RELIABILITY ANALYSIS OF CHINA'S HIGHWAY CONCRETE PAVEMENT DESIGN PROCEDURE

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This paper presents the application of reliability concepts to China's current highway concrete pavement design procedure. The fatigue life of slab is assumed to follow two-parameter Weibull distribution and theoretical reliability formula is derived by using "interference theory". First-order approximation method is used to obtain the mean and variance of fatigue life from the means and variances of design parameters. Reliabilities are calculated at various traffic levels, variability levels, and slab thicknesses. Sensitivity analysis upon the results shows that pavement design reliability is significantly influenced by factors such as traffic, concrete modulus of rupture, slab thickness, and level of material variability. A probabilistic design procedure is developed by introducing a reliability factor in the present design process.

Pavement structures are of non-deterministic nature. Large random variabilities and uncertainties of almost every measurable parameter are associated with pavement design, construction, and performance. However, current highway concrete pavement design practices in China are deterministic in that all design factors are assumed to be exact quantities. To partially account for the uncertainties, several empirical safety and judgement factors have been applied in forms of reducing the working strengths of materials and magnifying the computed stresses. This traditional design methodology has two drawbacks: first, the variable nature of the design parameters is not quantitatively taken into account, ignoring the fact that the actual pavement performance is strongly influenced by the variability; second, the empirical safety factors are chosen based on personal judgement and experience without directly considering the magnitude of the uncertainties involved, which will generally result in an overdesign or underdesign.

Many researches have been carried out in applying reliability to pavement design since early 1970's (2, 5), and many design procedures have been revised into probabilistic procedures. The application of reliability concepts to pavement design is an essential step toward improving the existing deterministic procedure to a more realistic one.

In this paper, reliability analysis is made for current highway concrete pavement design procedure in China. And the developemnt of a probabilistic approach is presented.

Current Procedure

Present highway concrete pavement design specification in China adopts an analytical approach (8). It takes the fatigue cracking in the slab as the critical damage condition, and the design is based on the criteria of flexural stress due to combined effects of repeated traffic loading and thermal gradients. The design process is illustrated in Figure 1 and can be summarised as the following:

1. Collect traffic related parameters, and calculate accumulative applications of equivalent standard axle load(ESAL) during design life through traffic conversion and prediction model.
2. Choose and evaluate parameters about pavement structure, materials, and environment, such as slab thickness, h, concrete modulus of rupture, MR, concrete modulus of elasticity, Ec, resilient modulus of subgrade, Es, and the annual equivalent fatigue thermal gradient, Tgy.
3. Estimate the loading stress σ_p by means of stress nomograph with given standard axle load, slab thickness h, and relative stiffness Ec/Es. Compute the equivalent fatigue warping stress σ_t through a series of equations.
4. Use the following fatigue equation to obtain the allowable loading stress $[\sigma_p]$ from σ_t and n:

$$\frac{\sigma_p + \sigma_t}{MR} = 0.944 - 0.0768\ (1 - R)\log n \qquad (1)$$

where $R = \sigma_t/(\sigma_p + \sigma_t)$. Compare the allowable stress, $[\sigma_p]$, with computed stress, σ_p' (multiplied by dynamic factor and safety factor). If the difference is more than 5 percent, repeat the analysis with a new pavement structure until $[\sigma_p]$ is about the same as σ_p'.

The above design methodology is deterministic in nature. All the design parameters are assumed to be exact quantities, and a unique pavement system is obtained for the unique set of input vari-

