INTERPRETATION OF FALLING WEIGHT DEFLECTOMETER RESULTS USING PRINCIPLES OF DIMENSIONAL ANALYSIS

Anastasios M. Ioannides **Ernest J. Barenberg**
University of Illinois, Urbana, IL

and
Jo A. Lary
Pavement Consultants, Inc., Seattle, WA

This Paper discusses a rigorous, theoretically sound and efficient backcalculation procedure, applicable to two-layer, rigid pavement systems. A number of practical illustrative examples demonstrate that the method presented simplifies considerably the effort required in interpreting nondestructive testing data. A unique feature of this approach is that in addition to yielding the required backcalculated parameters, it also allows an evaluation of the degree to which the *in situ* system behaves as idealized by theory, and provides an indication of possible equipment shortcomings. The method is extremely powerful and versatile, and may be extended to include other loading conditions, as well as multi-layer systems (flexible pavements). When the backcalculation is performed on a personal computer, execution time per deflection basin is trivial (≈1 sec.). Application of the procedure to actual field data from recent projects has confirmed that it yields very realistic, consistent and reliable results.

INTRODUCTION

The highway system of the United States was completed some twenty years ago, and consists of more than 4 million miles of pavements. Current state and local government expenditures for the maintenance of this network amount to about $15 billion per year, while the projected national outlay for pavement rehabilitation by the turn of the century is estimated at a staggering $400 billion *(1)*. It is, therefore, imperative to develop rational, effective and efficient methods of monitoring pavement performance, so that serviceability over the maximum possible length of time at a minimum cost is ensured.

Pavement evaluation methodologies based on destructive testing are undoubtedly attractive in view of their direct and fundamental nature, but have not gained much popularity, because they are also costly and time consuming. On the other hand, nondestructive testing (NDT) devices have attracted considerable attention in the last two decades, since they offer such advantages as mobility, ease of use, and speed, at a relatively low cost. Such equipment includes the Dynaflect *(2)*, the Road Rater *(3)*, and more recently the Falling Weight Deflectometer, or FWD *(4)*.

The development of commercially available NDT devices has prompted increased research activities, aiming at applying theoretically sound, mechanistic concepts to the structural evaluation of existing pavements. This can be achieved by matching the theoretically predicted response of the system, typically in the form of a deflection basin, with the corresponding behavior observed *in situ*, through the selection of appropriate system parameters, such as layer thicknesses and moduli. "Backcalculated" parameters can later be used to evaluate the performance of the pavement that may be expected under a wide range of conditions.

For the analytical part of the backcalculation process, Burmister's layered elastic theory *(5; 6)*, and variants thereof (see *(7)*, for example) have been the predominant tools used. This is primarily due to the efficiency with which such procedures can be adapted for execution on a personal computer. Finite element (f.e.) programs offer a very attractive alternative to these numerical integration solutions, in view of their versatility and ability to handle more realistic loading and boundary conditions. Their major drawback, however, is that they demand considerably more computer resources (execution time, memory, disk storage, etc.), and yield meaningful results only when applied by experienced users.

This Paper discusses a rigorous, theoretically sound and efficient backcalculation procedure, applicable to two-layer, rigid pavement systems. A number of practical illustrative examples demonstrate that the method presented simplifies considerably the effort required in interpreting NDT data. A unique feature of this approach is that in addition to yielding the required backcalculated parameters, it also allows an evaluation of the degree to which the *in situ* system behaves as idealized by theory, and provides an indication of possible equipment shortcomings when these arise in the field.

CURRENT RIGID PAVEMENT BACKCALCULATION SCHEMES

The majority of currently available backcalculation procedures for slab-on-grade systems can be classified as either iterative or database applications. The former term describes those procedures that employ layered elastic programs, such as BISAR *(8)*, CHEVRON *(9)*, etc. Through an iterative search scheme, they produce a set of backcalculated pavement system parameters, which result in an adequate match between observed and theoretical deflection basins. These procedures are of universal applicability: they are generally developed for an arbitrary n-layered system (n is typically set to 4 or 5), and can be used to evaluate both flexible and rigid pavements. Their two major drawbacks are as follows:

(a) They are fairly time consuming to execute on a personal computer. A typical backcalculation for a two-layer slab-on-grade system using BISDEF *(10)* takes about 600 CPU sec. to complete *(11)*.

(b) Seed moduli, i.e. initial estimates of the layer stiffnesses, must be provided. The iterative search procedure employed is quite sensitive to these inputs, and their selection calls for extreme care and long experience on the part of the user.

In contrast, database backcalculation approaches are less attractive from a sound engineering viewpoint, although they may result in accurate pavement system parameters, and are usually faster than the iterative search schemes. The concept on which these are founded is a very simple one: if one analyzes *a priori* all possible pavement systems, then any given system encountered in practice will already have been included in the database developed. Thus, at the development stage of such backcalculation schemes, computer codes for pavement analysis are executed for an enormous number of cases, to generate an adequately broad database of responses. This allows the evaluation of any other arbitrarily selected pavement, on the basis of interpolation. Therefore, the speed of execution witnessed by the end-user of the backcalculation routine, conceals the enormously time consuming effort expended by the developer of the database. Any attempt on the part of the latter to reduce the number of cases included in the database, jeopardizes the adequacy and accuracy of the interpolation performed by the user.

Anderson *(12)* developed such a database based on the CHEVRON code, containing close to half a million deflections, having executed the program tens of thousands times, for a 3-layer system. The resulting procedure, called COMDEF, can be a few hundred times faster than BISDEF. DBCONPAS is a similar scheme, developed for two-layer rigid pavements by Tia and his co-workers *(13)*. The pertinent database was generated by executing several hundred times the FEACONS III f.e. plate theory code *(14)*. The accuracy of the numerical solution controls in this case the accuracy of the backcalculation scheme. Furthermore, the ability of the f.e. method to accommodate a variety of load and boundary conditions, compounds the levels of the factorial of possible development runs, as well as the size of the resulting database. The major practical drawback of all database procedures, however, is that they can only be applied to the pavement system configurations for which they were developed, and yield accurate backcalculations only in the range of parameter values assumed in the development runs. Needless to say, of course, than an all-inclusive database is a futile task to pursue.

The "dual parametric approach" *(15; 16)* is a variant of the database procedures. The results generated by the repeated execution of the computer code at the development stage are interpreted in a graphical manner, using some geometric property of the deflection basin, such as Spreadability *(17)* or Area *(18; 19; 20)*. Figure 1, presented by Roman, *et al.* *(21)* illustrates the concept upon which such schemes are based. A pavement analysis computer code is executed a number of times for a given layered system configuration. In this case, a two-layer slab-on-grade system was considered, consisting of 6-in. thick, 15-ft square slabs. Finite element program ILLI-SLAB *(22)* was employed for the numerical analyses. By assuming a number of stiffness combinations for the layer and subgrade moduli, under a chosen load level, a number of data points are plotted on a chart of maximum deflection, D_0, versus Basin Area. These points are then connected by a series of intersecting stiffness "contours", as shown in Figure 1. Similar graphical constructions may be obtained using deflection basin properties other than Area. The implication is that any other system of the same layer configuration can be evaluated using such a grid, to yield "backcalculated" layer and subgrade stiffness values.

The drawbacks of this approach are quite evident:

(1) The grid developed is only applicable to the layer configuration and load level on which it was based. Thus, Figure 1 is only useful when considering 6-in. thick slabs-on-grade. Slab size and load level may introduce additional, if less important, approximations.

(2) A considerable number of executions of the analysis code at the development stage are necessary. Figure 1 involves 20 ILLI-SLAB runs, for which several hours must have been spent. For every other slab thickness considered, the 20-run factorial must be repeated.

(3) The method would be very awkward to extend to more than two layers, as it would require an even greater number of development runs. In addition, a multi-dimensional chart would have to replace the grid in Figure 1.

(4) The accuracy of the backcalculated parameters is strongly dependent on the fineness and other characteristics of the f.e. mesh used for the development runs. It is further controlled by the resolution afforded by the grid, such as shown in Figure 1.

CLOSED-FORM BACKCALCULATION PROCEDURE FOR RIGID PAVEMENTS

In developing a backcalculation procedure specifically for rigid pavement systems, it is very pertinent to keep in mind that within the broad spectrum of all pavement systems, slabs-on-grade occupy a very specific and unique position. This allows a number of simplifications not applicable to flexible pavement systems in general, which can be very effective in improving the versatility, speed and ease of application of the methodology envisaged. The most important such simplifications are:

(1) The use of plate theory, instead of layered elastic theory. The former ignores transverse deformations within the pavement thickness, and for a two-layer system reduces the number of independent variables from two to only one. According to Burmister's (flexible pavement) analysis, the ratios (h/a) of the top layer thickness to the radius of the applied load, and (E_1/E_2) of the two layer moduli are the pertinent inputs. In contrast, Westergaard's (rigid pavement) formulation involves only one fundamental governing ratio, namely (a/ℓ), where ℓ is the radius of relative stiffness of the pavement system. For most values of (a/ℓ) normally encountered in practice, the two theories yield identical results, as (E_1/E_2) tends to infinity.

(2) Slabs-on-grade can very often be reduced to two layers only, in which case there should be no need for seed moduli. Furthermore, closed-form theoretical solutions for such systems are readily available in the literature.

A closed-form backcalculation procedure, based on a consistent and theoretically rigorous approach utilizing the principles of dimensional analysis, has been presented by Ioannides *(23)*. The most elementary application of this method involves a two-layer system, consisting of a rigid pavement slab resting on an elastic solid (ES) or a dense liquid (DL) foundation. Solutions for these systems, in terms of deflection basins resulting from the application of a single, circular, interior load of uniform pressure intensity, have been presented by Westergaard *(24; 25)*, Hogg *(26)*, and Losberg *(27)*. The backcalculation process also utilizes the concept for determining the Area of the deflection basin, first proposed by Hoffman and Thompson *(28)*.

