of concrete is restrained by subgrade. At ends of the concrete slab, freely contracting strain is observed. However, deformation of concrete slab becomes to be restrained as a distance from the ends increases, and then remote from the ends the strain becomes constant. In accordance with the strain

Figure 1. Mechanism of restraint of CRCP before Cracking

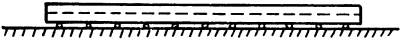
[I] State before contraction



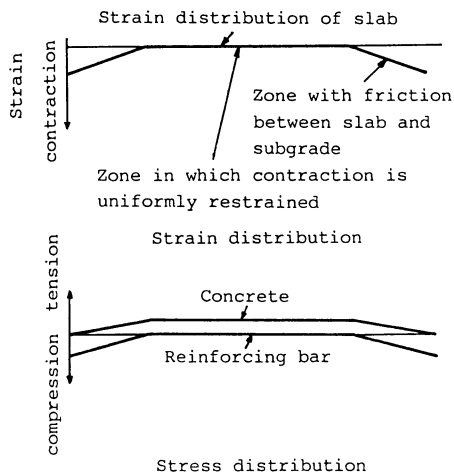
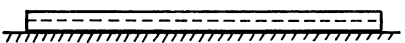
[II] State of contraction without restraint



[III] State of contraction restrained by reinforcing bar only



[IV] State of contraction by both subgrade and reinforcing bar



distribution described above, the maximum stress occurs in the interior portion of the concrete slab (that is, restrained region). As stress condition in CRCP is considered as mentioned above, many cracks with equal crack spacings will be caused by drying shrinkage and temperature drop.

An element of CRCP bounded by transverse cracks, within the fully restrained region, deforms due to drying shrinkage and temperature drop as shown in Figure 2. The right end of this figure shows the section at the crack and the left end designates the section at mid-length of the element. This figure also shows distribution of strain and stress in the element. They vary with distance from the crack; that is, the concrete slab can contract freely at the cracked section, and inside its movement is restrained by the subgrade. Remote from the cracked section, compressive strain is observed in the reinforcement and the bond stress between concrete and reinforcement is not caused because the reinforcement bonds the concrete fully. The strain and the stress varies outwards to the cracked section, especially the reinforcement strain changes to tension and reaches its maximum at the cracked section.

Figure 2. Deformation characteristics of an element between adjacent two cracks subjected to restraint force

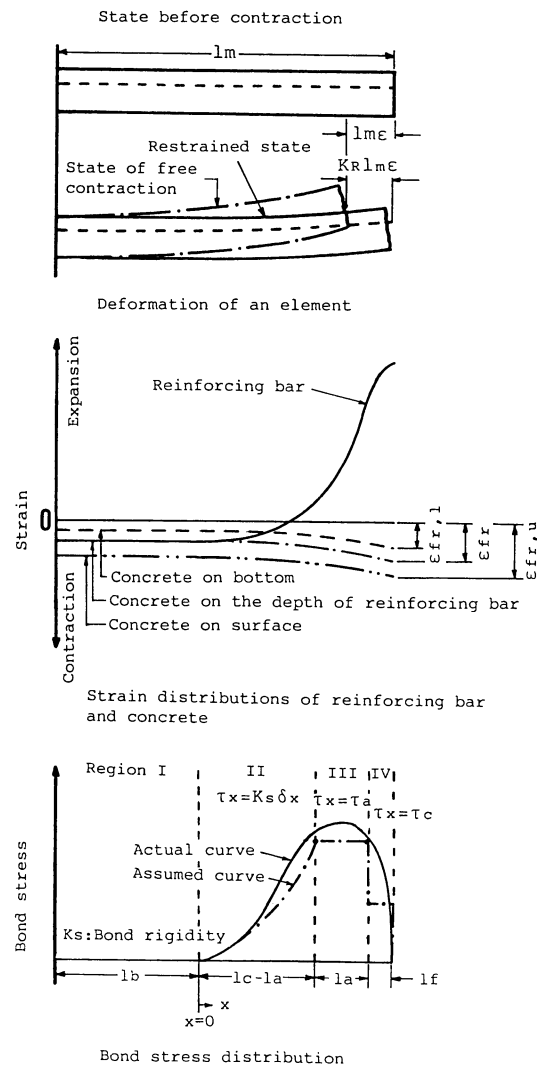
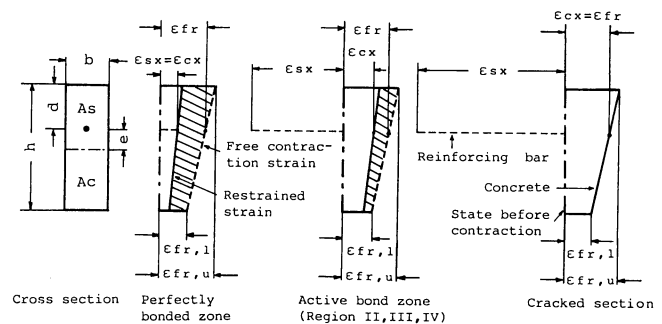


Figure 3. Strain distributions of concrete along slab thickness



Longitudinal strain in CRCP is considered to change linearly along the depth due to internal restraint (Figure 3). Strictly speaking, this is not necessarily true near the cracked section because the concrete can deform to some extent. However, external restraint by the subgrade is considered as dominant, so it is assumed that strain in CRCP changes linearly along the depth all over the slab; namely, irrespective of freely contracting area or restrained area.

Fundamental Equations

Let the origin of x-axis be the point that the relative movement between concrete and reinforcement (bond slip) occurs as shown in Figure 2. Taking the positive direction towards the cracked section, the following equilibrium equations are established.

$$P = P_c + P_s \quad (1)$$

$$P_c = A_c E_e (\epsilon_{cx} - \epsilon_{fr}) : \text{at the reinforcement} \quad (2)$$

$$P_c = A_c E_e \rho_u (\epsilon_{cx,u} - \epsilon_{fr,u}) : \text{on the surface} \quad (3)$$

$$P_c = A_c E_e \rho_l (\epsilon_{cx,l} - \epsilon_{fr,l}) : \text{at the bottom} \quad (4)$$

$$P_s = A_s E_s (\epsilon_{sx} - \epsilon_s, \Delta T) \quad (5)$$

where

- P : restraint force,
- P_s, P_c : tensile force in reinforcement, concrete at the depth of reinforcement,
- A_s, E_s : area, elastic modulus of reinforcement,
- A_c, I_c : area, inertial moment of concrete,
- E_e : effective modulus of concrete ($= E_c / (1 + \phi)$),
- E_c : elastic modulus, creep coefficient of concrete,
- $\epsilon_s, \Delta T$: reinforcement strain due to temperature change ($= \alpha_s \Delta T$),
- α_s : thermal expansion coefficient of reinforcement,
- α_c : thermal expansion coefficient of concrete,
- ΔT : temperature drop from the highest at the depth of reinforcement,
- $\Delta T_u, \Delta T_l$: temperature drop from the highest at the depth of surface, bottom,
- ϵ_{fr} : freely contracting strain due to temperature change and drying shrinkage in concrete at the depth of reinforcement ($= \epsilon_c, \Delta T + \epsilon_{sh}$),
- $\epsilon_{fr,u}$: freely contracting strain due to temperature change and drying shrinkage in concrete on the surface ($= \{\epsilon_c, \Delta T\}_u + \epsilon_{sh,u}$),
- $\epsilon_{fr,l}$: freely contracting strain due to temperature change and drying shrinkage in concrete at the bottom ($= \{\epsilon_c, \Delta T\}_l + \epsilon_{sh,l}$),
- $\epsilon_c, \Delta T$: free strain due to temperature change at depth of reinforcement ($= \alpha_c \Delta T$),
- $\{\epsilon_c, \Delta T\}_u$: free strain due to temperature change on the surface ($= \alpha_c \Delta T_u$),
- $\{\epsilon_c, \Delta T\}_l$: free strain due to temperature change at the bottom ($= \alpha_c \Delta T_l$),
- $1/\rho$: $1 + e / (I_c / A_c)$,
- $1/\rho_u$: $1 + e(e + d) / (I_c / A_c)$,

- $1/\rho_l$: $1 - e h - (e + d) / (I_c / A_c)$,
- b : spacing of reinforcement,
- h : slab thickness,
- e : distance of reinforcement from the centroid of the section,
- d : distance of reinforcement from the surface,
- ϵ_{cx} : strain in concrete at the depth of reinforcement,
- $\{\epsilon_{cx}\}_u$: strain in concrete on the surface,
- $\{\epsilon_{cx}\}_l$: strain in concrete at the bottom,
- ϵ_{sx} : strain in reinforcement.