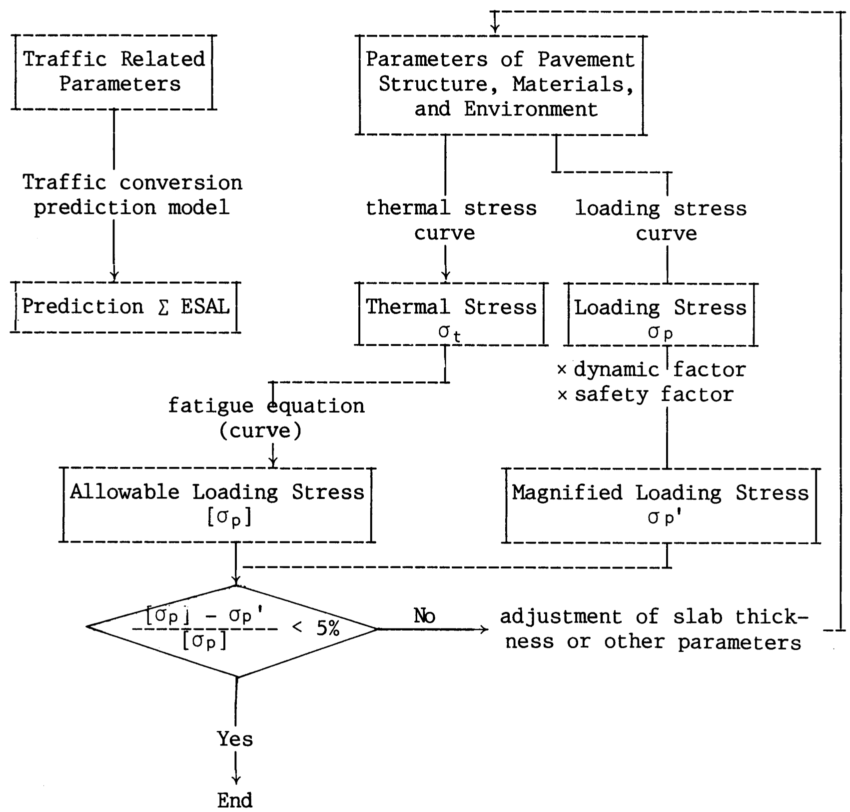


Fig.1 Flow chart of current design process

ables. The reality is, however, that various kinds of variations exist in design parameters: there are variations due to material property (such as concrete modulus of rupture), inavoidable construction deficiencies (such as slab thickness), and forecasting uncertainties with traffic load and environmental conditions (such as predicted ESAL applications). Moreover, there are performance deviations (lack-of-fit) due to the inability of the design equations to predict precisely the mean performance of replicate projects, which may be attributed to deficiencies in the experimental design and/or data analyses from which the design equations were derived. These variabilities are not directly taken into account in current design, and the safety factor is only a partial consideration. To make the design more realistic, probabilistic concepts should be applied so that the variations and uncertainties can be identified and quantitatively evaluated, thus more optimum design can be achieved by designing a pavement at a specific level of variability and a desired level of reliability.

Reliability Application to Pavement Design

Definition of Reliability

Pavement design reliability is defined to be the probability that the pavement will provide adequate service under the encountered load and environmental conditions over the period of time for which it has been designed. This definition only conveys general meaning for the subject. "Adequate service" may have different meanings according to the design criteria and the controlling critical damage condition in different design methods. Pavement failure occurs when the specified distress appears or the specified value of serviceability drops below its terminal level, which means that pavement can no longer maintain adequate service (provided that pavement serviceability is mainly influenced by that distress or value) and repair maintenance or replacement is needed.

For China's current highway concrete pavement design procedure, in which fatique cracking caused by repeated traffic loading and fluctuating thermal warping stresses is the critical damage condition, reliability can be defined to be the probability that fatigue fracture will not occur in the slab under given load and environmental conditions during its intended design life.

The above definition is much more specific than the first, but is still not enough to be used for reliability calculations. For practical reliability evaluation, the number of ESAL applications as related to fatigue cracking is chosen as the "performance indicator", and reliability can be written as:

$$R = P\,(N > n) \tag{2}$$

where N is the number of ESAL applications which the pavement can withstand before fracture, i.e. the fatigue life;and n is the predicted total number of ESAL applications which will be applied to the pavement. Reliability is expressed as a percentage, which hints statistically that R% of the slabs that would exist along a pavement project would be expected to remain uncracked at the end of the intended design life if they have the same design and the traffic is applied to all.

Both N and n are random variables as shown in Figure 2. The fatigue life, N, varies tremendously between different slabs because of the random nature of the design parameters on which it depends. While uncertainties exist in estimating the number of ESAL applications that will travel over the pavement, i.e. there is a difference (forecasting error) between predicted and actual n. The estimated accumulative ESAL applications, n, which will be applied to the pavement, is a negative factor or "load effect" upon fatigue cracking; while the fatigue life, N, which is the ability the pavement possesses , can be regarded as the "resistance" against fatigue cracking. As illustrated in Figure 2, the density functions of N and n have an overlap (interference area), which is a function of the probability of failure. "Interference theory" (4) can be applied to calculate the reliability:

$$R = 1 - \int_{-\infty}^{+\infty} f_n(n)\ F_N(n)\ dn \qquad (3)$$

where f_n is the probability density of n, and F_N is the distribution function of N. Pavement design reliability can now be calculated with equation 3 if we know the probability distributions and the statistical parameters of N and n.