Application of dimensional analysis indicates that for a given sensor arrangement, there is a unique relation between this Area and the radius of relative stiffness of the slab-subgrade system. Figure 2 shows the AREA versus ℓ curves, for four different loading and support conditions, assuming four sensors at 12-in. spacing are employed. Under these conditions, the basic steps of the backcalculation procedure are as follows:

(1) Drop the weight, and record the applied load, P, and the resulting deflections, D_0, D_1, D_2 and D_3, at 0, 12 24 and 36 in. from the center of the loading plate, respectively.

(2) Calculate the area of the deflection basin, in inches, as follows:

$$\text{AREA [in.]} = 6\ [\ 1 + 2\left[\frac{D_1}{D_0}\right] + 2\left[\frac{D_2}{D_0}\right] + \left[\frac{D_3}{D_0}\right]\ ] \tag{1}$$

(3) Enter Figure 2 with this AREA value, and pick up the corresponding radius of relative stiffness value, ℓ (also expressed in inches). The latter is defined by:

For the DL foundation:

$$\ell = \ell_k = \left(\frac{Eh^3}{12(1-\mu^2)k}\right)^{1/4} \tag{2a}$$

For the ES foundation:

$$\ell = \ell_e = \left(\frac{Eh^3(1-\mu_s^2)}{6(1-\mu^2)E_s}\right)^{1/3} \tag{2b}$$

where:

E: slab Young's modulus;
E_s: soil Young's modulus;
h: slab thickness;
μ: slab Poisson ratio;
μ_s: soil Poisson ratio; and
k: modulus of subgrade reaction.

In the discussion below, the subscripts k or e are sometimes dropped from ℓ for brevity, but this need not cause any confusion.

(4) Backcalculate subgrade support values, as follows:

For the DL foundation:

$$k = \left[\frac{d_i}{D_i}\right]\left[\frac{P}{\ell^2}\right] \tag{3a}$$

For the ES foundation:

$$C = \frac{E_s}{(1-\mu_s^2)} = \left[\frac{d_i}{D_i}\right]\left[\frac{2P}{\ell}\right] \tag{3b}$$

In these expressions, d_i denotes the four nondimensional sensor deflections corresponding to the measured deflections, D_i:

$$d_i = \frac{D_i D}{P\ell^2} = \frac{D_i k \ell^2}{P} \qquad (i=0,3) \tag{4}$$

where D is the slab flexural stiffness, given by:

$$D = \frac{Eh^3}{12(1-\mu^2)} \tag{5}$$

and ℓ is either ℓ_k or ℓ_e, depending on the subgrade idealization considered. The nondimensional deflections are known functions of the ratio (a/ℓ) only. Therefore, in the case of a constant plate load radius these are uniquely defined by ℓ. Figure 3 shows the variation with ℓ of the dimensionless deflections, d_i, for a circular load, radius a = 5.9055 in. (= 300 mm). The curves corresponding to d_0 are defined by the interior loading maximum deflection formulae presented by Westergaard *(25; 29)* for the DL, and Losberg *(27)* for the ES foundations, respectively. The remainder of the curves in Figure 3 were derived during the present investigation. For a chosen value of the subgrade Poisson's ratio, (say, μ_s = 0.4 to 0.5), Equation (3b) can be rewritten to yield the foundation modulus, E_s:

$$E_s = (1-\mu_s^2) \left[\frac{d_i}{D_i}\right] \left[\frac{2P}{\ell}\right] \tag{6}$$

(5) Backcalculate the slab flexural stiffness, as follows:

$$D = \frac{Eh^3}{12(1-\mu^2)} = \left[\frac{d_i}{D_i}\right] P\ell^2 \tag{7}$$

Thus, if the slab thickness, h, is known, the slab modulus can be calculated using:

$$E = \left(\frac{12(1-\mu^2)}{h^3} \right) \left[\frac{d_i}{D_i}\right] P\ell^2 \tag{8}$$

Alternatively, if the slab modulus, E, is known, one can backcalculate thickness, h, from:

$$h = \left(\left[\frac{12(1-\mu^2)}{E} \right] \left[\frac{d_i}{D_i}\right] P\ell^2 \right)^{1/3} \tag{9}$$

For the slab Poisson ratio, μ, a value of 0.15 may be used.

Note that using these backcalculation equations (Equations 3 through 9), four determinations of each pavement system parameter (k, E_s, h or E) are possible, each corresponding to one measured deflection, D_i. This provides a control on the accuracy of individual sensor readings, as well as a measure of *in situ* material variability, and of the departure of the real system from the idealized conditions assumed in theory.

APPLICATION OF CLOSED-FORM BACKCALCULATION PROCEDURE

To illustrate the application of the procedure described, a set of FWD data collected from a Portland cement concrete (PCC) pavement section along Interstate 80 in Illinois, will be used in backcalculating the pavement parameters. The radius of the load plate was the standard a = 5.9055 in. (300 mm), and the recorded load, P, was 7792 lbs. Sensors were located at 0, 12, 24 and 36 in. from the center of the plate. Recorded deflections, in inches, were as follows:

D_0 = 0.0030; D_1 = 0.0028; D_2 = 0.0024; D_3 = 0.0021.

Substituting these deflections in Equation (1) yields:

$$AREA = 6\ [\ 1 + 2\ (2.8/3.0) + 2\ (2.4/3.0) + (2.1/3.0)\ ] = 31.00 \text{ in.}$$

From Figure 2, the corresponding radius of relative stiffness values are:

DL: ℓ_k = 39 in.; ES: ℓ_e = 28 in.

Entering Figure 3 with these ℓ-values, one obtains the following nondimensional deflections:

DL Foundation:

d_0 = 0.124; d_1 = 0.116; d_2 = 0.102; d_3 = 0.084.

ES Foundation:

d_0 = 0.190; d_1 = 0.176; d_2 = 0.154; d_3 = 0.130.

Thus, Equations (3a) and (3b) give the following backcalculated subgrade support values (assuming μ_s = 0.45 for the ES):

For the DL Foundation:

$k = (0.124/0.0030)\ (7792 / 39^2)$
= 212 psi/in. based on D_0;

$k = (0.116/0.0028)\ (7792 / 39^2)$
= 212 psi/in. based on D_1;

k = 218 psi/in. based on D_2; and

k = 205 psi/in. based on D_3.

The mean of these backcalculated k-values is 212 psi/in. and their coefficient of variation (COV) is 2.15%.

For the ES Foundation:

$E_s = (1-0.45^2)\ (0.190/0.0030)\ (2\text{x}7792/28)$
= 28,111 psi based on D_0;

$E_s = (1-0.45^2)\ (0.176/0.0028)\ (2\text{x}7792/28)$
= 27,900 psi based on D_1;

E_s = 28,481 psi based on D_2; and

E_s = 27,477 psi based on D_3.

Thus, the average backcalculated value of the elastic solid foundation modulus is 27,992 psi, with a COV of 1.30%.

Assuming μ = 0.15 and E = 4,500,000 psi, Equation (9) is used to backcalculate the slab thickness, h.

For the DL Foundation:

$$h = \left(\{12(1-0.15^2)/4{,}500{,}000\}\ (0.124/0.0030)\ 7792\ (39)^2 \right)^{1/3}$$

$= 10.85$ in. based on D_0.

Similarly,

$h = 10.86$ in. based on D_1;

$h = 10.95$ in. based on D_2; and

$h = 10.73$ in. based on D_3.

In this case, the mean slab thickness is found to be 10.85 in., and the corresponding COV 0.72%.

For the ES Foundation:

$$h = \left[\{12(1-0.15^2)/4{,}500{,}000\} \quad (0.190/0.0030)\ 7792\ (28)^2 \right]^{1/3}$$

$= 10.03$ in. based on D_0;

while,

$h = 10.00$ in. based on D_1;

$h = 10.07$ in. based on D_2; and

$h = 9.95$ in. based on D_3.

The backcalculated values of h for the elastic solid foundation have a mean of 10.01 in., and a COV of 0.44%.

Alternatively, if the thickness is known to be 10.0 inches, Equation (8) is used with $\mu = 0.15$ to backcalculate the slab modulus.

For the DL Foundation:

$$E = \{12(1-0.15^2)/10.0^3\}\ (0.124/0.0030)\ 7792\ (39)^2$$

$= 5{,}746{,}145$ psi based on D_0.

The other backcalculated E-values are:

$E = 5{,}759{,}385$ psi based on D_1;

$E = 5{,}908{,}335$ psi based on D_2; and

$E = 5{,}560{,}786$ psi based on D_3.

Hence, an average value of $E = 5{,}743{,}662$ psi is determined (COV = 2.15%, identical to that for k-value).

For the ES Foundation:

$$E = \{12(1-0.15^2)/10.0^3\}\ (0.190/0.0030)\ 7792\ (28)^2$$
$= 4{,}538{,}323$ psi based on D_0.

The corresponding values for the other deflections are:

$E = 4{,}504{,}200$ psi based on D_1;

$E = 4{,}598{,}037$ psi based on D_2; and

$E = 4{,}435{,}954$ psi based on D_3.

The mean slab modulus is 4,519,128 psi and the COV is 1.30% (which is identical to that for E_s).