As Equation (1) is also valid at $x + dx$ and the restraint force is constant all over in CRCP, the relationship

$$\frac{d\epsilon_{cx}}{dx} = - \frac{n\rho}{\rho} \frac{d\epsilon_{sx}}{dx} \quad (6)$$

is obtained. On the other hand, the following two equations are well known on the relationship between bond slip δx and strains, and between bond stress τ_x and strain.

$$\frac{d^2 \delta x}{dx^2} = \frac{d\epsilon_{sx}}{dx} - \frac{d\epsilon_{cx}}{dx} \quad (7)$$

$$\frac{d\epsilon_{sx}}{dx} = \frac{U_s}{A_s E_s} \cdot \tau_x \quad (8)$$

where

- n : elastic modulus ratio ($= E_s / E_e$),
- p : steel ratio ($= A_s / A_c$),
- U_s : perimeter of reinforcement

The fundamental equation on the bonding

$$\frac{d^2 \delta x}{dx^2} = (1 + \frac{n\rho}{\rho}) \cdot \frac{U_s}{A_s E_s} \cdot \tau_x \quad (9)$$

can be derived from the above Equations (6), (7) and (8) in consideration of concrete strain distribution along the depth, position of reinforcement and creep characteristics of concrete. Therefore, calculation of the strain distribution in reinforcement is essential to consider the structural behaviour of CRCP. In other words, this results in a problem how to evaluate the bond stress between reinforcement and concrete.

Stress in Reinforcement

As the relationship between τ_x and δx is expressed as nonlinear in Figure 14, a complicated numerical analysis will be needed to solve Equation (9) strictly. Therefore, this kind of analysis may be impractical for design purpose. As a result, the element has been assumed to be divided into four regions.

- Region I : reinforcing bar bonds concrete perfectly ($\tau_x = 0$)
- Region II : bond stress-slip relationship is approximated to be linear ($\tau_x = K_s \delta x$, K_s : bond rigidity),

Region III : bond stress is approximated to be constant ($\tau_x = \tau_a = K_s \{\delta_x\}_{x=lc-la}$)

Region IV : bond stress is approximated to be constant ($\tau_x = \tau_c = \xi \tau_a$)

This zoning is conducted based on a variation of bonding characteristics along x-axis with the view of calculating the reinforcement stress not only faithfully to the phenomena but easily. (2) In this way, it becomes possible to evaluate bond stress distribution which varies with the restraint force.

Based on the above-mentioned zoning, boundary conditions for solving Equation(9) is considered as follows;

$$x = 0 : \delta_x = 0, d\delta_x/dx = \Delta \varepsilon = 0$$

$$x = l_c - l_a : \delta_x = \{\delta_x\}_{x=l_c-l_a}, \\ d\delta_x/dx = \{d\delta_x/dx\}_{x=l_c-l_a}$$

$$x = l_c : \delta_x = \{\delta_x\}_{x=l_c}, \\ d\delta_x/dx = \{d\delta_x/dx\}_{x=l_c}$$

Both the bond slip and the reinforcement strain can be obtained by solving Equation(9). The reinforcement strain is

1. Region II ($0 < x < lc-la$)

$$\varepsilon_{sx} = \varepsilon_{s0} + \frac{\Delta \varepsilon}{1+np/\rho} \cdot \{\cosh(\sqrt{A_1}x) - 1\} \quad (10)$$

2. Region III ($lc-la < x < lc$)

$$\varepsilon_{sx} = \varepsilon_{s0} + \frac{\Delta \varepsilon}{1+np/\rho} \cdot [g_c - 1 + \sqrt{A_1}g_s \{x - (lc-la)\}] \quad (11)$$

3. Region IV ($lc < x < lc+lf$)

$$\varepsilon_{sx} = \varepsilon_{s0} + \frac{\Delta \varepsilon}{1+np/\rho} \cdot [g_c - 1 + \sqrt{A_1}g_s \{l_a + \xi(x-lc)\}] \quad (12)$$

where

$$A_1 = (1+np/\rho) \cdot \frac{U_s}{A_s E_s} \cdot K_s, g_c = \cosh\{\sqrt{A_1}(lc-la)\},$$

$$g_s = \sinh\{\sqrt{A_1}(lc-la)\}, \xi = \tau_c/\tau_a,$$

s_0 : reinforcement strain in Region I,

On the other hand, the equation of equilibrium

$$A_s E_s [\{\varepsilon_{sx}\}_{x=lc+lf} - \varepsilon_{s, \Delta T}] \\ = A_c E_c \rho (\varepsilon_{s0} - \varepsilon_{fr}) + A_s E_s (\varepsilon_{s0} - \varepsilon_{s, \Delta T})$$

is valid between in Region I and at the cracked section. By substituting this equation into Equation(12) and letting $x = lc+lf$, the equations for calculating the reinforcement strain are derived.

$$\varepsilon_{s0} = \frac{np}{\rho} \cdot \frac{\Delta \varepsilon}{1+np/\rho} \{g_c - 1 \\ + \sqrt{A_1}g_s l_a (1 + \xi \frac{lf}{l_a})\} + \varepsilon_{fr} \quad (13)$$

$$\{\varepsilon_{sx}\}_{x=lc+lf} = \Delta \varepsilon \{g_c - 1 + \sqrt{A_1}g_s l_a (1 + \xi \frac{lf}{l_a})\} + \varepsilon_{fr} \quad (14)$$

Equation(13) for in Region I, Equation(14) for at the cracked section, respectively. Therefore, strain distribution in reinforcement can be calculated when the bond slip length (i.e. length of Region II and Region III, lc) is obtained; for l_a and lf are given from the bonding characteristics described later.

lc can be derived from the following process by considering the condition of compatibility on deformation. The reinforcement deforms due to restraint force as shown in Figure 2. The reinforcement deformation must coincide with the restrained deformation of concrete obtained by using a degree of restraint K_R and freely contracting strain. Accordingly, the compatibility condition

$$K_R \varepsilon_{lm} + \int_0^{lm} (\varepsilon_{sx} - \varepsilon) dx = 0 \quad (15)$$

where

ε : reinforcement and concrete strains when the free deformation of concrete is restrained by the reinforcement only

$$\varepsilon = \frac{1}{1+np/\rho} \cdot (\varepsilon_{fr} + \frac{np}{\rho} \cdot \varepsilon_{s, \Delta T}),$$

$\varepsilon = \varepsilon_{fr}$, when steel ratio is small,

lm : a half of crack spacing,

K_R : degree of restraint. $K_R=1$ means full restraint; that is, CRCP cannot deform.