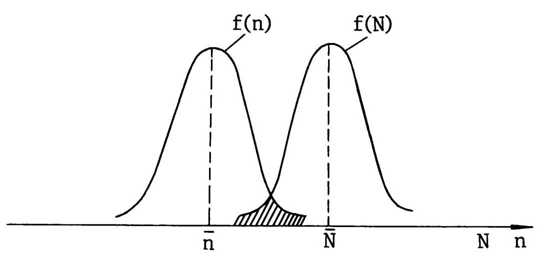


Figure 2 Probability densities of N and n

Probability Distribution of Fatigue Life

In some previous reliability analyses made for pavements (2, 5), fatigue life, N, was assumed to be lognormally distributed. This assumption makes the calculation of reliability very convenient. But the critical damage condition adopted in these design methods is unacceptable serviceability, not fatigue cracking,i.e. fatigue life as used in those analyses means the number of load applications that a given pavement can withstand to a specified minimum serviceability index. As pointed out by many researchers (6, 7), the hazard function, which is defined as the probability of failure after survival through n cycles, decreases with increasing life in the case of lognormal distribution. This fact is generally contradictory to the expected fatigue behaviour of concrete as cracking is concerned.

The Weibull distribution was found heuristically by Swedish engineer W. Weibull in 1939, and was later shown to be the third asymptotic distribution of smallest values by Gumbel. Its distribution function is given by the following (when minimum life is assumed to be zero):

$$F_N(n) \equiv P\ (N \leq n) = 1 - \exp[-\ (\frac{\eta}{\theta})^{\beta}\ ] \qquad (4)$$

where N is random variable denoting fatigue life; n stands for value of N; β is shape parameter, and θ is scale parameter.

Weibull distribution can be obtained under physically valid assumptions which bear the similar mechanism as fatigue damage. The hazard function of Weibull distribution is increasing when the shape parameter, β , is smaller than one. It is assumed in this paper that the fatigue life of concrete pavement slab follows two-parameter Weibull distribution.

As to the estimated traffic , n, some pavements have shown premature failure because of unforeseen traffic loadings. On the other hand, some pavements have lasted longer than expected because of an overestimation of the traffic loadings. It is assumed that the forecasting error in predicting the accumulative number of ESAL applications is normally distributed.

Knowing the probability distributions of N and n, reliability can therefore be expressed as (4):

$$R = P(N > n) \qquad (5)$$

$$= 1 - \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\{-\ \frac{(\eta-\mu_n)^2}{2\sigma_n^2}\} \cdot \{\ 1 - \exp[(-\ \frac{\eta}{\theta})^{\beta}]\}$$

$$= \Phi_{0,1}(1 - \frac{\mu}{\sigma}) - \frac{\theta}{\sqrt{2\pi}\sigma} \int_{0}^{+\infty} \exp\{-\eta^{\beta} - \frac{1}{2}[\frac{\theta}{\sigma}y - \frac{\mu}{\sigma}]^2\}dy$$

where $\Phi_{0,1}$ stands for the standard normal function, μ and σ is respectively the mean and standard error of n, β and θ is respectively the shape parameter and scale parameter of N's Weibull distribution.

Variance Model

Since N is assumed to follow two-parameter Weibull distribution , the probability function is uniquely defined by its shape parameter, β , and scale parameter, θ. These two parameters can be obtained from the mean and standard deviation of N, which depend on the variability of the factors that influence them. N is the function of several design variables:

$$N = f\ (h,\ MR,\ Es,\ Ec, \ldots\ldots) \qquad (6)$$

where h is the slab thickness, MR is the concrete modulus of rupture, Ec is the concrete modulus of elasticity, and Es is the resilient modulus of subgrade.

Theoretically, the mean and variance of a variable that is a function of several random variables can be determined by evaluating the integrals of the joint probability distribution of all the random variables. But this is rather difficult, and several approximation methods have been developed. Among these methods is the first-order approximation method. A function of random variables may be expressed in terms of the Taylor series expansion evaluated near the variable means:

$$f(x_1,x_2,\ldots,x_n) = f(\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_n) + \sum_{i=1}^{n}(x_i - \bar{x}_i)\frac{\partial f}{\partial x_i}\bigg|_{x_i=\bar{x}_i} + R \quad (7)$$

where R is the residual.