The results of the brief statistical analysis of the parameters backcalculated above provide some useful insights pertaining to the testing procedure adopted, and the observed behavior of the *in situ* pavement system. In all cases, a coefficient of variation substantially smaller than 10%, say, was obtained, indicating that no major blunders were involved in the individual sensor readings. If one of the sensors had malfunctioned, the backcalculated parameters based on that particular deflection reading would have been significantly different from the remainder, thus alerting the engineer to the probability of an error. In addition, Figure 3 indicates that d_0 and d_1 are relatively insensitive to changes in the value of ℓ, compared to d_2 and d_3. Thus, it may be concluded that the latter are more reliable backcalculation tools. On the other hand, the actual measured deflections D_0 and D_1 are probably more accurate than D_2 and D_3, owing to their larger magnitude and the equal sensitivity of all sensors, as used in conventional practice. Therefore, it may be desirable to use sensors of increased sensitivity for measuring deflections further away from the center of the applied load.

It must be stressed that the fundamental relation between AREA and ℓ, involves the radius of the applied load, a, assumed to be equal to 5.9055 in. (300 mm) in Figure 2. Calculations performed during this investigation (not presented in this Paper) indicate that the values of backcalculated parameters are quite sensitive to the load plate radius. Consequently, care should be exercised in the field to achieve good seating of the loading plate, as well as of the sensors.

Turning now to the *in situ* pavement behavior, it is observed that either foundation model would be adequate in this case, as indicated by the low coefficients of variation obtained. Nonetheless, since the corresponding COV values for the elastic solid are smaller in every case, it may be argued that this would be a more realistic representation of the competent pavement system considered here. This is supported by core thickness and slab modulus test data, which indicate that the slab thickness is very close to its nominal thickness of 10 in., while its E-value is closer to 4.5 million than to 5.7 million psi.

It must be borne in mind, however, that engineers are interested in backcalculated parameters, only inasmuch as these will be employed later to determine pavement responses, i.e. stresses and deflections. Table 1 presents some typical comparisons of such use in evaluating interior loading responses. These were obtained using the appropriate Westergaard *(25)* and Losberg *(27)* equations, with the mean values of the backcalculated parameters for the DL and the ES foundations, respectively. In view of the fact that *in situ* parameters are often used for a wide variety of loading radii, and since k (at least) is not an intrinsic soil property but varies with the applied tire-print size and slab stiffness, load radii of 10.0 and 2.0 in., respectively, are also considered. Referring to the ratio of the DL to the ES response, expressed as a percentage in Table 1, it is observed that agreement between the two foundation idealizations is best when considering maximum deflections, fairly good when comparing maximum bending stress, and unsatisfactory when contrasting maximum subgrade stresses. This is hardly surprising, since the backcalculation process involves measurements of deflections, and can, therefore, be expected to lead to more consistent predictions of that response. In contrast, the nature of the foundation, inherent in the subgrade idealization adopted, is expected to influence most drastically the response which it is related most closely to, namely subgrade stresses.

Note that in calculating the maximum bending stress in the slab, better agreement between the two foundation assumptions is obtained when the backcalculation involves a known value of E (Series I) than a known value of h (Series II). It may be worthwhile to consider modifying the current practice, which generally assumes that the thickness of the slab is known. A given error in the estimate of this property will have a significantly more detrimental effect on the backcalculated parameters, than the same error committed in the assumed value of the slab modulus would.

Finally, the comparison percentages in Table 1 are not very sensitive to the load radius effect, under conditions of full contact and linear elasticity (competent pavement) considered here. These percentages increase only slightly as the load radius increases. This, however, should not be interpreted as encouraging the indiscriminate use of the backcalculated parameters. In particular, one should avoid employing these to analyze pavements of different properties (most notably different thicknesses), or applying them to cases where the two fundamental assumptions of full contact and linear elasticity are violated. Significant amounts of additional experimental research are required before the relation of backcalculated parameters to intrinsic soil properties is unraveled fully.

GRAPHICAL APPLICATION OF CLOSED-FORM BACKCALCULATION PROCEDURE

To facilitate the general application of the method described above by practicing engineers, a number of additional charts have been prepared. These may be used in conjunction with Figure 2 in the backcalculation process. Two of these charts are shown in Figure 4. These apply for any value of E and h, but are based on the four sensor arrangement discussed earlier. They further assume that a = 5.9055 in., P = 9,000 lbs, and $\mu_s = 0.45$. Their use is illustrated below, with reference to the I-80 data analyzed above.

The measured deflections (corresponding to P = 7792 lbs) are first adjusted by linear extrapolation to yield the following values (in inches), expected under a 9000 lbs load:

$D_0 = 0.003465$; $D_1 = 0.003234$; $D_2 = 0.002772$; $D_3 = 0.002426$.

Since these are merely scaled-up versions of the original measurements, application of Equation (1) results, once again, in AREA = 31.00 in., while from Figure 2, the corresponding ℓ-values remain $\ell_k = 39$ in. and $\ell_e = 28$ in. Entering Figure 4(a) the former of these values along the abscissa and $D_0 = 0.003465$ in. along the ordinate, the intersection point yields (by interpolation), k = 225 psi/in. (cf. 212 psi/in. by the analytical approach). Similarly, Figure 3(b) (using $\ell_e = 28$ in.) results in $E_s = 29{,}000$ psi, compared to 28,111 psi, obtained above.

Now, in order to determine the slab property of interest, use may be made of Equation 2, for the radius of relative stiffness. Considering the DL foundation, for example, and assuming $\mu = 0.15$, Equation 2 may be rewritten as:

$$Eh^3 = 11.73\ k\ell^4 \tag{10}$$

Since both k and ℓ are known at this point, the value of (Eh^3) is readily determined, as follows:

$$Eh^3 = 11.73 \times 250 \times (39)^4 = 67.842\text{x}10^8 \text{ lb-in.}$$

Thus, assuming E = 4,500,000 psi results in:

$$h = \left\{ 67.842\text{x}10^8/4.5\text{x}10^6 \right\}^{1/3} = 11.47 \text{ in.}$$

(cf. 10.85 in. analytically).

Clearly, this graphical approach is coarser, but leads to reasonable estimates of the subgrade support parameters, without recourse to the equations presented above. It is observed that both the k- and h-values backcalculated using the graphical procedure are about 6% higher than the corresponding parameters from the analytical approach. Note that the algebraic approach would result in identical backcalculated values for the original data and the scaled-up (or scaled-down) values. The only source of error in this scaling process may be in not retaining enough significant digits in the adjusted deflections.

The charts in Figure 4, serve as an illustration of the sensitivity required from the deflection sensors. Thus, a slab resting on a stiff foundation needs to be loaded with a fairly high load, so that the measured deflection is an adequate backcalculation tool. They also underline the fact that in the approach described, the backcalculated foundation modulus is independent of the assumed slab property value (i.e. h or E). This is the direct consequence of maintaining constant the plate radius (a = 5.9055 in.), as well as the radius of relative stiffness (given pavement system). Thus, the fundamental ratio of (a/ℓ) is constant. This is known to be the single controlling variable under elastic, full contact conditions *(30)*.

To illustrate this point, consider the following data collected from a continuously reinforced concrete (CRC) pavement along I-57 in Illinois. Early morning deflections (in inches) measured under a cool temperature, were as follows (adjusted to P = 12,000 lbs):

$D_0 = 0.0243$; $D_1 = 0.0165$; $D_2 = 0.0123$; $D_3 = 0.0870$ (AREA = 22.4 in.).

Using ILLI-BACK, a computerized adaptation of the approach outlined above which is currently under development, results in the following mean values for the backcalculated parameters, assuming h = 8.0 in. and μ = 0.15:

k = 192 psi/in. (COV = 19%); ℓ_k = 16.97 in.

E_s = 13,937 psi (COV = 17%); ℓ_e = 9.62 in.

DL: E = 365,050 psi (COV = 19%)

ES: E = 178,006 psi (COV = 17%).

The test was repeated at high noon, when the temperature was much warmer, and the following data were collected (again, in inches, adjusted to P= 12,000 lbs):

D_0= 0.00966; D_1= 0.00867; D_2= 0.00684; D_3= 0.00500 (AREA = 28.58 in.).

The corresponding mean backcalculated parameters are as follows:

k = 207 psi/in. (COV = 3%); ℓ_k = 26.58 in.

E_s = 18,152 psi (COV = 5%); ℓ_e = 20.84 in.

DL: E = 2,370,092 psi (COV = 3%)

ES: E = 2,360,408 psi (COV = 5%).

These results indicate that the rise in temperature between the two tests affected primarily the slab modulus value. Expansion of the CRC pavement caused the cracks to close, resulting in improved load transfer and an increase in the apparent slab stiffness. On the other hand, the effect on the foundation modulus, particularly on the k-value, is considerably less pronounced, as might be expected. Furthermore, higher coefficients of variation were obtained in the cool morning, indicating that the pavement behaves like neither of the foundation idealizations, and that it is probably in a non-elastic, incompetent state. The much lower coefficients of variation obtained under the warmer temperatures suggest that the pavement has returned to a more elastic mode of behavior. The fact that its behavior appears to be closer to the dense liquid than the elastic solid assumption may be an indication that some soil yielding may still persist. This is supported by the low radius of relative stiffness values obtained even at high noon.

ADDITIONAL ILLUSTRATIVE EXAMPLES

The backcalculation method presented by Ioannides *(23)* is very versatile and extremely powerful, in view of its rigorous, fundamental and theoretical basis. To illustrate this fact, a large set of FWD data collected from two recent projects in Washington State are analyzed and discussed below. Backcalculations presented were also conducted using ILLI-BACK.

The first test site was a 420-ft northbound section of Interstate 5, located near Seattle, WA. The pavement consists of four lanes, bounded by asphalt concrete (AC) shoulders on either side. The slab length was 15 ft, while the lane width was 12 ft. A series of FWD drops were performed at the interior of slabs in two of the pavement lanes. "Row 2" designates the outer-most lane, while "Row 5" indicates tests in the next lane in from the right-hand-side shoulder. The pavement design was as follows: 9 in. of PCC slab; 2 in. crushed surfacing top course; 5 in. of special ballast; silty sand subgrade.