has to be valid. By substituting Equation(10) through (13) into Equation(15), Equation(16) is obtained.

$$\exp\{\sqrt{A_1}(lc-la)\} = - \frac{2\{\varepsilon_{fr} - (1-K_R)\varepsilon\}}{\frac{\Delta \varepsilon}{1+np/\rho} \cdot (\frac{1}{l_m} \cdot \eta_1 + \frac{np}{\rho} \cdot \eta_2)} \quad (16)$$

In this process, some simplification has been adapted by neglecting the infinitesimal terms expressed as follows.

$$g_c = g_s = 1/2 \cdot \exp\{\sqrt{A_1}(lc-la)\} \quad (17)$$

$$|\varepsilon_{fr} - (1-K_R)\varepsilon| \gg \frac{\Delta \varepsilon}{1+np/\rho} \cdot (\frac{lc+lf}{l_m} + \frac{np}{\rho}) \quad (17')$$

The bond slip length lc is derived from Equation(16).

$$lc = l_a + \frac{1}{\sqrt{A_1}} \cdot \ln \left| \frac{2\{\varepsilon_{fr} - (1-K_R)\varepsilon\}}{\frac{\Delta \varepsilon}{1+np/\rho} \cdot (\frac{1}{l_m} \cdot \eta_1 + \frac{np}{\rho} \cdot \eta_2)} \right| \quad (18)$$

where

$$\eta_1 = \frac{1}{\sqrt{A_1}} + l_a \{1 + (\frac{l_f}{l_a})\} + \frac{1}{2} \sqrt{A_1} l_a^2 \{1 + 2(\frac{l_f}{l_a}) + \xi (\frac{l_f}{l_a})^2\}$$

$$\eta_2 = 1 + \sqrt{A_1} l_a \{1 + \xi (\frac{l_f}{l_a})\}$$

On the other hand, the maximum reinforcement stress at the cracked section is calculated by

$$\{\sigma_s\}_{x=l_c+l_f} = E_s [\{\epsilon_{sx}\}_{x=l_c+l_f} - \epsilon_{s,\Delta T}]$$

By substituting Equation(14), (16) and (17) into the above equation and neglecting an infinitesimal term ($\Delta\epsilon = 0$), Equation(19) is obtained.

$$\{\sigma_s\}_{x=l_c+l_f} = -E_s \left[\left(1 + \frac{np}{\rho}\right) \cdot \frac{\eta_2}{l_m} \cdot \frac{1}{\eta_1} + \frac{np}{\rho} \cdot \eta_2 \right. \\ \left. \cdot \{\epsilon_{fr} - (1 - k_R)\epsilon\} + (\epsilon_{fr} - \epsilon_{s,\Delta T}) \right] \quad (19)$$

It is clear from Equation(19) that the reinforcement stress can be determined by freely contracting strain, degree of restraint, bonding characteristics, crack spacing, position of reinforcement, material characteristics, and so on. In order to obtain both the bond slip length and the reinforcement stress, the maximum crack spacing l_{max} is necessary. l_{max} can be calculated as described in the following section.

Maximum Crack Spacing

The maximum tensile stress in CRCP will occur in Region I where the reinforcement bonds the concrete fully. Strictly speaking, this is true when stress of concrete caused by curling, warping and local restraint from the subgrade is negligible. The maximum tensile force P_c caused in the concrete is expressed as a difference in the tensile forces in reinforcement between in Region I and at the cracked section.

$$P_c = A_s E_s [\{\epsilon_{sx}\}_{x=l_c+l_f} - \epsilon_{s0}] \\ = A_s E_s \cdot \frac{\Delta\epsilon}{1+np/\rho} \cdot \{g_c \cdot 1 + \sqrt{A_1} g_s l_a \{1 + \xi (\frac{l_f}{l_a})\}\} \quad (20)$$

The cracking is assumed to occur when the maximum tensile stress on the surface reaches tensile strength of the concrete f_t . The maximum crack spacing is defined as that caused in this way. At that time, the following relationship is found between P_c and f_t .

$$f_t = \frac{P_c}{A_c} \left\{1 + \frac{e(e+d)}{l_c A_c}\right\} = \frac{P_c}{A_c} \cdot \frac{1}{\rho_u} \quad (21)$$

By substituting Equation(20) into Equation(21) and using Equation(16), the practical equation on the maximum crack spacing is obtained.

$$l_{max} = 2 \cdot l_m \\ = \frac{np}{\rho_u} \cdot \left\{ \epsilon_{fr} - \frac{2}{\epsilon_{c,max}} (1 - k_R) \epsilon - \frac{\rho_u}{\rho} \right\} \cdot \frac{\eta_1}{\eta_2} \quad (22)$$

where

$$\epsilon_{c,max} = f_t / E_e$$

In the process of derivation, the infinitesimal term on $\Delta\epsilon$ is neglected; that is,

$$\frac{\eta_1}{np/\rho_u} \cdot \epsilon_{c,max} \gg \frac{\eta_2}{1+np/\rho} \cdot (l_c + l_f) \cdot \Delta\epsilon \\ - \{\epsilon_{fr} - (1 - k_R)\epsilon\} - \frac{\rho_u}{\rho} \cdot \epsilon_{c,max} \gg \frac{np/\rho}{1+np/\rho} \cdot \Delta\epsilon$$

Maximum Crack Width

1. At Depth of Reinforcement

Equation(23) show the maximum crack width at the depth of reinforcement.

$$w = 2 [\{\epsilon_{sx}\}_{x=0} - \{\epsilon_{cx}\}_{x=0}] \cdot l_b \\ + 2 \int_0^{l_c+l_f} (\epsilon_{sx} - \epsilon_{cx}) dx \quad (23)$$

As the relationship

$$\{\epsilon_{sx}\}_{x=0} = \{\epsilon_{cx}\}_{x=0} = \epsilon_{s0}$$

is valid in Region I, Equation(24) is obtained.

$$w = 2 \int_0^{l_c+l_f} (\epsilon_{sx} - \epsilon_{cx}) dx \quad (24)$$

By rearranging the equilibrium condition at any section,

$$P = A_s E_s \epsilon_{s,max} \\ = A_c E_c \rho (\epsilon_{cx} - \epsilon_{fr}) + A_s E_s (\epsilon_{sx} - \epsilon_{s,\Delta T})$$

where

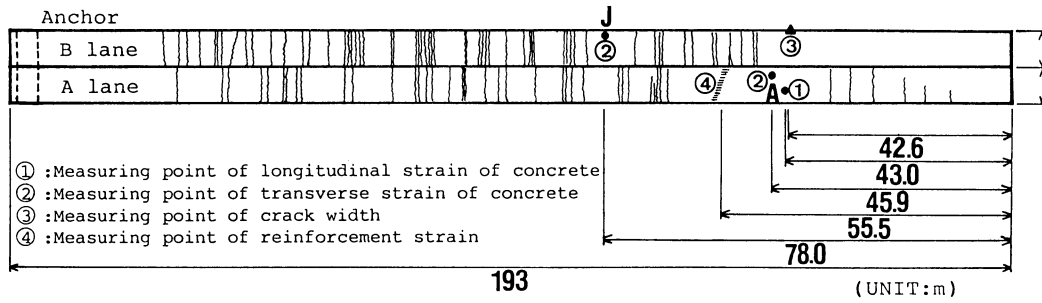
$\epsilon_{s,max}$: the maximum stress related strain at the cracked section.

