

A COMPUTER PACKAGE FOR STRUCTURAL ANALYSIS OF CONCRETE PAVEMENTS

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A computer package based on the finite element method was developed for the structural analysis of concrete pavements. The package can be applied to multiple jointed slabs on a liquid, solid or layer foundation. The procedures for developing the package are presented and its capabilities and limitations described. Salient features of the package include the development of an iterative method to reduce the computer storage, the analysis of partial contact by a method of successive approximation, and the consideration of shear and moment transfer across the joints and the looseness of dowel support. Examples are given to illustrate the application of the package to the analysis of concrete pavements.

During the past decade, the author has been engaged in the development of finite element computer programs for the structural analysis of concrete pavements. These programs can be used for multiple jointed slabs on liquid foundations (1,2,3), solid foundations (4,5,6,7), or layer foundations (8,9,10). They were written individually for specific purposes with considerable duplications and inconsistencies in program formulation. As the methods used in these programs had been updated and improved since their publications, an effort was made recently to rewrite these programs and combine them into a new coherent package for the structural analysis of concrete pavements. The purpose of this paper is to present the method used in developing the package; describe its capabilities, limitations and special features; illustrate its applications to the analysis of concrete pavements; and interpret the results obtained.

The package consists of a large number of subroutines. By simply calling the appropriate subroutines, it can be applied to multiple jointed slabs on Winkler's liquid foundations, Boussinesq's solid foundations, or Burmister's layer foundations. Within the storage capacity of a computer, any number of slabs with both shear and moment transfers across the joints can be analyzed. The package has been extensively tested and a user's manual is available.

Although several finite element computer programs (11,12) are currently available for the analysis of jointed slabs, they are all based on the

assumption of liquid foundations. The program involving solid and layer foundations as described herein is original and was developed by the author after more than 10 years of research.

Description of Method

Figure 1 shows a four slab system with load transfer across the joints. For simplicity, only a

Figure 1. A four slab system.

11	13	15	23	25
SLAB 2			SLAB 4	
10	12	14	22	24
3	6	9	18	21
2	5	8	17	20
SLAB 1			SLAB 3	
1	4	7	16	19

very limited number of finite elements is used. Slab 1 is divided into four rectangular elements, slabs 2 and 3 are divided into two elements each, while the entire slab 4 is considered as one element. There are a total of 25 nodes. Because each node has three displacements (a vertical deflection and two rotations) and three forces (a vertical force and two moments), the stiffness matrix of the system, which relates nodal displacements to nodal forces has a dimension of 75 by 75. The relationship between displacements and forces can be written as

$$[K]\{\delta\} = \{F\} \quad (1)$$

in which $[K]$ = stiffness matrix of the system, $\{\delta\}$ = nodal displacements, and $\{F\}$ = externally applied nodal forces.

Stiffness Matrices

The stiffness matrix of the system includes three components: slab, foundation and joint. The stiffness matrix of each component must be determined and then superimposed to form the stiffness matrix of the system. As these matrices are symmetric, only the upper half of the matrices need be considered.

Slab. The stiffness matrix of slab can be obtained from the classical theory of thin plate by assuming that the plane before bending remains plane after bending. Equations for computing the stiffness matrix of plate were presented by Zienkiewicz (13). Because the forces at one node are affected by the displacements at another node only when the two nodes are located in the same element, the stiffness matrix of slab is banded with many zero entries outside the half band. For the system shown in Figure 1, the maximum difference between the two nodal numbers in the same element is 4, so the minimum half band width required is $(4+1) \times 3 = 15$. A small half band width is highly desirable because it reduces both the computer time and storage in solving the simultaneous equations.

Foundation. The stiffness matrix of foundation involves only vertical forces and vertical deflections, and the effect of moments and rotations is neglected. To be consistent with this assumption, externally applied loads should be distributed to the nodes by statics, so that only vertical forces are considered while the contribution by moments is ignored.

In the liquid, or Winkler, foundation, it is assumed that a spring exists under each node. Thus the force at a given node depends on the deflection of the node itself, independent of the deflection at any other node. Consequently, the stiffness matrix of liquid foundation is a diagonal matrix with a half band width of unity. The diagonal entry for each node is the spring constant obtained by multiplying the modulus of subgrade reaction with the area surrounding the node. These spring constants vary from node to node depending on the size of elements used. For example, when four slabs meet at a point, such as nodes 9, 14, 18 and 22, it is assumed that one spring exists under each node with a spring constant equal to one-fourth of the element area multiplied by the modulus of subgrade reaction.