An 840-ft section of the westbound lanes Interstate 90, near Spokane, WA, constituted the second test site. This is a three-lane pavement, bounded by a 3-ft PCC gutter and a 7-ft AC shoulder on the right, and an 11-ft AC shoulder on the left. "Row 4" and "Row 1" refer to the two series of interior loading FWD tests conducted, the first being in the outer-most lane, while the latter in the next lane in from the right. Once again, slabs were 15 ft long by 12 ft wide. The actual design for I-90 consisted of the following layers: 7.8 in. of PCC slab; 2.4 in. of crushed surfacing top course; 3.0 in. of special gravel base; sandy subgrade with some silt.

The FWD test program comprised 12 interior load locations for I-5, and 13 interior load locations for I-90. At each location, 8 drops were performed, two for each of the four load levels, nominally at 6, 9, 12 and 15 kips. To reduce potential seating errors, the first of each pair of drops was ignored. The discussion below considers only 48 drops for I-5 and 52 drops for I-90.

To determine the *in situ* thickness of the PCC slab and its variability, 40 cores were extracted from various I-5 locations. Of these, 28 were considered most representative of the conditions at the load locations. The thicknesses obtained ranged between 8.25 and 9.625 in., with a mean of 9.058 in. and a COV of 4.6%. Similarly, 51 cores were drilled into I-90, yielding a thickness range of 6.5 to 8.5 in., a mean of 7.662 in. and a COV of 3.1%. Thus, the average core thickness for the two test sites was 100.64% and 98.23% of the corresponding design thickness, respectively.

In addition, unconfined compressive strength (q_u) tests (ASTM Designation C 42) were performed on four cores from each of the two test sites. The reported mean q_u-values were 11,406 psi and 8,984 psi for the I-5 and I-90 test sites, respectively. The corresponding COV values were 1.5 and 5.8%, respectively. Finally, split tensile strength (σ_t) tests (ASTM C496) were conducted on 5 cores from I-5 and 4 cores from I-90. The reported mean σ_t strengths were 923 and 752 psi, respectively, with COV values of 8.5% and 5.3%.

The conventional way to estimate the modulus of elasticity of concrete, E *(31)* involves using the following empirical regression formula, developed by Pauw *(32)*:

$$E = 33\ \gamma_c^{1.5}\ (q_u)^{0.5} \tag{11}$$

where γ_c is the unit weight of concrete, in pcf. E and q_u are expressed in psi. Hognestad *(33)* suggested the following expression, instead:

$$E = 1{,}800{,}000 + 500\ (0.85\ q_u) \tag{12}$$

The units of both E and q_u are psi. Still another relation has been derived by ERES Consultants, Inc., as quoted by Herrin, *et al.* *(20)*, and may be written as:

$$E = \left(\frac{9\ (q_u)^{0.5}}{209} \right)^{1.3587} \qquad (13)$$

Here, q_u is entered in psi and E is calculated in million psi (Mpsi). Using these expressions (assuming where necessary that γ_c = 145 pcf), the following estimates of the average E-values at the two test sites may be made:

	I-5	I-90
Equation (11):	E = 6.15 Mpsi	E = 5.46 Mpsi
Equation (12):	E = 6.65 Mpsi	E = 5.62 Mpsi
Equation (13):	E = 7.95 Mpsi	E = 6.76 Mpsi

Correlations involving the split tensile strength exhibit an even wider range of predictions.

To backcalculate the required pavement parameters the current version of computer program ILLI-BACK was used. This is coded in ANSI-77 FORTRAN and is compiled using IBM Professional FORTRAN software (PROFORT). A typical run of ILLI-BACK takes about 1 CPU sec. for each deflection basin analyzed, on an IBM-AT computer equipped with a math co-processing chip. The program accepts raw FWD data (in metric units), converts these into English units, performs the backcalculation, as well as some statistical analysis of the results, and produces a detailed output, consisting of one 8-1/2 by 11 in. page for each drop. In this are given the backcalculated individual k and E_s values for each of the four deflections, as well as their mean, standard deviation and COV. For cases when the slab thickness is assumed to be known, as was done in this study, ILLI-BACK also produces slab modulus values for each of the four deflections, for each of the two foundation idealizations. Again, mean, standard deviation and COV values are also given. Figure 5 shows a typical output for one of the I-5 Row 2 drops. Similar outputs were obtained for all the other drops examined.

Table 2 presents a summary of the results for the 48 drops conducted at I-5 Row 2, assuming the slab thickness was 9.058 in. (i.e. equal to the average core thickness). Note that each entry for k, E_s, E for the DL, and E for the ES subgrade in this Table is the mean of the four individual values corresponding to each of the four sensor deflections. The spread of these individual values for each of the two subgrade assumptions is indicated by the two COV columns in Table 2. For each of the load levels, rows for the mean, standard deviation and COV values of the parameters tabulated are also given in Table 2. Similar tables were compiled for other assumed thicknesses (minimum, maximum and average core thickness), for each Row, and for each test site.

All the results obtained in this study are summarized in Table 3. This includes the mean values of the most pertinent parameters from Table 2 (and similar tables not presented here), to allow some broad conclusions to be drawn. The following observations can be made on the basis of the data in this Table:

(1) For the competent pavements considered here ($\ell_k \geq 30$ in.), no consistent load level effect is evident. In other words, the supporting foundation system does not exhibit stress dependence.

(2) The COV values for the backcalculated parameters generally decrease as the load level increases. This is due to the sharper sensor resolution feasible with higher deflections, and points to the need for more accurate sensors. It appears advisable to use one of the higher load levels for routine backcalculations.

(3) The COV values give an indication of the sensitivity of each parameter to the sources of variability involved. Thus, the applied load has a COV generally smaller than 2%, while the basin AREA shows a COV of less than 5%. Nonetheless even this small variation in AREA, results in COV values on the order of 10 to 15% for ℓ_k and ℓ_e, and 20 to 40% or more for the backcalculated parameters, k, E_s, E_{DL} and E_{ES}. This points, once again, to the need for very accurate deflection measurements *(34)*. A small variation in the deflections may cause only a small change in AREA, but may lead to significant differences in the value of the backcalculated parameters. On the other hand, alternative geometric properties of the deflection basin may be considered as substitutes for AREA (see *(35)*, for example).

(4) The backcalculated foundation stiffnesses (k or E_s values) are independent of the assumed slab thickness value. Yet, when combined with the h-value and the independently backcalculated slab E-value, they yield the radius of relative stiffness predicted by the AREA curves in Figure 2. This confirms that the backcalculation method employed is robust and reliable.

(5) The backcalculated slab moduli are in substantial agreement with the values expected on the basis of laboratory testing, particularly in view of the range in the E-values predicted by each of the formulae quoted above, as well as the uncertainty in the value of h. In general, the slab modulus backcalculated on the basis of the elastic solid assumption is about 10% lower than the corresponding value based on the dense liquid idealization. This effect needs to be investigated further; it appears, however, that it may be attributable to the shear interaction provided by adjacent foundation elements in the elastic solid subgrade.

(6) The most significant variation observed in Table 3 occurs in the value of the slab modulus, in response to the assumed slab thickness value. Consider I-5 Row 5, for example, assuming the dense liquid idealization. The mean of the four slab modulus values, obtained when the average core thickness of 9.058 in. is used, is 5.74 Mpsi. The maximum backcalculated E-value is obtained

using the minimum core thickness of 8.250 in., and is 8.86 Mpsi, or 154% of the mean value. Similarly, the minimum E-value, based on the maximum core thickness of 9.625 in., is 4.45 Mpsi, or 78% of the mean stiffness. Thus, a 1.375 in. (15%) change in the assumed thickness value results in a 4.41 Mpsi (77%) difference in the backcalculated E-value. This stresses the need to modify conventional practice, so that E is assumed and h is backcalculated, as suggested above. Field slab modulus values may be estimated using a number of NDT methods, most notably the Schmidt Rebound Hammer. Any errors in the estimated E value will be attenuated rather than amplified when h is backcalculated.

SUMMARY AND CONCLUSION

The principles of dimensional analysis have long been recognized as powerful tools in interpreting experimental data, particularly in those fields where theoretical, closed-form solutions are scarce. This is true of many engineering disciplines, most notably fluid mechanics. In the area of transportation facilities, use of dimensional analysis, while prominent in the works of Westergaard, Burmister, Bradbury, Losberg and other pioneers, has largely been ignored in the last 25 years. In a series of recent publications, Ioannides *(30; 36; 37; 38)* has sought to rectify this situation, and has demonstrated the great potential of this approach to a wide variety of applications in pavement studies. These principles lead to a closed-form backcalculation procedure for rigid pavements *(23)*, which is outlined and discussed in this Paper. A large number of illustrative examples are presented, including some involving the use of a computerized adaptation of the approach (program ILLI-BACK).

The major strengths of the method discussed, compared to other currently available backcalculation schemes are as follows:

(a) It is founded upon a rigorous, sound and fundamental theoretical basis, and is extremely powerful and versatile. For example, in order to examine loading at other locations, such as at the slab edge or corner, the procedure would merely require the derivation of the corresponding AREA versus ℓ curves, similar to those in Figure 2, the latter being applicable to interior loading only. With few modifications, the procedure may be extended to multi-layer systems, including flexible pavements, as well.

(b) When the backcalculation is performed on a personal computer, execution time per deflection basin is trivial (≈1 sec.). This permits the interpretation of a vast amount of NDT data in a very reasonable time. As a corollary, the method makes it feasible for the first time to have a practical backcalculation procedure attached to the testing device in the field, providing instant checks on the accuracy of the deflection results generated, while there is still time and opportunity for remedial action.