Equation(25) is obtained.

$$\epsilon_{cx} = -np/\rho \cdot \epsilon_{sx} - np/\rho \cdot (\epsilon_{s,max} + \epsilon_{s,\Delta T}) + \epsilon_{fr} \quad (25)$$

If Equation(25) is substituted into Equation(24), Equation(17) is used, and a term on $\Delta\epsilon$ is neglected, the maximum crack width at the depth of reinforcement can be obtained as follows.

Figure 4. Configuration, dimension and cracking pattern of full-scale test pavement



$$w = -\frac{2(1 + np/\rho)\eta_1}{2\eta_1/l_{max} + np/\rho \cdot \eta_2} \cdot \{\epsilon_{fr} - (1 - K_R)\epsilon\} \quad (26)$$

2. Crack Width on Surface

Crack width on the surface is expressed by integrating the strain difference between in reinforcement and on concrete surface.

$$w_u = 2[\{\epsilon_{sx}\}_{x=0} - \{\epsilon_{cx,u}\}_{x=0}] \cdot l_b + 2 \int_0^{c+l_f} (\epsilon_{sx} - \epsilon_{cx,u}) dx \quad (27)$$

As well as Equation(25), $\epsilon_{cx,u}$ is expressed as

$$\epsilon_{cx,u} = -np/\rho_u \cdot \epsilon_{sx} + np/\rho_u \cdot (\epsilon_{s,max} + \epsilon_{s,\Delta T}) + \epsilon_{fr,u} \quad (28)$$

In Region I,

$$P_o = A_c E_c \rho_u [\{\epsilon_{cx,u}\}_{x=0} - \epsilon_{fr,u}] = A_c E_c \rho [\{\epsilon_{sx}\}_{x=0} - \epsilon_{fr}] \quad (29)$$

is valid. By rearranging this, the following equation is obtained.

$$\{\epsilon_{cx,u}\} = \epsilon_{fr,u} + \rho/\rho_u \cdot (\epsilon_{s0} - \epsilon_{fr}) \quad (30)$$

By substituting Equation(28) and Equation(30) into Equation(27), the maximum crack width on the surface is derived as follows.

$$w_u = \frac{1+np/\rho_u}{1+np/\rho} \cdot w + \left[\frac{1-\rho/\rho_u \cdot np}{1+np/\rho} \cdot \{\epsilon_{sx}\}_{x=c+l_f} + \frac{1+np/\rho_u}{1+np/\rho} \cdot \epsilon_{fr} - \epsilon_{fr,u} \right] \cdot l_{max} \quad (31)$$

where

w : a crack width at the depth of reinforcement.

3. Crack Width at Bottom

As well as the crack width on the surface, the following three equations are derived.

$$w_l = 2[\{\epsilon_{sx}\}_{x=0} - \{\epsilon_{cx,l}\}_{x=0}] \cdot l_b + 2 \int_0^{c+l_f} (\epsilon_{sx} - \epsilon_{cx,l}) dx \quad (32)$$

$$\epsilon_{cx,l} = -np/\rho_l \cdot \epsilon_{sx} + np/\rho_l \cdot (\epsilon_{s,max} + \epsilon_{s,\Delta T}) + \epsilon_{fr,l} \quad (33)$$

$$\{\epsilon_{cx,l}\}_{x=0} = \epsilon_{fr,l} + \rho/\rho_l \cdot (\epsilon_{s0} - \epsilon_{fr}) \quad (34)$$

Based on these, the maximum crack width at the bottom is derived.

$$w_l = \frac{1+np/\rho_l}{1+np/\rho} \cdot w + \left[\frac{1-\rho/\rho_l \cdot np}{1+np/\rho} \cdot \{\epsilon_{sx}\}_{x=c+l_f} + \frac{1+np/\rho_l}{1+np/\rho} \cdot \epsilon_{fr} - \epsilon_{fr,l} \right] \cdot l_{max} \quad (35)$$

Full Scale Test Pavement

Full Scale test pavements including CRCP have been constructed in a testing site of New Tokyo International Airport. The CRCP is 193m long and 7m wide as shown in Figure 4. At one end of the

Figure 5. Details of cross section

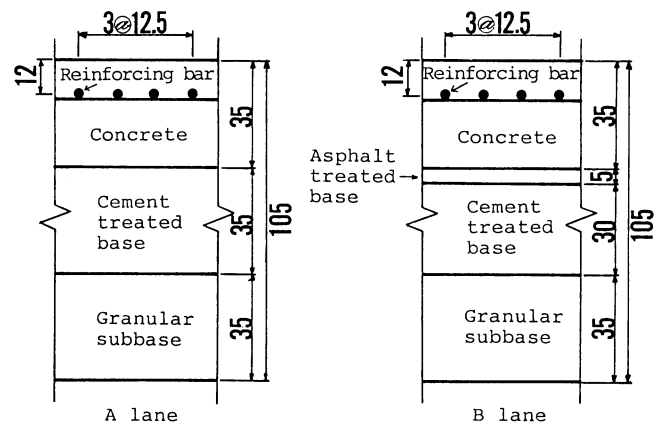
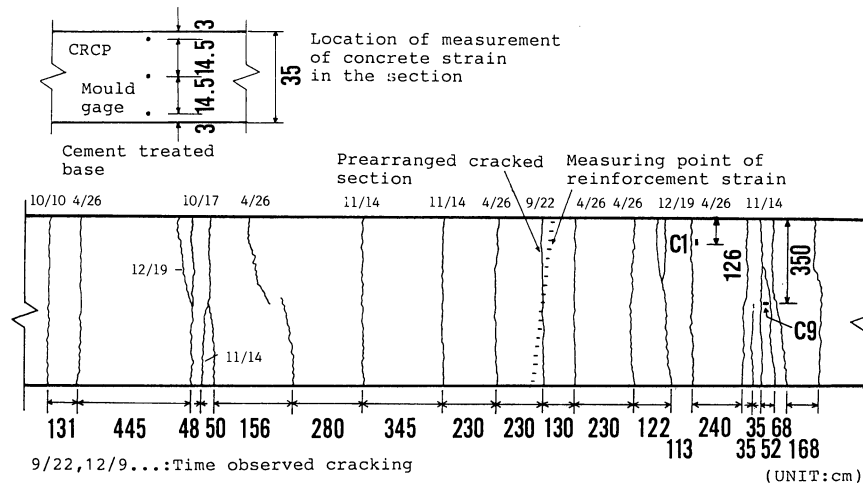


Figure 6. Crack pattern and location of strain measurement in the neighborhood of prearranged cracked section (Portion of A in Figure 4)



pavement an anchoring structure is installed to restrain the movement of this portion of CRCP. Two kinds of sections shown in Figure 5 are adopted for testing. One of the test pavements (A lane) is composed of 35cm thick concrete slab, 35cm thick cement treated base and 35cm granular subbase resting on subgrade. In the other, 5cm thick asphalt treated layer has been placed on the cement treated base (B lane). Reinforcing steel bars of which diameter₂ is 19mm and nominal yield strength is 40kgf/mm², are installed longitudinally at the depth of 12cm from the surface. As their spacing is 12.5cm, the steel ratio ρ is 0.65%.