The determination of the stiffness matrix of solid, or Boussinesq, foundation is not so simple, especially when the slabs are jointed. First, the well known Boussinesq equation, which relates the nodal deflection to the nodal force, is used to generate the flexibility matrix, which is then inverted to obtain the stiffness matrix of foundation. Because the force at a given node is affected by the deflections at all nodes, the half band width covers the entire matrix and equals to the total number of equations. For the system shown in Figure 1, a half band width of 75 is required. The procedure for computing the stiffness matrix of solid foundation follows essentially that of Cheung and Zienkiewicz (14) and Wang, et al. (15). However, their procedure only applies to a single slab. For multiple slabs, it is necessary to distribute the stiffness matrix over those nodes meeting at the same point. Take Figure 1 for example. Although there are 25 nodes on the slabs, only 16 nodes exist on the foundation, because many of the nodes, such as 9, 14, 18 and 22, are actually located at the same point and should be considered as one node.

Therefore, the flexibility matrix of foundation has a dimension of 16 by 16. After inversion, the 16 by 16 stiffness matrix must be enlarged to a 25 by 25 matrix by distributing the stiffness to all nodes sharing the same point. For instance, the stiffness entry (3,9) is evenly divided among (3,9), (3,14), (3,18), (3,22), (10,9), (10,14), (10,18) and (10,22); the stiffness entry (9,9) is evenly divided among (9,9) (14,14), (18,18) and (22,22). When two or more nodes meet at a joint, the deflections of the slab at the nodes are different but the deflection of the foundation is the same.

In the layer, or Burmister, foundation, the flexibility matrix of foundation is obtained from the Burmister theory. In this theory, the load is distributed over a circular area but can be easily converted to a concentrated load by letting the radius of contact approach zero, or

$$\lim_{a \rightarrow 0} \frac{J_1(ma)}{ma} = 0.5 \quad (2)$$

in which a = radius of loaded area; J_1 = Bessel function of the first kind and order 1, which appears in the Burmister equation for surface deflection; and m = an integration parameter. Therefore, the procedure for layer foundations is the same as that for solid foundations except that the relationship between deflection and force of the former is obtained from the Burmister theory, whereas that of the latter from the Boussinesq equation.

Joint. The stiffness of joint is represented by a shear spring constant and a moment spring constant. Taking nodes 7 and 16 in Figure 1 as an example, the shear and moment transfer across the joint can be expressed as

$$P_7 = (w_{16} - w_7)C_w \quad (3)$$

$$M_7 = (\theta_{16} - \theta_7)C_\theta \quad (4)$$

$$P_{16} = (w_7 - w_{16})C_w \quad (5)$$

$$M_{16} = (\theta_7 - \theta_{16})C_\theta \quad (6)$$

in which P = shear force across joint; w = vertical deflection; C_w = shear spring constant, which is defined as the shear force per unit length of joint required to effect a unit difference in deflection between two adjoining slabs; M = moment across joint; θ = rotation; and C_θ = moment spring constant; which is defined as the moment per unit length required to effect a unit difference in rotation. These forces and moments can be placed on the force vector at the right hand side of equation 1 and the vertical deflections and rotations are then transferred to the left to form the stiffness matrix of joint. The half band width of the stiffness of joint depends on the maximum difference between the two nodal numbers on the opposite sides of joint. In Figure 1, the maximum difference is 9, so the minimum half band width required is $(9+1) \times 3 = 30$.