(c) Each sensor reading provides an independent estimate of the backcalculated parameters. This allows the engineer to investigate the sources of variability in field measurements and materials. Either of the two major subgrade models, namely the dense liquid or the elastic solid foundation, may be employed, providing a rare opportunity for meaningful comparisons between the two. The departure of *in situ* behavior from the theoretical assumptions is reflected in the backcalculation statistics obtained. This affords an estimate of the relative location of the actual pavement system behavior within the spectrum defined by the two extreme soil idealizations.

(d) There is no need for the provision of seed moduli in this approach. These moduli greatly affect the accuracy of other backcalculation schemes. If the slab thickness is known, however, the slab modulus can be estimated. Conversely, if the slab modulus is known, the slab thickness is backcalculated. Results presented above indicate that the latter is a more advisable route to follow, although this option is unavailable in other current procedures. Contrary to what is generally assumed in practice, thickness exhibits considerable variability in the field, and this has a large impact on the backcalculated slab stiffness values. In addition, the backcalculation of the subgrade modulus, being independent in the proposed method of the assumed slab h or E, is entirely insensitive to any errors that may occur in estimating these slab properties.

(e) The method is general in nature, and is not based on a limited number of particular cases as are database approaches. This permits very useful and theoretically valid inferences to be made for a given pavement system by examining data collected from a wide variety of other dissimilar systems.

(f) Application of the procedure to actual field data from recent or on-going projects has confirmed that it yields very realistic, consistent and reliable results. Calculations performed follow a definite and closed loop, all but eliminating the probability of calculation errors.

(g) A number of interesting topics may be examined in the future using this method. These include: the effect of number and spacing of sensors; correlation between backcalculated and intrinsic system properties, particularly with respect to the dynamic effect of the NDT procedure; examination of the effect of friction and bonding between layers, and comparison with results from an equivalent combined thickness approach; field determination of properties necessary in other aspects of pavement design, such as dowel concrete interaction, amount of load transferred by the critical dowel as well as by the entire dowel assembly, etc.

ACKNOWLEDGEMENTS

Parts of this research effort were supported by funding provided by the Departments of Transportation of the states of Illinois and Washington, and by the University of Illinois Advanced Construction Technology Center, which is sponsored by the Army Research Office as part of the University Research Initiative Program. This support is gratefully acknowledged. The Washington data examined were kindly released by Mr Newton C. Jackson of the Washington State DOT. The laboratory testing program reported was conducted at the University of Washington by Prof. J.P. Mahoney and his co-workers. Their invaluable contribution is greatly appreciated.

DISCLAIMER

The contents of this Paper reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of any of the funding agencies. This Paper does not constitute a standard, specification, or regulation.

REFERENCES

1. Transportation Research Board (1984). "America's Highways: Accelerating the Search for Innovation," Special Report 202, Transportation Research Board, National Research Council, Washington, D.C.
2. Scrivner, F.H., Swift, G., and Moore, W.M. (1966). "A New Research Tool for Measuring Pavement Deflection," Highway Research Record No. 129, Highway Research Board, National Research Council, Washington, D.C., pp. 1-11.
3. Wang, M.C., Larson, T.D., Bhajandas, A.C., and Cumberledge, G. (1978). "Use of Road Rater Deflections in Pavement Evaluation," TRRGG, Transportation Research Board, National Research Council, Washington, D.C., pp. 32-38.
4. Bohn, A., Ullidtz, P. Stubstad, R., and Sorensen, A. (1972). "Danish Experiments with the French Falling Weight Deflectometer," Proceedings, The University of Michigan Third International Conference on the Structural Design of Asphalt Pavements, Sept., Vol. I, Ann Arbor, MI, pp. 1119-1128.
5. Burmister, D.M. (1943). "The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Runways," Proceedings, Highway Research Board, Vol. 23, pp. 126-144, Discussion: pp. 144-148.
6. Burmister, D.M. (1945). "The General Theory of Stresses and Displacements in Layered Soil Systems," Journal of Applied Physics, Vol. 16, Part I: No. 2, Feb., pp. 89-96; Part II: No. 3, Mar., pp. 126-127; Part III: No. 5, May, pp. 296-302.
7. Ashton, J.E., and Moavenzadeh, E. (1967). "Analysis of Stresses and Displacements in a Three-Layer Viscoelastic System," Proceedings, The University of Michigan Second International Conference on the Structural Design of Asphalt Pavements, Aug. 7-11, Ann Arbor, MI, pp. 209-219.
8. Peutz, M.G.F., Van Kempen, H.P.M., and Jones, A. (1968). "Layered Systems Under Normal Surface Loads," Highway Research Record No. 228, Highway Research Board, National Research Council, Washington, D.C., pp. 34-45.
9. Michelow, J. (1963). "Analysis of Stresses and Displacements in an N-Layered Elastic System Under a Load Uniformly Distributed on a Circular Area," California Research Corporation, Richmond, California, Sept. 24.
10. Bush, A.J., and Alexander, D.R. (1985). "Pavement Evaluation Using Deflection Basin Measurements and Layered Theory," Transportation Research Record 1022, Transportation Research Board, National Research Council, Washington, D.C., pp. 16-29.
11. Van Cauwelaert, F.J., Barker, W.J., White, T.D., and Alexander, D.R. (1988). "A Competent Multilayer Solution and Backcalculation Procedure for Personal Computers," Presented at the First International Symposium on Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM, June 29-30, Baltimore, MD.
12. Anderson, M. (1988). "A Data Base Method for Backcalculation of Composite Pavement Layer Moduli," Presented at the First International Symposium on Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM, June 29-30, Baltimore, MD.
13. Tia, M., and Eom, K.S. (1988). "Development of DBCONPAS Computer Program for Estimating of Concrete Pavement Parameters from FWD Data," Presented at the First International Symposium on Nondestructive Testing of Pavements and Backcalculation of Moduli, Poster Session I, ASTM, June 29-30, Baltimore, MD.
14. Tia, M., Armaghani, J.M., Wu, C.L., Lei, S., and Toye, K.L. (1987). "FEACONS III Computer Program for Analysis of Jointed Concrete Pavements," Transportation Research Record 1136, Transportation Research Board, National Research Council, Washington, D.C., pp. 12-22.
15. Vaswani, N.K. (1971). "Method for Separately Evaluating Structural Performance of Subgrades and Overlying Flexible Pavements," Highway Research Record No. 362, Highway Research Board, National Research Council, Washington, D.C., pp. 48-62.
16. Little, D.N. (1980). "The Dual Parametric Approach to the Analysis of Surface Deflection Data," Texas Transportation Institute, Texas A&M University System, College Station, June.
17. Lary, J.A., and Mahoney, J.P. (1984). "Seasonal Effects on the Strength of Pavement Structures," Transportation Research Record 954, Transportation Research Board, National Research Council, Washington, D.C., pp. 88-94.
18. ERES Consultants, Inc. (1982). "Nondestructive Structural Evaluation of Airfield Pavements," Prepared for U.S. Army Engineer Waterways Experiment Station, Champaign, IL.

19. Foxworthy, P.T. (1985). "Concepts for the Development of a Nondestructive Testing and Evaluation System for Rigid Airfield Pavements," Thesis presented to the University of Illinois, at Urbana, IL, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
20. Herrin, S.M., Darter, M.I., Barenberg, E.J., and Shahin, M.Y. (1986). "Airfield Pavement Evaluation and Management at Dulles International Airport," Transportation Research Record 1060, Transportation Research Board, National Research Council, Washington, D.C., pp. 53-61.
21. Roman, R.J., Shahin, M.Y., and Freeman, T.J. (1986). "Analysis of Concrete Pavement Performance and Development of Design Procedures," Transportation Research Record 1099, Transportation Research Board, National Research Council, Washington, D.C., pp. 37-44.
22. Ioannides, A.M., Thompson, M.R., and Barenberg, E.J. (1985). "Finite Element Analysis of Slabs-On-Grade Using a Variety of Support Models," Proceedings, Third International Conference on Concrete Pavement Design and Rehabilitation, Purdue University, April 23-25, pp. 309-324.
23. Ioannides, A.M. (1988). "Dimensional Analysis in NDT Rigid Pavement Evaluation," Submitted for publication, Journal of Transportation Engineering, ASCE, July.
24. Westergaard, H.M. (1926). "Stresses in Concrete Pavements Computed by Theoretical Analysis," Public Roads, Vol. 7, No. 2, April, pp. 25-35.
25. Westergaard, H.M. (1939). "Stresses in Concrete Runways of Airports," Highway Research Board Proceedings, 19th Annual Meeting, pp. 197-202; Discussion: pp. 202-205.
26. Hogg, A.H.A. (1938). "Equilibrium of a Thin Plate, Symmetrically Loaded, Resting on an Elastic Foundation of Infinite Depth," Philosophical Magazine, Series 7, Vol. 25, March, pp. 576-582.
27. Losberg, A. (1960). "Structurally Reinforced Concrete Pavements," Doktorsavhandlingar Vid Chalmers Tekniska Högskola, Göteborg, Sweden.
28. Hoffman, M.S., and Thompson, M.R. (1981). "Mechanistic Interpretation of Nondestructive Pavement Testing Deflections," Civil Engineering Studies, Transportation Engineering Series No. 32, Illinois Cooperative Highway and Transportation Research Program Series No. 190, University of Illinois, at Urbana, IL.
29. Ioannides, A.M., Thompson, M.R., and Barenberg, E.J. (1985). "Westergaard Solutions Reconsidered," Transportation Research Record 1043, Transportation Research Board, National Research Council, Washington, D.C., pp. 13-23.
30. Ioannides, A.M. (1986). Discussion of "Response and Performance of Alternate Launch and Recovery Surfaces that Contain Layers of Stabilized Material," by R.R. Costigan and M.R. Thompson, Transportation Research Record 1095, Transportation Research Board, National Research Council, Washington, D.C., pp. 70-71.
31. American Concrete Institute (1963). "Building Code Requirements for Reinforced Concrete," ACI Standard 318-63, June, p. 50.
32. Pauw, A. (1960). "Static Modulus of Elasticity of Concrete as Affected by Density," ACI Journal, Proceedings, Vol. 57, December, pp. 679-687.
33. Hognestad, E. (1951). "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," Bulletin No. 399, University of Illinois Engineering Experiment Station, Urbana, IL, November.
34. Irwin, L.H., Yang, W.S., and Stubstad, R.N. (1988). "The Importance of Deflection Reading Accuracy and Layer Thickness Accuracy in Back-Calculation of Pavement Layer Moduli," Presented at the First International Symposium on Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM, June 29-30, Baltimore, MD.
35. Hill, H.J. (1988). "Early Life Study of the FA409 Full-Depth Asphalt Concrete Pavement Sections," Thesis presented to the University of Illinois, at Urbana, IL, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
36. Ioannides, A.M. (1987). Discussion of "Thickness Design of Roller-Compacted Concrete Pavements," by S.D. Tayabji and D. Halpenny, Transportation Research Record 1136, Transportation Research Board, National Research Council, Washington, D.C., pp. 31-32.
37. Ioannides, A.M. (1988). "The Problem of a Slab on an Elastic Solid Foundation in the Light of the Finite Element Method," Proceedings, 6th International Conference on Numerical Methods in Geomechanics, Paper No. 408, April 11-15, Innsbruck, Austria, pp. 1059-1064.
38. Ioannides, A.M., and Salsilli-Murua, R.A. (1989). "Temperature Curling in Rigid Pavements: An Application of Dimensional Analysis," accepted for presentation and publication at the 68th Annual Meeting of the Transportation Research Board, Washington, D. C., January. □