Strain in the concrete has been measured both longitudinally and transversely at the depth of 3cm both from the surface and from the bottom, and at the mid-depth with₂ mould gages having elastic modulus of 400kgf/cm². Temperature at the same depth has also been monitored. They are recorded

Figure 8. Relationship between elastic modulus under compression and age

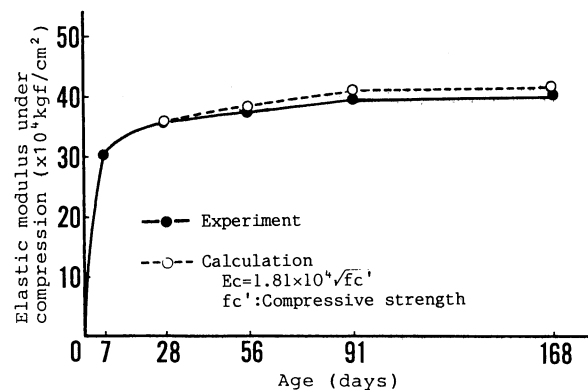
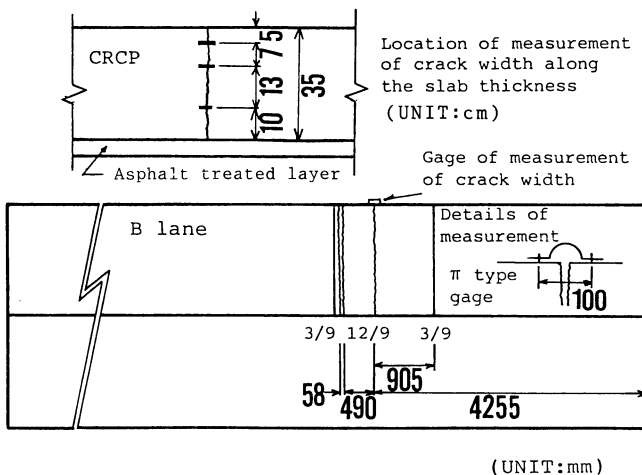


Figure 7. Location and method of measurement of crack width along the slab thickness



continuously just after casting concrete.

Strain in the reinforcement has been measured at every 5cm in the neighborhood of the prearranged cracked section as shown in Figure 6. Gages for measuring the reinforcement strain have been welded on ditches (4cm wide, 3cm deep, 21cm long) made both on the surface and at the bottom of the reinforcing steel. These gages were cured both under the temperature from 120°C to 190°C and under the pressure of 5kgf/cm² in order to conduct a long term monitoring reliably and to observe the actual bond characteristics of the reinforcement to the concrete. Correction factor for temperature has been obtained on all the gages beforehand.

Crack width along slab thickness has been monitored continuously since its occurrence as shown in Figure 7 with 10cm long π type gages.

Material characteristics of the concrete is water cement ratio of 48%, slump₂ of 2.5cm and design flexural strength of 50kgf/cm². Figure 8 shows changes in a compressive elastic modulus of the concrete with age in the case of standard curing. The elastic modulus of the concrete cured in-situ is

Figure 9. Relationship between creep coefficient and effective age

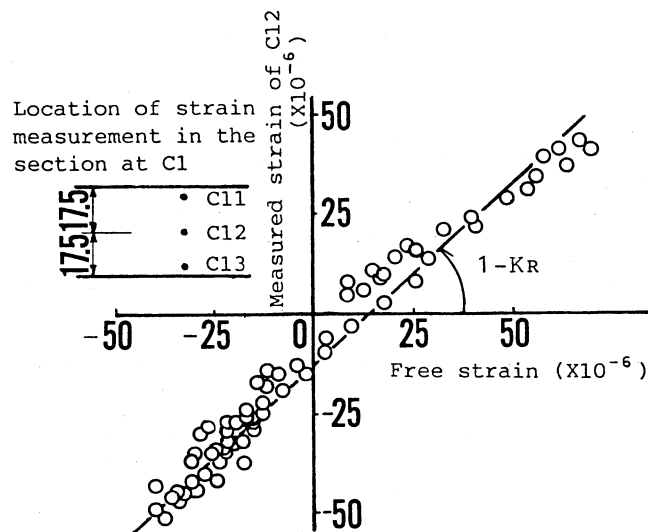


approximated to be same as that in the case of standard curing, so the relationship shown in the figure is used in the following analysis. As shown in Figure 9, a creep coefficient of concrete is decided with respect of an annual average humidity of 74% according to the reference. (3) The effective age is calculated by considering both temperature and age.

Results and Discussions

Measured strain in CRCP is shown below firstly. In the following two figures, variation of strain up to seven days from the placement of the concrete in order to eliminate the effect of drying shrinkage. Measured transverse strain in the concrete slab at the mid-depth is shown in Figure 10, together with the calculated free strain ϵ_c , ΔT due to temperature change (thermal expansion

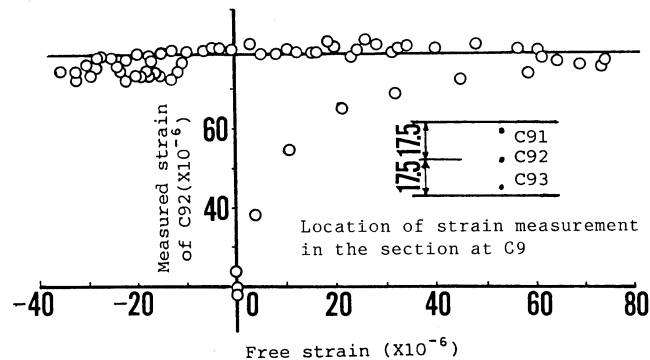
Figure 10. Relationship between measured transverse strain and free strain induced by temperature change at C1 in Figure 6



coefficient $\alpha_c: 10 \times 10^{-6}/^{\circ}\text{C}$). This strain is measured at the point about 1.26m distant from the longitudinal joint (shown as C1 in Figure 6). From this figure it is found that the measured strain varies linearly with the calculated free strain. The coefficient of restraint KR, which is defined by $KR = 1 - \epsilon_m/\epsilon_c$, ΔT (ϵ_m : measured strain, ϵ_c , ΔT : calculated free strain due to temperature change), is calculated to be almost 0.1. This means that the measured strain can be considered as the free strain.

Figure 11 shows the relationship of the measured longitudinal strain to the calculated free strain as well as in Figure 10. It has been obtained at the mid-depth in the position about 40m apart from the end designated as C9 in Figure 6. It is well known from the survey of CRCP that remote from the free end (30 or 40m apart from the end) crackings are caused with equal spacings and restraint from the subgrade is constant in this area. The measured strain has not been affected by the temperature changes once the temperature has been settled down. It means that the slab does not deform longitudinally.

Figure 11. Relationship between measured longitudinal strain and free strain induced by temperature change at C9 in Figure 6



In the case of CRCP with asphalt treated base (B lane), the coefficient of restraint in the transverse direction measured at the point 30cm apart from the longitudinal joint in the central zone is 0.0. The coefficient of restraint in the longitudinal direction is also calculated to be 1.0 as well as that shown in Figure 11.

Bond rigidity between the reinforcement and the concrete, which is one of the important factors included in the equations described beforehand, is discussed in the following section. Figure 12 shows typical examples of strain distribution in the reinforcement measured at the age of 4 and 10 days in the neighborhood of the cracked section. The free strains in the concrete at the depth of the reinforcement are about -70×10^{-6} , about -130×10^{-6} , respectively. The stress related strain can be obtained by subtracting the temperature induced strain from this actual strain and the stress in the reinforcement is calculated by multiplying the stress related strain by the elastic modulus.