In the case of first-order approximation, the mean and variance of the variable may be derived:

$$E\{f(x_1,x_2,\ldots,x_n)\} = f(\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_n) \quad (8)$$

$$\sigma^2\{f(x_1,x_2,\ldots,x_n)\} = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\bigg|_{x=\bar{x}_i}\right)^2\cdot\sigma^2(x_i) + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{\partial f}{\partial x_i}\bigg|_{x_i=\bar{x}_j}\right)\left(\frac{\partial f}{\partial x_j}\bigg|_{x_i=\bar{x}_j}\right)\cdot\rho_{ij} \quad (9)$$

where E and σ stands for mean and standard deviation, and ρ_{ij} is the covariance of x_i and x_j.

It should be noted that equations 8 and 9 are in most engineering cases good approximation. This method have been compared elsewhere (1, 2) with simulation results ($<\pm$ 10%) and results by Rosenblueth point-estimation method ($<\pm$ 5%). Because of its relative ease in use and the complexity of equation 6, the first-order approximation method is adopted in this analysis as the variance model.

The mean and variance of N can be therefore be calculated from the means and variances of the parameters by the following:

$$E(N) = N\{E(MR),E(h),E(Ec),E(Es),\ldots\ldots\} \quad (10)$$

$$\sigma^2(N) = \left(\frac{\partial N}{\partial MR}\right)^2\cdot\sigma^2(MR) + \left(\frac{\partial N}{\partial h}\right)^2\cdot\sigma^2(h) + \left(\frac{\partial N}{\partial Ec}\right)^2\cdot\sigma^2(Ec) + \left(\frac{\partial N}{\partial Es}\right)^2\cdot\sigma^2(Es) + \left(\frac{\partial N}{\partial MR}\right)\cdot\left(\frac{\partial N}{\partial Ec}\right)\cdot\rho_{MR,Ec} \quad (11)$$

where $\rho_{MR,Ec}$ is the covariance of MR and Ec.

On the other hand , E(N) and $\sigma^2(N)$ can be expressed as the function of β and θ from Weibull distribution:

$$E(N) = \theta\cdot\Gamma\left(\frac{1}{\beta}+1\right) \quad (12)$$

$$\sigma^2(N) = \theta^2\left[\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right] \quad (13)$$

The above equations can be used to evaluate β and θ from E(N) and $\sigma^2(N)$.

Reliability Implementation and Sensitivity Analysis

Variances of Design Parameters

Almost every measurable parameter associated with the pavement design process tends to vary in a random manner. But only those parameters which are important to the design and those which have significant variances are treated as random variables in this paper. These include concrete modulus of rupture, MR, slab thickness, h, concrete modulus of elasticity, Ec, and resilient modulus of subgrade, Es.

The above parameters were measured in field study during the construction of test pavement in Shanxi Province. And data of these parameters have been obtained from many construction projects and laboratory results. On the basis of these data, ranges of coefficients of variations of design parameters are suggested at three variability levels as shown in Table 1. To give a better indication of the overall range of variability, the extreme high and low variabilities were used. The values of CV are generally a little higher than those used in foreign reliability analysis corresponding to the present pavement construction level in China.

Reliability Calculation

Knowing the means and variances of design parameters, the mean and variance of fatigue life, N, can be obtained from the first-order approximation model, the shape parameter β and scale parameter θ of N's Weibull distribution can be calculated from equations 12 and 13. Together with the mean and variance of predicted ESAL applications, n, the pavement design reliability can be calculated by numerical integration of equation 5.