TABLE 1 SENSITIVITY OF INTERIOR LOADING RESPONSES

SERIES	LOAD RADIUS (in.)	MAX. DEFLECTION (mils) DL	ES	%	MAX. SUBGRADE STRESS (psi) DL	ES	%	MAX. BENDING STRESS (psi) DL	ES	%
I	2.0000	3.0189	3.0464	99.10	0.6390	1.8401	34.73	130.36	139.03	93.76
I	5.9055	2.9885	3.0118	99.23	0.6325	1.7023	37.16	91.08	93.04	97.89
I	10.0000	2.9381	2.9563	99.38	0.6219	1.5679	39.66	72.09	71.08	101.42
II	2.0000	3.0200	3.0452	99.17	0.6392	1.8386	34.77	153.45	139.33	110.14
II	5.9055	2.9896	3.0106	99.30	0.6328	1.7010	37.20	107.21	93.24	114.98
II	10.0000	2.9392	2.9552	99.46	0.6221	1.5668	39.71	84.86	71.24	119.12

Note: For both Series, P = 7792 lbs; μ = 0.15; μ_s = 0.45; k = 212 psi/in.; E_s = 27,992 psi
For Series I, E = 4.5 Mpsi; DL: h = 10.85 in.; ES: h = 10.01 in
For Series II, h = 10.00in.; DL: E = 5.74 Mpsi; ES: E = 4.52 Mps

DL: Dense Liquid Foundation; ES: Elastic Solid Foundation; % : (DL/ES) x 100%

TABLE 3 SUMMARY OF BACKCALCULATION RESULTS

(A) SITE 1: I-5 (SEATTLE, WA)

LOAD (lbs)	AREA (in.)	ℓ_k (in.)	ℓ_e (in.)	k (pci)	E_s (psi)	E DL (Mpsi)	E ES (Mpsi)
(a) ROW 2: h = 9.058 in. (avg.)							
6243	29.9	34.3	25.8	296	31388	5.82	5.13
8663	29.7	33.1	25.2	292	30495	5.23	5.52
12000	30.0	33.6	25.4	272	28716	5.18	4.61
15443	29.8	32.6	25.0	277	28738	4.83	4.33
(b) ROW 5: h = 9.058 in. (avg.)							
6223	31.6	45.1	33.2	109	15383	6.72	5.53
8523	31.3	41.5	30.5	125	16331	5.48	4.53
11680	31.2	41.3	30.6	125	16271	5.36	4.48
15197	31.1	40.4	29.8	134	17222	5.38	4.52
(c) ROW 5: h = 8.250 in. (min.)							
6223	31.6	45.1	33.2	109	15383	8.86	7.34
8523	31.3	41.5	30.5	125	16331	7.28	6.00
11680	31.2	41.3	30.6	125	16271	7.09	5.95
15197	31.1	40.4	29.8	134	17222	7.09	5.96
(d) ROW 5: h = 9.625 in. (max.)							
6223	31.6	45.1	33.2	109	15383	5.61	4.61
8523	31.3	41.5	30.5	125	16331	4.58	3.78
11680	31.2	41.3	30.6	125	16271	4.47	3.74
15197	31.1	40.4	29.8	134	17222	4.45	3.76
(e) COEFFICIENTS OF VARIATION, %: ROW 2: h = 9.058 in. (avg.)							
1.6	4.2	15.0	14.0	43.1	32.0	26.6	23.3
1.1	3.5	11.5	10.9	37.7	27.9	26.6	25.3
0.5	3.0	12.1	10.4	32.2	24.6	25.3	23.2
0.5	3.1	11.6	10.2	30.9	23.2	23.2	21.5

(B) SITE 2: I-90 (SPOKANE, WA)

LOAD (lbs)	AREA (in.)	ℓ_k (in.)	ℓ_e (in.)	k (pci)	E_s (psi)	E DL (Mpsi)	E ES (Mpsi)
(a) ROW 1: h = 7.662 in. (avg.)							
6438	28.4	29.1	22.4	272	25808	5.12	4.75
8795	28.9	29.2	22.5	266	25390	5.16	5.78
11957	29.1	29.4	22.8	255	24427	5.11	4.78
15428	29.2	29.9	23.0	254	24637	5.41	4.98
(b) ROW 4: h = 7.662 in. (avg.)							
6056	28.5	28.7	21.3	277	25924	4.74	4.13
8520	28.0	27.6	20.7	287	26373	4.36	3.81
11799	28.5	28.5	21.3	269	25096	4.53	4.03
15233	28.8	29.0	22.0	266	25006	4.76	4.25
(c) COEFFICIENTS OF VARIATION, %: ROW 1: h = 7.662 in. (avg.)							
2.1	4.8	16.1	17.4	28.3	20.2	39.3	40.4
1.0	4.5	13.2	16.1	24.4	17.7	32.6	35.9
1.0	4.1	13.7	15.5	24.0	17.2	33.6	33.8
0.9	3.9	13.8	14.7	22.9	15.8	32.8	31.7