The bond stress between the reinforcement and the concrete is distributed in the neighborhood of

Figure 12. Strain distributions in reinforcing bar induced by restraint force

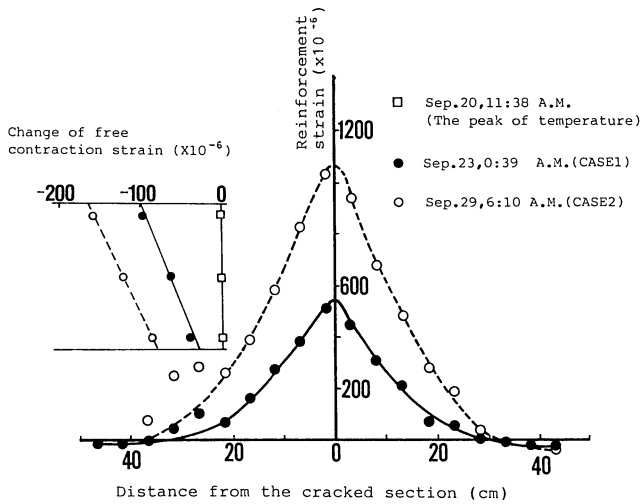
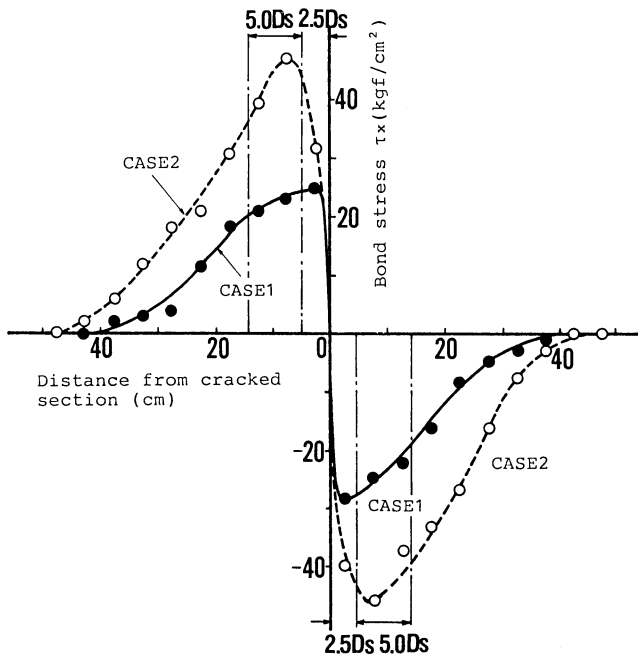


Figure 13. Bond stress distributions along the longitudinal direction



the crack as shown in Figure 13. This is calculated using strain data in Figure 12 by $\tau_x = (A_s E_s / U_s) \Delta \epsilon_s / \Delta x$ (Δx : a distance between strain measuring points, $\Delta \epsilon_s$: strain changed in Δx). It is clear that the amount of the bond slip increases with approaching the cracked section. The bond slip area is thought to be divided into three regions described above; namely,

1. the bond stress increases with the bond slip (Region II).
2. the bond stress keeps constant irrespective of

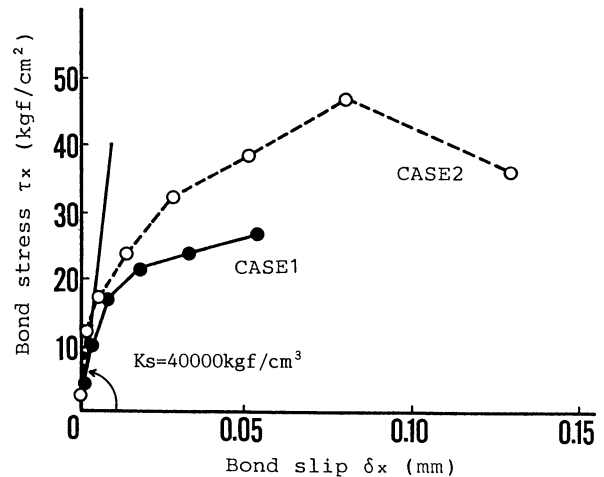
an increase of the bond slip (Region III).

3. the bond stress decreases with an increase of the bond slip (Region IV).

In order to apply the theoretical equations to the actual phenomena, it is assumed that Region III is within the area showing more than 90% of the maximum bond stress, Region IV is from the out side of Region III to the cracked section. The bond stress in each Region is considered as constant. The bond stress including the assumed stress in Region III and IV will increase if the stress in the reinforcement increases. $\tau_a = K_s \{\delta x\} x = l_c - l_a$ is derived by solving Equation(9) under the given boundary conditions. $\tau_c = \xi \cdot \tau_a$, $\xi = 0.5$ is also obtained by considering that the bond stress is 0 at the cracked section. Based on the measured data, it is assumed that $l_a = 5.0D_s$, $l_f = 2.5D_s$ (D_s : diameter of reinforcement).

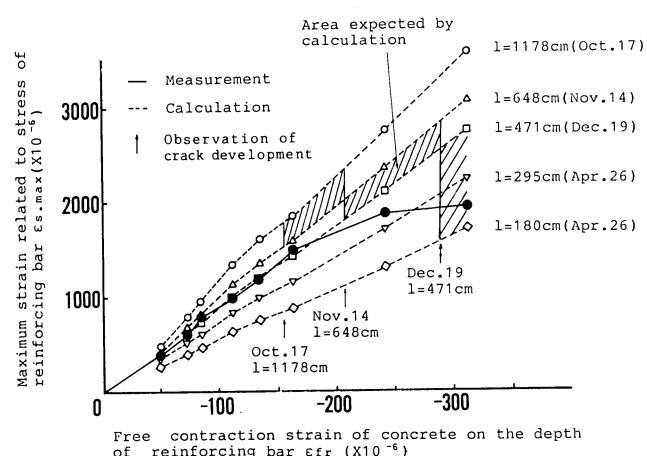
Figure 14 shows a typical example of the relationship between the bond stress and the bond slip. This is calculated from Figure 13 and Figure 12 by neglecting the change in the strain of the concrete in Region II, III and IV because it is much smaller than that in the reinforcement strain. The bond rigidity K_s is considered to be approximately constant (40000 kgf/cm^3) irrespective of the amount of restraint force when the bond slip is small.

Figure 14. Typical bond stress-slip relationship



Based on the above discussion, freely contracting strain in the concrete at the depth of reinforcement is shown with the measured maximum reinforcement strain related to stress in Figure 15. Five types of calculated strain are also plotted in the figure because the date of crack occurrence is uncertain; that is, survey on cracking has been only conducted 7, 14, 28, 56, 91, 180 and 220 days after the placement of the concrete (the concrete is cast on September, 19). The crack spacing used in this calculation is actually observed as shown in Figure 6, which is an average of crack spacings on both sides of the crack. The tensile strength of the concrete is thought to decrease because the concrete is subject to continuous restraint tensile force, and because the stress due to an internal restraint

Figure 15. Comparison of measured and calculated maximum strain of reinforcement



occurs in the concrete. However, it is impossible to evaluate the strength with accuracy, so the tensile strength f_t is assumed to be 30 kgf/cm^2 and $\Delta \epsilon$ is assumed to be 1×10^{-6} .

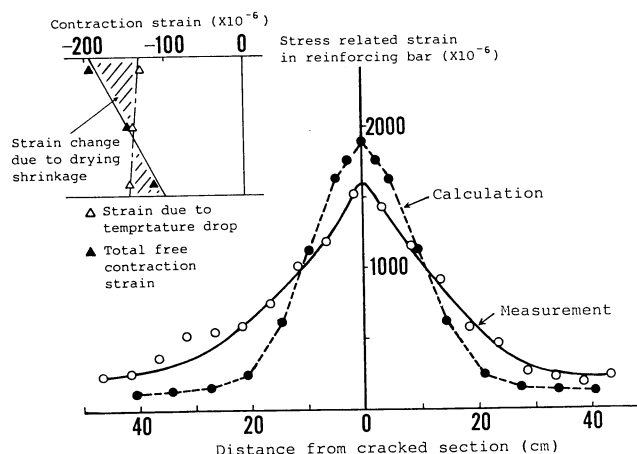
The measured strain in the reinforcement increases linearly as the freely contracting strain increases up to -160×10^{-6} . Over this value, the strain increases with the freely contracting strain, but its incremental ratio exhibits a tendency to decrease. This is brought by that the restraint force has been released by development of cracking.