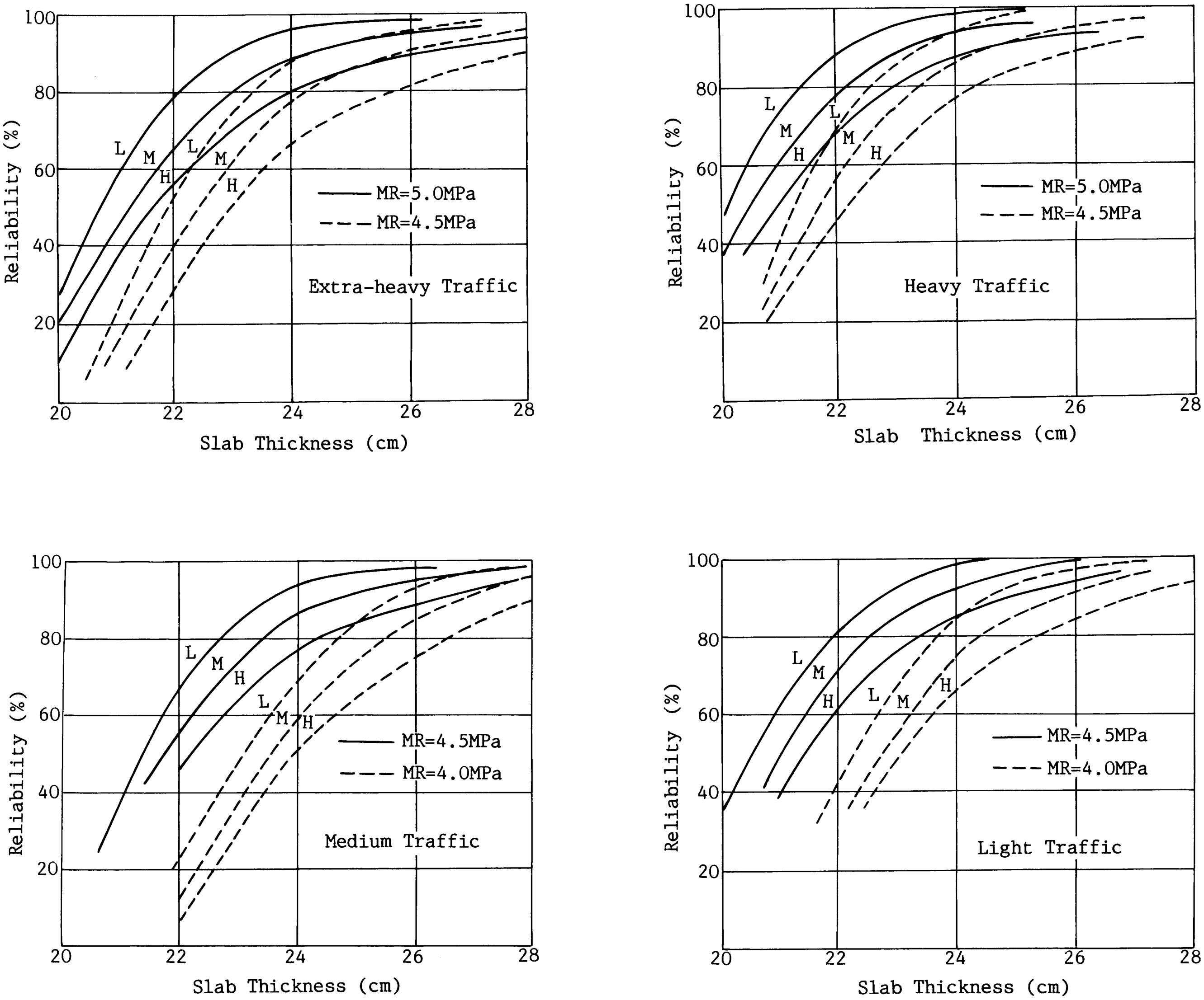
Table 1 Suggested ranges of CV values

Design Parameters	Variability Level: Low	Medium	High
MR	0.10-0.12	0.15-0.18	0.21-0.25
h	0.01-0.03	0.05-0.08	0.10-0.12
Ec	0.10-0.20	0.25-0.35	0.45-0.50
Es	0.10-0.15	0.20-0.30	0.40-0.55

Computer program was prepared for the calculation of reliability following the above process. Different sets of design parameters were inputted at four levels of traffic and three levels of variability. Considering the influence of concrete modulus of rupture, two different values of MR were used for each traffic level. The inputs are listed in Table 2 (the inputted CV values are taken from Table 1). And the computed reliability results are shown in Figures 3 to 6 by thickness-reliabilty relationship curves.

Table 2 Input parameters in reliability computation

Design Parameters	Traffic Level: Extra-Heavy	Heavy	Medium	Light
Initial Traffic (ESAL per day)	2500	1200	400	150
Design Life (years)	40	30	30	20
Dynamic Factor	1.15	1.15	1.20	1.20
MR (MPa)	4.5 5.0	4.5 5.0	4.0 4.5	4.0 4.5
Ec ($\times 10^3$ MPa)	28 31	28 31	27 28	27 28



Note: L, M and H stand for respectively low, medium, and high variability.

Fig.3 Slab thickness-reliability relationships

It should be noted that the above results are obtained without considering the impacts of design equations' lack-of-fit error because there is few data which can be used to quantify this error. The obtained reliabilities are valid only when the design equations are assumed to be precise.