TABLE 2 MEAN BACKCALCULATED PARAMETERS FROM I-5 ROW 2 USING AVERAGE THICKNESS

STATION	LOAD (lbs)	D_0 (mils)	D_1 (mils)	D_2 (mils)	D_3 (mils)	AREA (in.)	ℓ_k (in.)	ℓ_e (in.)	k (psi/in.)	E_s (psi)	E DL (Mpsi)	COV DL (%)	E ES (Mpsi)	COV ES (%)
6000	6070	2.441	2.323	2.047	1.693	32	43	32	166	23940	9.3	2.5	7.6	2.4
17000	6150	3.031	2.835	2.559	2.165	32	43	32	135	19451	7.5	0.9	6.2	0.8
31000	6182	2.126	1.85	1.575	1.339	29	30	23	392	37866	4.8	8.4	4.5	3
34000	6197	2.047	1.732	1.457	1.181	28	27	20	487	44406	4.1	8.7	3.7	3.1
3000	6213	3.425	3.11	2.756	2.323	31	37	28	163	19695	4.9	3.2	4.2	1
40000	6213	4.843	4.764	3.622	2.913	30	36	27	125	14653	3.3	5.3	2.9	7.1
9000	6213	2.638	2.402	2.126	1.811	31	38	28	205	25250	6.6	3.3	5.6	1.1
20000	6229	2.205	2.008	1.732	1.457	30	36	27	276	31873	7	3.5	6.1	0.3
23000	6293	1.969	1.732	1.457	1.26	29	30	23	412	40766	5.6	8	5.2	2.9
37000	6356	1.89	1.693	1.417	1.102	29	30	23	442	43664	5.9	3.6	5.5	2.3
26000	6388	1.969	1.732	1.457	1.26	29	30	23	418	41383	5.7	8	5.3	2.9
12000	6420	2.362	2.047	1.811	1.535	29	31	24	330	33712	5.1	8.3	4.7	3.6
MEAN	6243.					29.91	34.25	25.83	295.9166	31388.	5.8166	5.308	5.125	2.541
ST. DEV	97.86					1.255	5.133	3.624	127.4561	10033.	1.5469	2.684	1.191724	1.714
COV %	1.567					4.196	14.98	14.03	43.07164	31.965	26.595	50.57	23.25316	67.45
6000	8486	3.622	3.268	2.835	2.441	30	35	27	232	26531	5.6	4.9	4.9	1.6
3000	8565	4.764	4.331	3.858	3.268	31	38	28	156	19215	5.1	3.2	4.3	1.1
17000	8581	4.409	3.937	3.583	3.11	31	38	28	170	20824	5.4	5.1	4.6	3
31000	8613	3.11	2.677	2.362	2.008	29	31	24	353	35103	4.9	9	4.5	4
9000	8613	3.819	3.425	2.953	2.559	30	34	26	236	26148	5.1	5.8	4.5	2.1
20000	8613	3.15	2.874	2.48	2.047	30	35	27	269	31001	6.7	2.7	5.9	0.7
34000	8676	3.031	2.559	2.165	1.732	28	27	20	462	42151	3.9	8.5	3.4	3
26000	8724	2.874	2.48	2.165	1.772	29	30	23	412	39707	5	8	4.7	2.8
40000	8724	6.575	5.984	4.843	3.78	29	30	23	177	17360	2.3	2.6	2.1	3
23000	8756	2.795	2.52	2.205	1.85	30	35	27	311	35591	7.5	4.1	6.6	1
12000	8772	3.307	3.031	2.638	2.244	31	37	28	236	28730	7.2	3.1	6.2	0.4
37000	8835	2.874	2.48	2.047	1.693	28	27	21	495	43587	4.1	9.2	4	2.3
MEAN	8663.					29.66	33.08	25.16	292.4166	30495.	5.2333	5.516	4.641666	2.083
ST. DEV	96.98					1.027	3.796	2.733	110.1494	8517.6	1.3900	2.433	1.172218	1.075
COV %	1.119					3.463	11.47	10.86	37.66867	27.930	26.561	44.10	25.25426	51.61
31000	11902	4.528	3.937	3.425	2.874	29	31	24	335	33387	4.7	7.9	4.3	2.9
17000	11950	6.299	5.709	5.118	4.488	31	39	29	156	19805	5.6	4.2	4.7	2.4
37000	11950	4.055	3.622	2.992	2.48	29	30	23	386	38003	5.1	5.8	4.8	0.5
3000	11966	6.811	6.22	5.551	4.685	31	39	29	147	18505	5.1	2.6	4.3	0.9
6000	11982	5.276	4.803	4.213	3.583	31	37	28	206	24765	6.1	3.5	5.2	0.9
20000	11998	4.724	4.173	3.622	2.992	30	32	24	302	31272	5	6.3	4.5	1.6
40000	11998	8.819	7.795	6.299	4.921	29	27	21	216	19276	1.9	5.8	1.8	1.4
12000	11998	4.961	4.409	3.937	3.307	30	35	26	244	27642	5.7	5.2	5	2.1
34000	12029	4.291	3.74	3.11	2.559	29	28	22	419	38391	4.1	7.9	4	1.6
9000	12045	5.394	4.882	4.331	3.701	31	37	28	200	24225	6	3.9	5.2	1.5
26000	12061	4.173	3.661	3.15	2.598	29	31	24	370	36819	5.1	6.8	4.8	1.8
23000	12125	4.094	3.74	3.228	2.795	31	37	27	273	32505	7.7	4.1	6.7	1.2
MEAN	12000					30	33.58	25.41	271.1666	28716.	5.175	5.333	4.608333	1.566
ST. DEV	56.49					0.912	4.071	2.660	87.37451	7058.9	1.3077	1.643	1.071181	0.645
COV %	0.470					3.042	12.12	10.46	32.22170	24.581	25.270	30.81	23.24445	41.22
40000	15319	10.866	9.528	7.677	5.984	28	27	21	223	20511	2	5.1	1.8	1.1
17000	15335	8.031	7.283	6.535	5.748	31	39	29	155	19827	5.7	4.2	4.8	2.4
31000	15366	5.866	5.157	4.528	3.819	30	32	25	301	31698	5.3	7	4.8	2.5
3000	15382	8.78	7.953	7.087	6.024	31	37	28	155	18876	4.8	3.6	4.1	1.4
26000	15414	5.394	4.843	4.173	3.425	30	33	25	317	34135	6.1	4.4	5.5	0.7
9000	15462	6.969	6.299	5.63	4.764	31	37	28	197	23995	6	3.6	5.2	1.5
6000	15462	6.929	6.181	5.433	4.606	30	34	26	231	25775	5.1	5.4	4.5	1.9
20000	15478	6.378	5.472	4.724	3.858	29	28	22	354	32849	3.6	8.7	3.5	2.9
37000	15478	5.394	4.764	3.976	3.268	29	30	23	392	37727	4.7	6.4	4.4	0.8
12000	15494	6.496	5.827	5.118	4.37	30	35	27	236	27031	5.7	5	5	1.8
23000	15525	5.669	4.882	4.291	3.701	29	31	24	347	34597	4.9	9.4	4.5	4.3
34000	15605	5.669	4.882	4.173	3.346	29	28	22	414	37845	4	8.1	3.9	2.2
MEAN	15443					29.75	32.58	25	276.8333	28738.	4.825	5.908	4.333333	1.958
ST. DEV	79.72					0.924	3.773	2.549	85.57631	6654.5	1.1188	1.910	0.931247	0.963
COV %	0.516					3.106	11.58	10.19	30.91257	23.155	23.189	32.33	21.49033	49.17

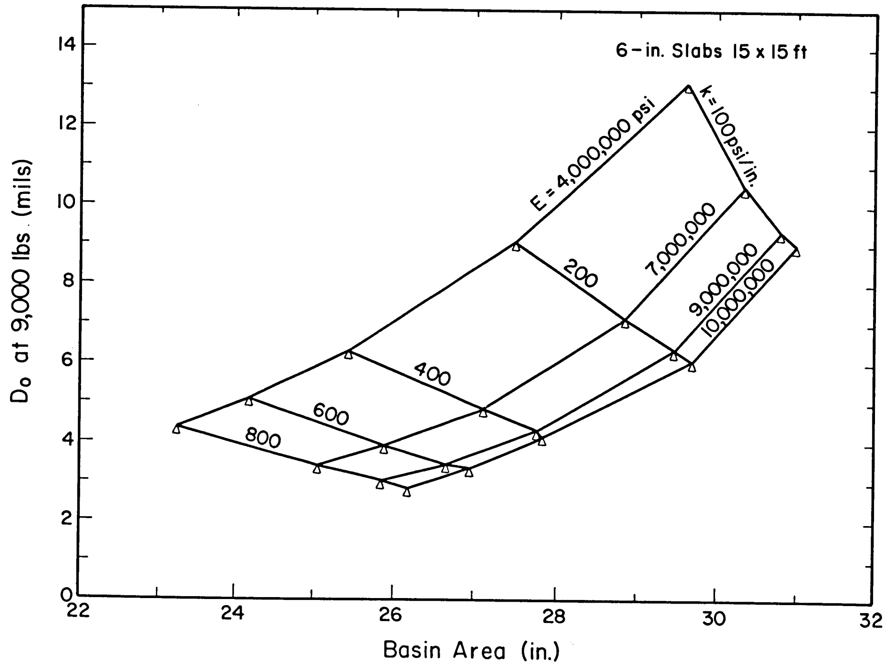

FIGURE 1 Typical Maximum Deflection Versus Basin Area Backcalculation Grid [After Roman, *et al.* *(21)*]

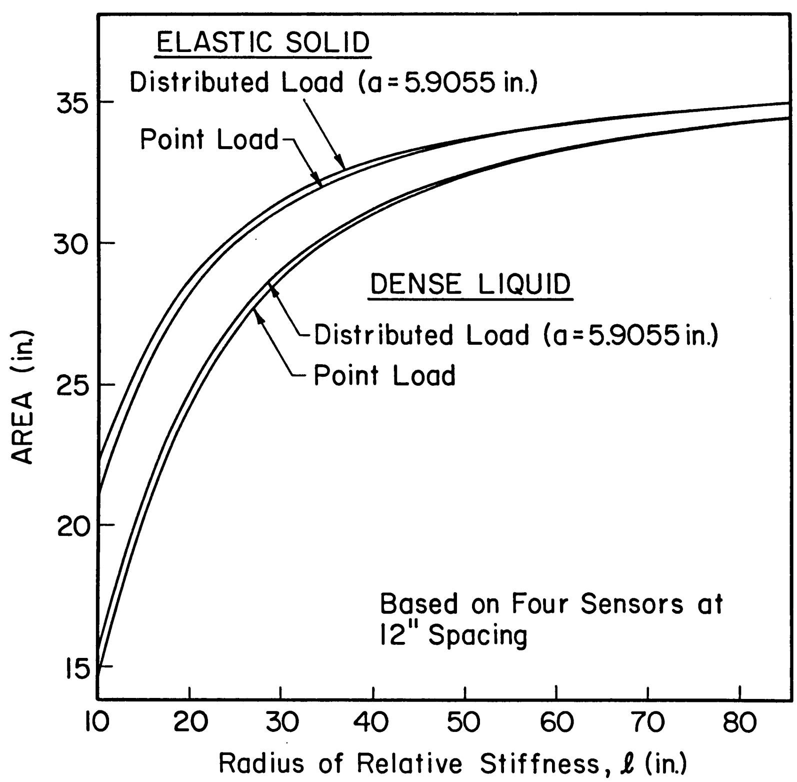

FIGURE 2 Variation of AREA with ℓ [After Ioannides *(23)*]