On the other hand, the calculated strain in the reinforcement will decrease as well as the measured strain after the crack occurs. A difference between them tends to increase with an increase of the freely contracting strain. The reason of this is as follows. As the survey on cracking has not carried out between December, 19 and April, 26 (shown in Figure 6), the cracks may have occurred within this period (in the severest season for cracking, especially January and February). This means the crack spacings must have been smaller than those anticipated beforehand. The calculated reinforcement strain would have been close to the measured one if the cracking found in the survey on April, 26 had occurred immediately after December, 19 and the crack spacing had been 1.8m or 2.95m. The maximum difference between the measured strain and the calculated strain is within 25% when ϵ_{fr} is up to 250×10^{-6} , but they are thought to coincide with each other in consideration of complexity of cracking mechanism.

As the reinforcement is installed in the upper portion of the concrete slab, the maximum reinforcement strain does not exceed about 1200×10^{-6} theoretically when the maximum stress on the concrete surface reaches 30 kgf/cm^2 . However, this is neglected in the calculation. However, the reinforcement strain of larger than this value has been obtained actually. It may come from that the cracking has occurred soon (3 days) after the construction at the predetermined cracked section.

Now consider the case that a freely contracting strain due to temperature change and drying shrinkage of the concrete occurs as shown in Figure 16. The figure shows the distribution of both the

Figure 16. Comparison of measured and calculated strain distributions in reinforcement



measured and the calculated strain related to stress in the reinforcement when the freely contracting strain is restrained perfectly. An accuracy of estimating crack width is affected by an accuracy of evaluating the strain distribution in the reinforcement since the crack width is a difference of the deformation between the reinforcement and the concrete, and deformation of the concrete is smaller than that of the reinforcement.

It is found that the calculated reinforcement strain is smaller than the measured strain when the strain is relatively small, but the calculation overestimates the strain at the cracked section conversely. However, the calculation is recognized to be valid as a whole. A difference in deformation of the reinforcement between in the calculation and in the measurement is only 3% (the deformation is 0.39mm in the calculation and 0.38mm in the measurement). Therefore, it is concluded that although there is a difference in the reinforcement strain distribution, the crack widths in the calculation and in the measurement are coincide well.

The reason of this is thought as follows; the bond rigidity K_s used in the calculation is assumed smaller than the actual one at the point where bond slip begins, and the actual bond rigidity becomes larger towards the cracked section. This means that it is unsuitable to use only one value as the bond rigidity all over the Region II. Therefore, an accuracy of the calculation will be improved if two different K_s is used. Namely, that is, one should be used for the neighborhood of the point where the bond slip starts, and the other should be used outside of this area.

Figure 17 shows variation of temperature and freely contracting strain in the concrete with time near the surface, at the mid-depth and near the bottom. Figure 18 shows changes of the average crack spacings with time observed corresponding to Figure 17. The maximum crack spacing and crack width on the surface and at the bottom calculated using data in Figure 17 are plotted in Figure 19. The line shows the changes of the calculated maximum crack spacing with time. It has been assumed in the calculation that the maximum crack spacing does not larger than

the minimum value which is calculated beforehand. It should be noted that the crack width does not correspond with the crack spacings shown as the line, but corresponds with that shown as points.

The crack spacing seems to converge upon a constant value both in the calculation and in the measurement about 91 days after the construction (on December, 19). But the minimum value of the calculated maximum crack spacing is nearly a half of the measured average crack width. This depends upon the fact that the calculated maximum stress in the reinforcement is larger than the measured one.

From the observation on cracking occurred in CRCP, the cracking can be classified into two types

Figure 17. Variation of temperature and free contraction strain of concrete with time

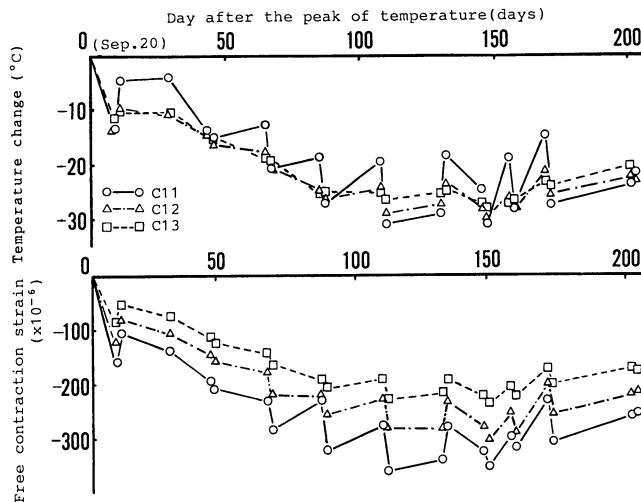
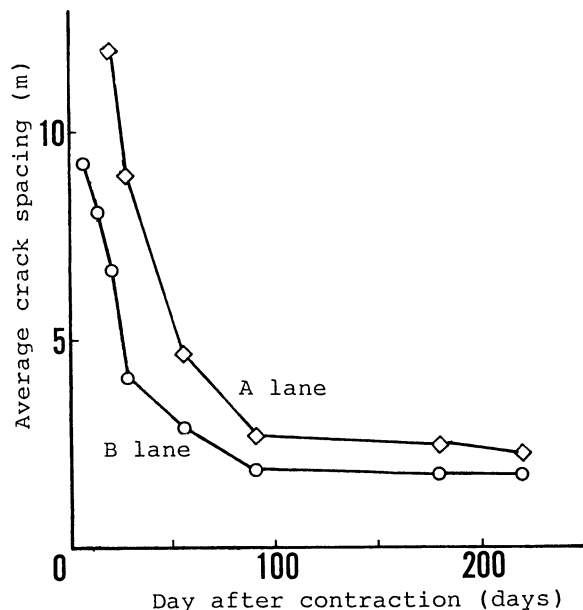


Figure 18. Observed time-dependent changes of average crack spacing



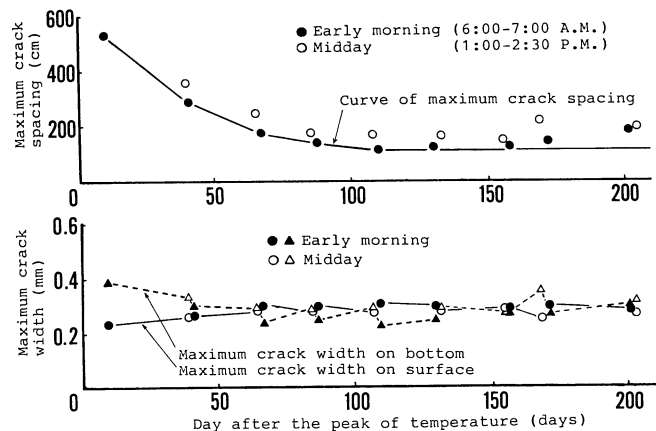
except for the cracking with uniform spacings that are expected to occur by the theory. One is the cracking where several "secondary" cracks with spacing of 30 to 50cm occurs around the "primary" crack. Although the secondary crack develops from the surface to the level of the reinforcement, the primary crack occurs all over the section. The other type is the cracking with crack spacings much larger than the calculated one.