Simultaneous Equations

Equation 1 can be solved by the Gauss elimination method. For the system shown in Figure 1, the total number of equations is 75. If the foundation is liquid, the half band width is 30, instead of 15, due to the presence of joints, so the dimension for the stiffness matrix is $75 \times 30 = 2250$. If the foundation is solid or layer, the half band width

is 75, so the dimension for the stiffness matrix is $75 \times 75 = 5625$. In practical applications, the number of nodes is much greater than 25. In its present dimensions, the package allows a maximum of 150 nodes. For solid foundations, the dimension of stiffness matrix is $450 \times 450 = 202,500$, which is equivalent to a computer storage of more than 800K. It can be seen that a major problem for analyzing solid or layer foundation is the computer storage. To solve this problem, an iteration method was developed. In this method, an arbitrary half band width can be assigned. Any entry in the stiffness matrix outside the half band is stored and moved to the right side of equation 1, or

$$[K_1]\{\delta\} = \{F\} - [K_2]\{\delta\} \quad (7)$$

in which $[K_1]$ = stiffness matrix within the assigned half band and $[K_2]$ = stiffness matrix outside the half band. Because the stiffness matrix outside the half band consists principally of the stiffness matrix of foundation with two-third of the entries as zeros, the storage requirement for $[K_2]$ is quite small compared to $[K]$. This iteration procedure can be applied to all three types of foundations. It was found that the solution converges quite rapidly if a large half band width is assigned. If the half band width is equal to the number of equations, no iteration will be required. However, if the half band width is too small, the solution may diverge and a larger half band width should be used.

The iteration method presented above is different from that presented in the previous papers (4,7) in that the previous method considers each slab independently while the new method considers the system as a single slab. The previous method works well when the efficiency of shear transfer, which is defined as a deflection ratio between the two adjoining slabs, is used. However, when dowel bars or spring constants are specified, the solution converges very slowly and in some cases may not converge at all. The present method has no problem on convergence but cannot be used for efficiency of shear transfer because the use of efficiency destroys the symmetry of stiffness matrix. In view of the fact that the use of a fixed efficiency for all nodes along a joint is totally unreasonable, only the cases of dowel bars and spring constants are considered. When dowel bars are used, they can be converted to spring constants by equation 11 in reference 7.

Partial Contact

One factor which may affect the design of concrete pavements is the contact condition between slab and foundation. Both Westergaard's analysis for liquid foundations (16) and Pickett's analysis for solid foundations (17) are based on the assumption that the slab and foundation are in full contact. This assumption is valid if there are no gaps between slab and foundation, because the weight of the slab naturally imposes a large precompression on the foundation, which will keep the slab and foundation in full contact. However, this is not true when the slab is subjected to warping or pumping, which results in a separation between slab and foundation. This computer package is particularly useful for evaluating the effect of contact condition on the design of concrete pavements.

For liquid foundation, the analysis follows essentially that presented in reference 2. First, the slab and foundation are assumed in full contact and the deflections of slab are determined. These deflections are checked against the gaps or precom-

pressions due to temperature warping or pumping to determine whether they are in contact. If any node is found not in contact, its reactive force is set to zero in the next iteration. The process is repeated until the same contact condition is obtained.

The above procedure cannot be applied to solid or layer foundations. The actual criterion for deciding the contact condition is whether any tension exists between the slab and foundation. In liquid foundations, tension is indicated if the node moves up and compression if the node moves down. By merely comparing deflections, the contact condition can be ascertained. In solid foundations, tension or compression at a given node is not only governed by the deflection of the node itself but also by the deflections of all other nodes. Tension can exist even if the slab moves down. Therefore, the reactive force, either tension or compression, should be used to determine the contact condition. First, the slab and foundation are assumed in full contact and the reactive force at each node is determined. If the reactive force is in tension and the tension is greater than the precompression (note that the term precompression used in solid or layer foundations indicates a force, while that in liquid foundations indicates a displacement), the node is not in contact and its reactive force is set to zero in the next iteration. When there is no reactive force at a given node, that node should be eliminated from the flexibility matrix and a new stiffness matrix of foundation is formed. If the slab and foundation are assumed not in contact and the deflection of slab is smaller than the gap plus the deflection of foundation, the assumption is correct and the slab and foundation are not in contact. Otherwise, the slab and foundation should be assumed in contact. It can be seen that two criteria are used for checking contact: if the slab and foundation are assumed in contact, the net effect of reactive force and precompression must be in compression; if the slab and foundation are assumed not in contact, the deflection of slab must be smaller than the gap and the deflection of foundation combined. If any one of the above criteria is violated, the contact condition should be changed and the iteration continues until the above two criteria are satisfied.