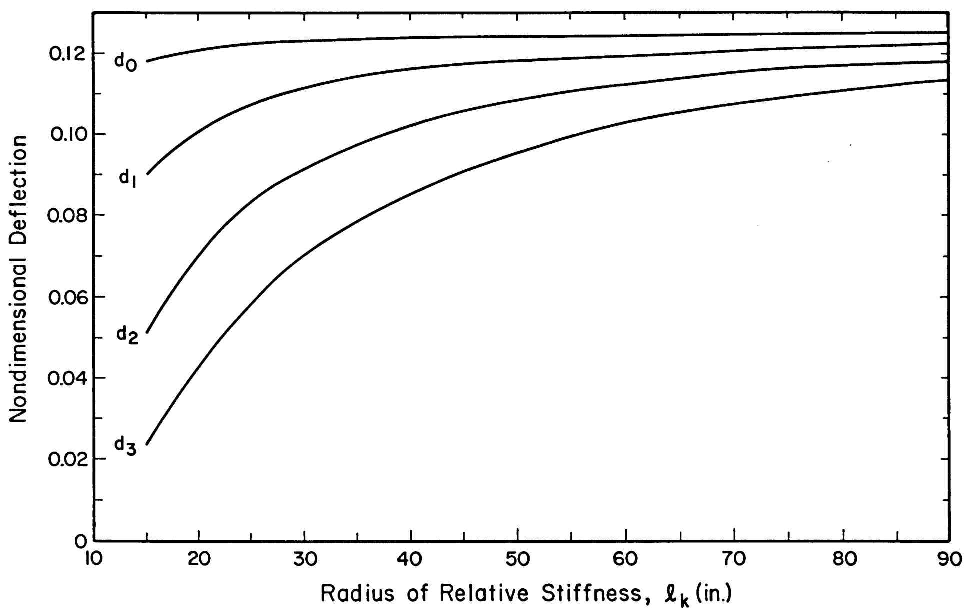

FIGURE 3 Variation of Dimensionless Deflections with ℓ:
(a) For Dense Liquid Foundation

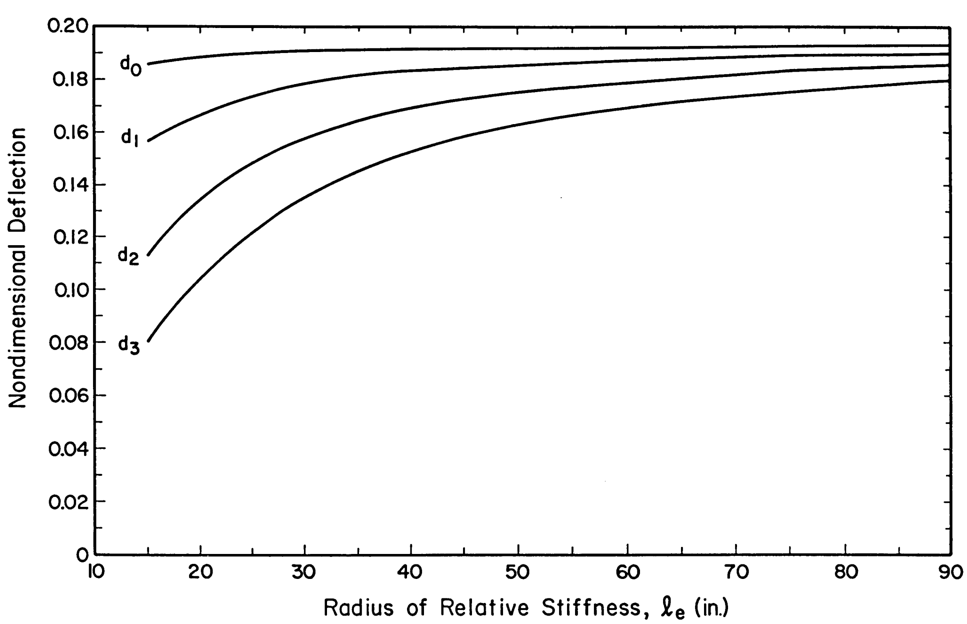

FIGURE 3 Variation of Dimensionless Deflections with ℓ:
(b) For Elastic Solid Foundation

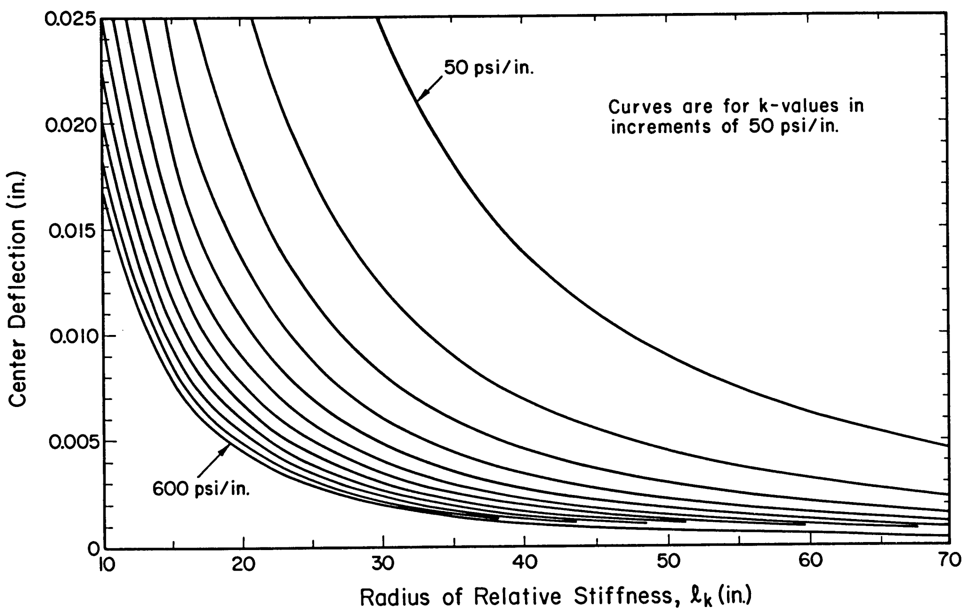

FIGURE 4 Chart for the Determination of Subgrade Modulus: (a) For Dense Liquid Foundation

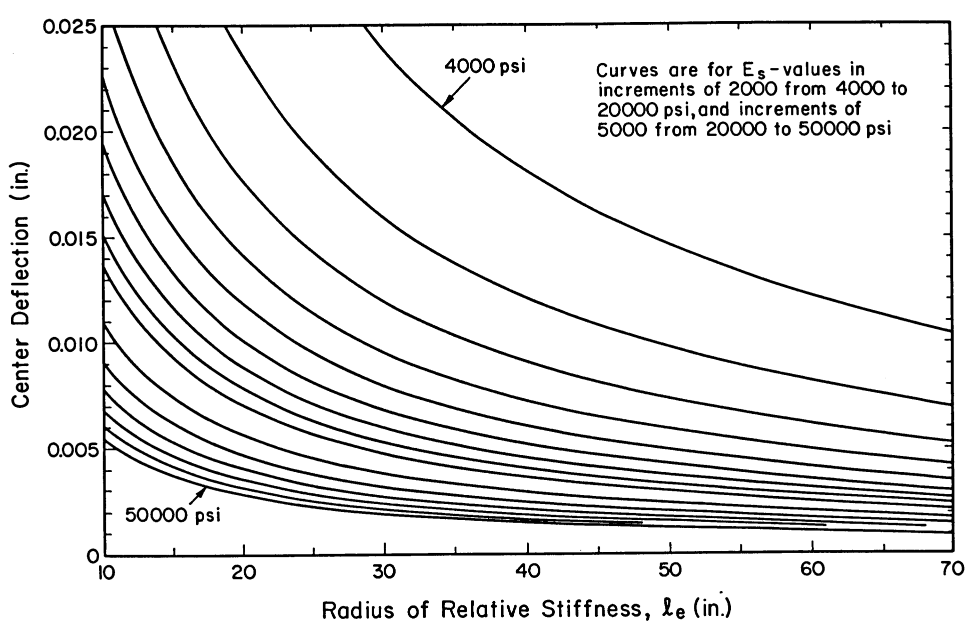

FIGURE 4 Chart for the Determination of Subgrade Modulus: (b) For Elastic Solid Foundation

```
**********************************************
*              PROGRAM ILLI-BACK             *
*   A CLOSED-FORM BACKCALCULATION PROCEDURE  *
*     CODED BY A.M. IOANNIDES - JUNE 1988    *
*             ALL RIGHTS RESERVED            *
*          NOT FOR PUBLIC DISTRIBUTION       *
*                VERSION 2.0                 *
* Date of Present Version: August 25, 1988   *
**********************************************
             I-5 ROW 2, STATION 3000

CURRENT KNOWN PARAMETERS AS ENTERED
 APPLIED LOAD (lbs) = 6213.3
 PLATE RADIUS (in.) = 5.9055
 SLAB POISSON RATIO =   0.15
 SOIL POISSON RATIO =   0.45
SLAB THICKNESS (in.) =  9.058

DEFLECTIONS AS ENTERED (in mils)
  D0=  3.425   D1=  3.110   D2=  2.756   D3=  2.323

 BASIN AREA (in inches) =          30.62069

RADIUS OF RELATIVE STIFFNESS (in inches)
  DENSE LIQUID  =        37.18299
  ELASTIC SOLID =        27.84477


 BACKCALCULATED SUBGRADE MODULUS VALUES

             DENSE LIQUID        ELASTIC SOLID
       (k values in psi/in.) (Es values in psi)

    FROM D0 =   161.89433         19725.93359
    FROM D1 =   167.81856         19961.91602
    FROM D2 =   165.16695         19521.00586
    FROM D3 =   155.62369         19570.12500
       MEAN =   162.62589         19694.74609
   ST. DEV. =     5.25934           198.25459
        COV =     3.23401 %           1.00664 %

BACKCALCULATED SLAB MODULUS VALUES

            DENSE LIQUID         ELASTIC SOLID
         (E values in psi)     (E values in psi)

    FROM D0 = 4884388.50          4214139.50
    FROM D1 = 5063124.50          4264553.50
    FROM D2 = 4983124.00          4170360.00
    FROM D3 = 4695202.00          4180853.50
       MEAN = 4906460.00          4207476.50
   ST. DEV. =  158669.91            42398.14
        COV =    3.23390 %           1.00769 %
```

FIGURE 5 Typical ILLI-BACK Output (I-5 Row 2, Station 3000)