In the former, the slab stiffness near the cracked section becomes smaller and then a large amount of the deformation of the slab can be absorbed within this area. Therefore, the stiffness of the slab as a whole is considered to become smaller. This means that it is necessary to give a proper consideration to the effect of the concentrated cracking for the purpose of analyzing the behaviour of CRCP strictly. However, in considering the crack width in the case of the maximum spacing, as the maximum stress in the reinforcement is not influenced by the spacing, the maximum crack width is independent of the spacing.

Controlling the crack width is important for keeping the sufficient load transfer at the cracked section and for decreasing the transverse flexural stress caused by aircraft loadings. This load transfer is conducted by interlocking action between aggregates in the concrete and its efficiency depends on the crack width. Accordingly, it is desirable to keep the crack width within the allowable one in whole cracked section for maintaining the good load transfer. It is considered valid empirically up to now to install the reinforcement in the upper portion of the concrete slab. In general, the crack will open wider on the surface if the contracting strain on the surface is larger than that at the bottom. For the purpose of preventing this tendency, by installing the reinforcement in the upper portion the almost uniform crack width throughout the section is intended to be realized.

When the strain increases towards the surface in the section as shown in Figure 17, which is observed in almost all cases throughout measurement, the crack widths are thought as approximately constant and are almost within 0.3mm (Figure 19). Accordingly, installing the reinforcement in the

Figure 19. Calculated results of maximum crack spacing and crack width



upper portion is recognized to be beneficial in these cases. On the other hand, it may possibly happen that the crack width in the lower portion is larger (Figure 20). In this case, installing the reinforcement in the upper portion is clearly disadvantageous. However, it is impossible to deal with the above-mentioned two cases by installing the reinforcement at a single level on the concrete slab. Therefore, the optimum depth of the installation should be decided in consideration of a variation in the freely contracting strain during a service life of that pavement.

Although it is difficult to reach a conclusion by Figure 19 and Figure 20, it is clear that the position of the reinforcement shown in Figure 5 is valid since changes in the crack width are small at least for six months from October to March.

Figure 20. Effect of distribution of total free contraction strain on the crack width along the slab thickness (B lane)

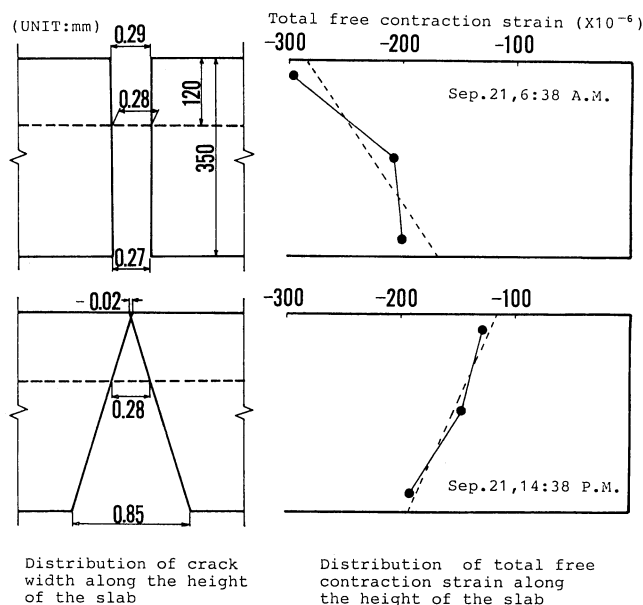
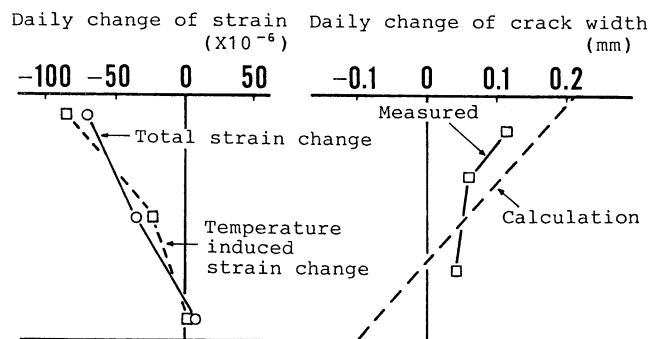


Figure 21. Comparison of measured and calculated distribution of crack width along the slab thickness induced by daily temperature change



The crack width varies with the daily temperature change as shown in Figure 21. Both the measured data and the calculated data is described in the figure. When freely contracting strain is larger in the upper portion, the crack width change increases towards the surface. However, the calculated crack width is larger than the measured one at any depth. For example, the calculated crack width, the measured crack width is 0.17mm, 0.11mm at 5cm below the surface, 0.10mm, 0.06mm at the depth of the reinforcement. This comes from the fact that the restraint force decreases and the crack width decreases since the secondary crack have occurred around the primary crack as shown in Figure 6. Incidentally, the stress in the reinforcement of more than 7500kgf/cm² is obtained by the calculation using the actual crack spacing. Needless to say, it exceeds the yield stress.

Conclusion

Determination of the amount and the position of the reinforcement and control of cracking have been carried out empirically. In this study, theoretical investigations on these problems are conducted to contribute the the development of the rational design method for CRCP. As a result, the following conclusions are obtained.

1. The authors have developed the fundamental formula based on the bond between the reinforcement and the concrete, considering the creep of the concrete and the position of the reinforcement in the section. For the purpose of obtaining the maximum reinforcement stress, the maximum crack spacing and the crack width along the depth at given vertical distribution of strain due to temperature change and drying shrinkage, the practical equations for design have been derived from this fundamental formula.

2. The reinforcement strain increases with freely contracting strain of the concrete, but it drops suddenly when cracks occur nearby. The equations represent the actual phenomenon like this well since the maximum difference between calculated and measured strain is within 25% at any crack spacings.

3. The calculated maximum crack spacing decreases with time and it tends to converge upon a certain value as well as the measured spacing. However, the calculated spacing is a half of the measured spacing; for the calculated reinforcement stress is larger than the measured one, and the deformation of the CRCP is absorbed within the cracked zone because some "secondary" cracks which develop from the surface to the reinforcement have occurred around the "primary" crack.

4. Judging from the fact that the calculated reinforcement strain distribution in the neighborhood of the crack coincides with the measured one well, the validity of the model which divides the bond slip zone into three regions is confirmed. Two kinds of bond rigidity should be used in Region II when much more accurate solution is required in the calculation.

5. Cracking behaviour observed in the CRCP is

recognized to be satisfactorily sound since the maximum crack width of less than 0.3mm at any depth has been obtained by the calculation using the freely contracting strain in the concrete. These data have been collected for about 200 days since the construction of the CRCP.

6. The result described in 5. has been obtained in the case that the contracting strain becomes larger in the upper portion and the reinforcements of which steel ratio is 0.65% are installed in the upper portion. Based on these, it is recognized that the empirical method that the reinforcement is embedded in the upper portion is supported by the newly developed simple method.

7. In the season when the temperature in the concrete fluctuates remarkably, the contracting strain in the upper portion may possibly become smaller due to sunlight. In this case, the crack width at the bottom will show an extremely large value if the reinforcement is installed in the upper portion.

8. The calculated crack width due to a daily temperature change only is found to be by about 60% larger than the measured width both on the surface and at the mid-depth. This results from that crack spacing used in the calculation is wider than the actual spacing and the effect of a decrease of the stiffness in the concentrated cracked zone is not taken into consideration.

9. Based on the above-mentioned results, it is concluded that the newly developed simple equations are valid as the design equations for control of cracking in CRCP.

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