Computer Program

The computer package is written in FORTRAN and consists of a short main program and 35 subroutines. In its present dimensions, it requires a storage of 678K, of which 141K is for object code and 537K for array area. It can be applied to a maximum of nine slabs, 12 joints and 150 nodes. Each slab can have a maximum of 15 nodes in the x direction and 15 nodes in the y direction. The package can be used for all three types of foundation, viz. liquid, solid and layer. If the user is interested only in one type of foundation, the package can be easily abstracted to include that foundation only and the required storage can be reduced.

Capabilities

The capabilities of the package are as follows:

1. A maximum of nine slabs with shear and moment transfer across the joints can be analyzed. Shear transfer can be effected by specifying a shear spring constant or by providing information on the size, spacing, Young's modulus and Poisson's ratio

of dowel bars together with the joint width and the modulus of dowel support. Moment transfer is specified by a moment spring constant. The looseness of dowel can be considered by specifying a gap between dowel and concrete. This gap can also be applied to the shear spring constant but not to the moment spring constant.

2. Each slab can have different thicknesses and sizes. In the same slab, thickness can vary from node to node. However, two adjoining slabs must have the same width, and all joints must be continuous throughout the slabs.

3. The slabs can have two rigid layers, either bonded or unbonded. Each layer has its own Young's modulus and Poisson's ratio. For bonded layer, an equivalent layer based on the transformed section concept (11) is used for determining the stiffness matrix of slab. For unbonded layer, the stiffness matrix of slab is the sum of both layers.

4. The load can be uniformly distributed over rectangular areas or concentrated at a given number of nodes. Each loaded area can have a different intensity of pressure. To use the programs for structures other than pavements, such as footings and raft foundations, moments about x and y axes can be applied at any node.

5. If symmetry with respect to one or both axes exists, only one-half or one-quarter of the slab system need be considered. This feature can save a great deal of computer time and storage.

6. Effect of temperature warping and gap between slab and foundation on the stresses and deflections can be analyzed.

7. The slab and foundation can be assumed in full contact at all nodes or not in contact at some designated nodes. The contact condition can also be evaluated automatically by iterations.

8. The package can analyze slabs on liquid foundations, solid foundations or layer foundations.

Limitations

The package has the following limitations:

1. When considering temperature warping, each slab is assumed to warp into a spherical surface independent of the others (2). This occurs only when there is no moment transfer across the joints and each slab can move up and down freely, as is in the case of lubricated dowel joints. It may not be applicable to hinged or longitudinal joints, where the edges of adjoining slabs are held tightly together. Fortunately, the most critical area for a pavement is not in the vicinity of a longitudinal joint and this inaccuracy should have very little effect on the final design. The analysis of temperature warping is based on the assumption that there is only one layer of slabs and that each slab is of uniform thickness. For slabs of nonuniform thickness, an average thickness must be assumed.

2. When slabs are composed of two layers, the joints through the two layers are exactly at the same locations. If the bottom layer has no joints and is much weaker than the top layer, this assumption should have very little effect on the stresses in the top layer. Furthermore, the use of equivalent layer for analyzing two bonded layers is very approximate, as will be illustrated later.

3. The package does not permit the use of the efficiency of shear transfer or infinitely stiff joints, where the deflections or rotations on the two sides of the joint are equal. When a very large spring constant is applied to a joint, the equation for each node on the two opposite sides is identical and the system of simultaneous equations becomes

singular. This situation will not occur in practice and can be easily detected if the printout shows that the sum of applied forces is significantly different from the sum of reactive forces.

4. The use of rectangular elements strongly limits the size of elements to be employed. If small elements are used in the main slab, elements in the adjoining slabs will be of the same small width. The limitation that the length-width ratio of any element be not greater than 4 or 5 requires that a lot of elements be used throughout the slab system. If many slabs are involved, a relatively large grid should be used to save the computer time and storage. Fortunately, a relatively large grid can still yield reasonable results, as will be illustrated later.

Verification

To insure the correctness of a computer program, it is desirable to check the numerical results with analytical solutions. Unfortunately, due to the complexity of jointed slabs, no analytical solutions are available except in the case of a single slab. However, the results obtained from the new package were compared with those determined from the previous programs developed by the author and found to be in good agreements. These previous programs were applied to various practical and hypothetical cases with reasonable results, as published in references 1 through 10. For the case of single slabs, the solutions based on liquid foundations checked closely with Westergaard's analytical solutions and Pickett and Ray's influence charts for interior loading (1) as well as with Westergaard's analytical solutions for temperature warping (2); those based on solid foundations checked with Pickett and Badaruddin's analytical solutions for edge loading (4); and by assuming each layer having the same elastic modulus and Poisson's ratio, solutions based on layer foundations checked with those based on solid foundations.