Sensitivity Analysis

A preliminary sensitivity analysis was undertaken to examine the effects of design parameters and their variations upon design reliability. The CV value of one parameter was varied each time while the other parameters kept constant and reliabilities were computed (base values: heavy traffic, medium variability, MR = 4.5 MPa, h = 24 cm). The results are shown in Table 3.

Table 3 Influences of parameter variability on reliability

CV(MR)	R(%)	CV(h)	R(%)	CV(Ec)	R(%)	CV(Es)	R(%)
0.10	91.90	0.01	88.04	0.10	88.04	0.10	86.84
0.13	89.76	0.03	88.01	0.20	87.44	0.18	86.84
0.16	86.84	0.06	86.84	0.30	86.84	0.25	86.84
0.20	83.70	0.09	85.61	0.40	86.23	0.38	86.84
0.25	79.73	0.12	83.70	0.50	84.98	0.55	86.83

From Figure 3 and Table 3, the following can be observed:

1. Material variability has a significant effect upon the pavement reliability. The variability makes the reliability decrease about 0.2 when

it changes from low to high level and the traffic-thickness-MR condition remains constant. Slab thickness requirements increase as material variability increases over a constant reliability level, and this extra thickness is much larger when the reliability level is high.

2. Reliability increases as thickness increases over an unchanged traffic-variability-MR condition, but the rate of reliability increase becomes slower when reliability gets higher.

3. Concrete modulus of rupture is an important factor influencing pavement reliability, especially under thin slab thickness and low reliability level.

4. As to material variability, reliability is most strongly influenced by the variations of MR, followed by the variations of slab thickness, h, and those of Ec. Reliability is nearly insensitive to the variations of Es.

Probabilistic Approach

In developing a reliability-based procedure for pavement design, reliability requirements of the design should be selected for pavements serving different levels of traffic. The selection should be made on the basis of cost- benefit analysis. Reliability levels listed in Table 4 is tentatively suggested according to traffic levels for the purpose of illustrating the development of the probabilistic approach.

Table 4 Suggested reliability levels

Traffic Level	Design Reliability (%)
Extra-heavy	95
Heavy	90
Medium	85
Light	60

There are many ways to introduce reliability in the pavement design process, such as through design curves, factors, equations or tables. It will be more convenient and more acceptable for the design engineers if the probability-based method keeps the same form and procedure with the present conventional design method. A "reliability factor", Fr, is introduced in this paper as the "carrier" of the reliability requirement. This reliability factor, though in similar form with safety factor being used in current specification, has completely different meanings. Fr is obtainer from the following steps:

1. Calculate the thermal stress, σ_t , and loading stress, σ_p , from imputted design parameters and their variances, thus obtain the mean and variance of fatigue life, E(N) and $\sigma^2(N)$.

2. Calculate the allowable load stress $[\sigma_p]$ from σ_t and n with equation 1.

3. Compute the design reliability, R. And the reliability factor corresponding to that value of reliability is given by: $Fr = [\sigma_p]/\sigma_p$.

Values of Fr were calculated for different traffic levels, variability levels and reliability levels. The obtained rsults of Fr are shown in Tables 5.

Table 5 Calculated reliability factors

Variability	Traffic Level			
	Extra-Heavy	Heavy	Medium	Light
Low	1.20	1.15	1.10	1.00
Medium	1.30	1.25	1.20	1.05
High	1.50	1.35	1.30	1.12

The probabilistic design procedure will have the same design process as illustrated in Figure 1, except that the multiplying safety factor is replaced by the reliability factor, which is determined by traffic level , variability level and reliability level.

Summary

In this paper, reliability analysis has been made for China's current highway concrete pavement design procedure by using interference theory and first order approximation variance model. Tentative sensitivity analysis has been undertaken to examine the effects of material variabilities upon reliability. And a probabilistic design procedure has been developed for the revision of current design specification. The main features of this procedure are:

1. Assuming the fatigue life of slab to follow two-parameter Weibull distribution.
2. Introducing a reliability factor, which has the same form with the existed safety factor, in the design process.

The limitation of this research should be recorgnized that the lack-of-fit error of design equations is not considered in the analysis. And further work is needed for this error to be quantified and added to the total variance.

Acknowledgement

The Authors wish to express sincere thanks to Professor Zukang Yao for his encouragement and help in reviewing and revising this manuscript.

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