Special Features

The package has two salient features which need further discussion. One is the arbitrary assignment of a half band width by the user, and the other is the automatic checking of contact between slab and foundation.

If the half band width is specified as zero, the package will determine the minimum band width and solve the simultaneous equations with no iterations required. When the computer storage is not a problem, this direct approach is recommended because it will give correct answers with no complications anticipated, although a smaller half band width may save some computer time. If the computer storage is a problem, the use of a largest possible half band width within the capacity of the computer is recommended because a small band width may cause the solution to diverge or converge at such a slow rate that more computer times are required. When iterations are not required, the sum of applied forces should be checked against the sum of reactive forces; if not nearly equal, something must be wrong. When iterations are required, the deflection at a designated node will be checked every other iteration until it converges to within a specified tolerance. Then, the sum of applied forces will check against the sum of reactive forces. If too much discrepancy exists, the specified tolerance will be reduced and iterations continue until the desired accuracy is obtained.

To check the contact condition between slab and foundation under a given load, it is necessary to know the contact condition before the load is applied. Therefore, the gaps at those nodes not in contact and the precompressions at those nodes in contact must be specified. Because these gaps and precompressions prior to the application of the load are difficult to determine, the best way is to divide the analysis into two steps. First, determine the gaps and precompressions between slab and foundation under weight, temperature warping and initial gaps, if any. The initial gap is the gap before the slab is constructed. In fact, there is no gaps between slab and foundation when the concrete is poured. Gaps develop only after pumping or plastic deformation of the foundation has taken place. These initial gaps can be visualized as a change in grade should the slabs be removed. As these gaps are difficult to predict anyway, a rough estimate of their possible occurrences near the pavement edges or joints is all that is needed. Next, using the gaps and precompression obtained in the first step, determine the stresses and displacements under the applied load. These two steps can be executed in the same run, one immediately after the other, so the gaps and precompressions determined in step 1 can be used in step 2. They can also be run separately by reading in the gaps and precompressions during step 2.

The reasons that warping and loading are analyzed separately in two steps are because these two cases do not occur at the same time in reality, and also the modulus of subgrade due to the slowly changing temperature gradient may be much smaller than that due to transient wheel loads. Although stresses due to temperature warping are quite significant in concrete pavements, current design practices only consider the stresses due to loading, so a separation of loading and warping stresses is needed. However, the package can determine the combined effect of warping and loading, if so desired.

Figure 2 shows a simplified flow chart.

Numerical Results

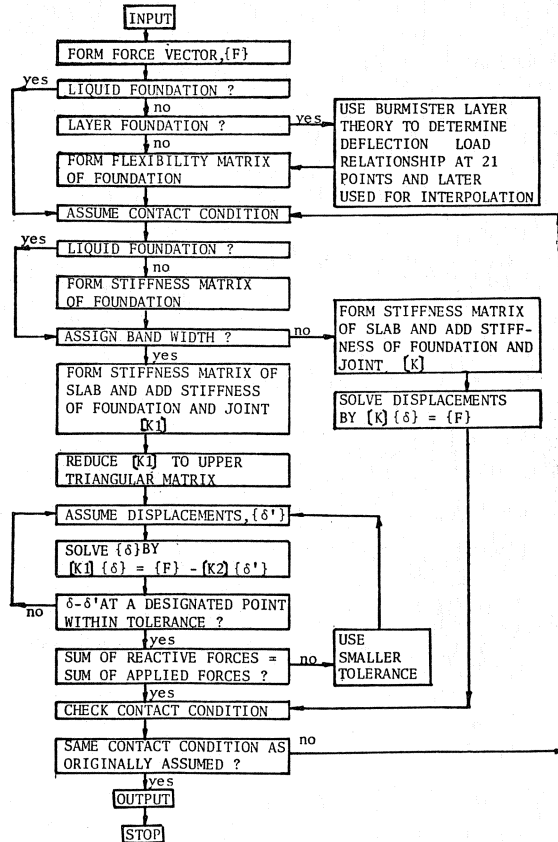
Several examples are given to illustrate the applications of the computer package and interpret the results. In all the examples, it is assumed that the concrete slab has a thickness of 8 in. (203 mm), a modulus of elasticity of 4×10^6 psi (28 GN/m²), and a Poisson's ratio of 0.15. If the foundation is of the solid type, as will be used in most of the examples, a Young's modulus of 1×10^4 psi (69 MN/m²) and a Poisson's ratio of 0.4 are assumed.

Comparison Between Plate and Layer Solutions

The plate theory is used for the structural analysis of concrete pavements, while the layer theory for that of asphalt pavements. When the modulus of elasticity of the pavement is much greater than that of the foundation and the load is applied in the interior of pavement far away from edges and corners, both theories should yield about the same results. Therefore, results obtained from Burmister's layer theory can be used to check those from slabs on solid foundations.

Figure 3 shows a large 24 ft (7.3 m) by 24 ft (7.3 m) slab on a solid foundation. A 9,000 lb (40 kN) load is applied at the center of slab over a square contact area with a contact pressure of 75

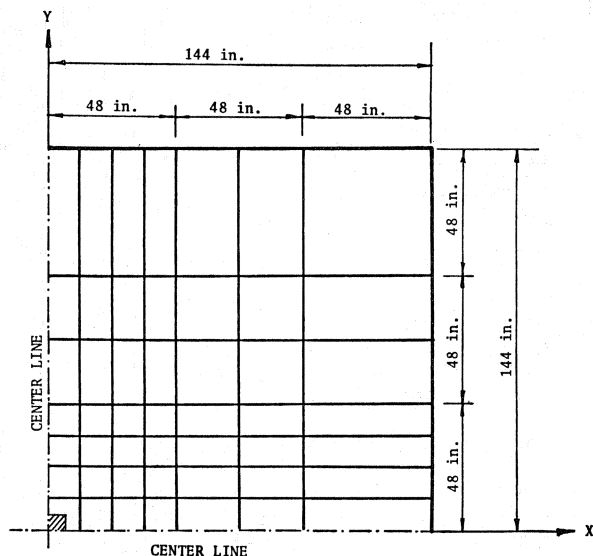
Figure 2. A simplified flow chart.



psi (518 kN/m²). Due to symmetry with respect to both x and y axes, only one-quarter of the slab is employed. The slab is divided into rectangular finite elements as shown. The stresses and deflections along the x axis at the first five nodes are compared with those obtained from the layer theory (18), and the results are presented in Table 1. In the layer solutions, the same load and contact pressure are used but the contact area is a circle instead of a square. It can be seen from Table 1 that both solutions check very closely with a discrepancy less than 1% under the load and not over 10% elsewhere. Note that in the plate solutions the stress and deflection on the top are the same as those at the bottom, but they are not the same in the layer solutions. Although the layer solutions are based on the assumption that the slab and foundation are bonded with rough interface while the plate solutions are unbonded with smooth interface, this difference should have very little effect on the solutions because of the weak foundation and the very small shear stresses induced at the interface in the layer solutions.

Table 2 shows a comparison between plate and layer solutions when the slab is composed of two bonded layers. The example is the same as that shown in Table 1 except that a second layer of slab having a thickness of 6 in. (152 mm), a Young's modulus of 2×10^6 psi (13.8 GN/m²) and a Poisson's ratio of 0.3 is bonded to the top slab. In the plate solutions, an equivalent layer based on the transformed section concept is used. It can be seen

Figure 3. A large slab with an interior load.
(1 in. = 25.4 mm)



that the differences between plate and layer solutions in Table 2 are much greater than those in Table 1. The use of equivalent layer results in a maximum tensile stress in the top layer 16.4% smaller than that obtained from the layer theory, indicating that the use of equivalent layer for computing the critical stress in concrete is on the unsafe side. The table also shows that the stresses in the bottom layer and the deflections obtained by the equivalent layer method check well with the layer solutions.

Comparison Between Solid and Layer Foundations

When a pavement is composed of two unbonded slabs on a solid foundation, two methods of analysis may be employed. The first method, which requires much less computer time, is to consider the pavement as two layers of unbonded slab on a solid foundation. The second method, which requires more computer time, is to consider it as one layer of slab on a layer foundation, the second layer of slab being considered as the top layer in the layer foundation. Table 3 shows the differences between these two solutions. The strong bottom layer of slab has a Young's modulus of 2×10^6 psi (14 GN/m^2) and a Poisson's ratio of 0.3, and the weak bottom layer of slab has a Young's modulus of 2×10^5 psi (1.4 GN/m^2) and a Poisson's ratio of 0.3. In both cases, the solutions of solid foundation check quite well with those of layer foundation except when the stresses are small. Therefore, if a rigid base course is placed under the concrete slab, it is reasonable to consider the base course as the bottom

Table 1. Comparison between plate and layer solutions - one layer of slab.

Distance (in.)			0	12	24	36	48
σ_x	Plate	Solid (psi)	171.89	58.60	13.79	-3.58	-11.27
	Layer	Top (psi)	-187.13	-57.89	-14.40	3.35	10.63
		Bottom (psi)	171.77	56.90	12.56	-4.66	-11.53
	Difference (%)		0.1	3.0	9.8	6.9	6.0
σ_y	Plate	Solid (psi)	171.89	101.74	61.54	37.87	24.10
	Layer	Top (psi)	-187.13	-106.68	-61.58	-38.08	-24.28
		Bottom (psi)	171.77	106.21	60.10	36.68	23.07
	Difference (%)		0.1	1.4	2.4	3.2	4.5
w	Plate	Solid (0.001 in.)	9.16	8.66	7.68	6.62	5.63
	Layer	Top (0.001 in.)	9.23	8.53	7.48	6.40	5.41
		Bottom (0.001 in.)	9.16	8.53	7.48	6.40	5.41
	Difference (%)		-0.8	1.5	2.7	3.4	4.1

Note: (1) 1 in. = 25.4 mm and 1 psi = 6.9 kN/m^2 .

- (2) In layer solutions, tension is positive and compression is negative; in plate solution, positive indicates tension at bottom and negative indicates tension at top.
- (3) σ_x = longitudinal or radial stress; σ_y = transverse or tangential stress; and w = vertical deflection.
- (4) Difference is a comparison between plate and layer solutions using the latter as a base, and only tensile stresses and surface deflection are compared.

Table 2. Comparison between plate and layer solutions - two layers of bonded slab.

Distance (in.)				0	12	24	36	48
Top Layer	σ_x	Plate	Top	-85.11	-38.22	-17.58	-7.71	-1.92
			Bottom (psi)	30.12	13.53	6.22	2.73	0.68
		Layer	Top (psi)	-112.02	-30.09	-15.03	-5.99	-0.78
			Bottom (psi)	36.02	8.18	3.76	1.21	-0.30
		Diff. in Tension(%)		-16.4	65.4	65.4	125.6	-
	σ_y	Plate	Top (psi)	-85.11	-57.32	-37.13	-26.03	-18.92
			Bottom (psi)	30.12	20.28	13.14	9.21	6.69
		Layer	Top (psi)	-112.02	-56.54	-36.08	-25.28	-18.36
			Bottom (psi)	36.02	20.44	11.91	8.05	5.71
		Diff. in Tension(%)		-16.4	-0.7	10.3	14.4	17.2
Bottom Layer	σ_x	Plate	Top (psi)	15.06	6.76	3.11	1.36	0.34
			Bottom (psi)	58.27	26.17	12.03	5.28	1.32
		Layer	Top (psi)	15.29	5.51	2.86	1.22	0.25
			Bottom (psi)	57.82	31.19	14.17	6.51	2.16
		Diff. in Tension(%)		0.8	-16.1	-15.1	-18.9	-38.9
	σ_y	Plate	Top (psi)	15.06	10.14	6.57	4.60	3.35
			Bottom (psi)	58.27	39.24	25.42	17.82	12.95
		Layer	Top (psi)	15.29	10.93	6.46	4.25	2.91
			Bottom (psi)	57.82	40.74	25.75	17.50	12.26
		Diff. in Tension(%)		0.8	-3.7	-1.3	1.8	5.6
w	Plate (0.001 in.)			6.12	5.96	5.60	5.17	4.72
	Layer (0.001 in.)			6.01	5.68	5.27	4.82	4.37
	Difference (%)			1.8	4.9	6.3	7.3	10.8

Note: (1) 1 in. = 25.4 mm and 1 psi = 6.9 kN/m².

(2) Tension is positive and compression is negative.

(3) σ_x = longitudinal or radial stress; σ_y = transverse or tangential stress; and w = vertical surface deflection.

layer of the slab instead of the top layer in the foundation.

Effect of Grid Sizes

Figure 4 shows four different grid sizes to be used for analysis. The slab, 12 ft (3.65 m) wide and 24 ft (7.3 m) long, is on a solid foundation. An 18,000 lb (80 kN) single axle load is applied at the mid span of the slab adjacent to the edge, which is considered as the most critical loading position. Each tire has a 6.65 in. (169 mm) by 9.67 in. (246 mm) contact area. The center to center spacing between the two dual tires is 13.5 in. (343 mm) and that between the two sets of dual is 6 ft (1.8 m). The outer face of exterior tire is located 2 in. (51 mm) from the pavement edge. Due to symmetry with respect to y axis, only one-half of the slab is considered.

Table 4 shows the effect of four different grid sizes on stresses and deflections. It can be seen that the results converge as the grid becomes smaller. The difference in deflections between the

very coarse and the very fine mesh is less than 8%, while the difference in maximum stress is 12.7%. Thus even very coarse mesh can produce reasonable results. Note that the results of the fine mesh are not significantly different from those of the very fine mesh. Therefore, the fine mesh will be used for the examples which follow.

Comparison Between Liquid and Solid Foundations

It is well known that the modulus of subgrade reaction k used with the liquid foundation is a fictitious quantity not characteristic of soil behaviors. However, due to simplicity in application, the liquid foundation and the associate k value have been used most frequently for the design of concrete pavements. It will be useful if the modulus of subgrade reaction can be related to Young's modulus and Poisson's ratio of the foundation. Vesic and Saxena (19) indicated that the value of k depends on the relative flexibility of the slab with respect to the foundation and that there is no single value of k that can give stresses

Table 3. Comparison between solid and layer solutions - two layers of unbonded slab.

Distance (in.)			0	12	24	36	48
Strong bottom layer	σ_x	Solid (psi)	144.53	51.62	14.20	-0.95	-8.12
		Layer (psi)	139.30	52.58	14.18	-2.92	-6.52
		Difference (%)	3.8	-1.8	0.1	-67.5	24.5
	σ_y	Solid (psi)	144.53	91.93	53.77	33.93	22.15
		Layer (psi)	139.30	91.22	54.58	31.43	25.43
		Difference (%)	3.8	0.8	-1.5	8.0	-12.9
	w	Solid (0.001 in.)	8.60	8.18	7.33	6.40	5.52
		Layer (0.001 in.)	8.36	7.95	7.12	6.21	5.34
		Difference (%)	2.9	2.9	2.9	3.1	3.4
Weak bottom layer	σ_x	Solid (psi)	168.66	57.80	13.87	-3.25	-10.89
		Layer (psi)	167.01	57.15	13.22	-3.05	-11.03
		Difference (%)	1.0	1.1	4.9	6.6	-1.3
	σ_y	Solid (psi)	168.66	105.89	60.65	37.43	23.88
		Layer (psi)	167.01	104.92	59.77	36.93	23.40
		Difference (%)	1.0	0.9	1.5	1.4	2.1
	w	Solid (0.001 in.)	9.09	8.60	7.64	6.60	5.62
		Layer (0.001 in.)	8.95	8.47	7.52	6.49	5.52
		Difference (%)	1.6	1.5	1.3	1.7	1.8

Note: (1) 1 in. = 25.4 mm and 1 psi = 6.9 kN/m².

(2) Positive stresses indicate tension at bottom and negative stresses indicate tension at top.

(3) σ_x = longitudinal stress in top layer; σ_y = transverse stress in top layer; and w = vertical deflection.

and deflections in a concrete pavement comparable to those obtained by considering the foundation as an elastic solid. For computing stresses, they suggested the use of the following equation

$$k = 3 \sqrt{\frac{E_f}{E_c}} \frac{E_f}{(1 + \nu_f^2) h} \quad (8)$$

in which E_f = Young's modulus of foundation; E_c = Young's modulus of concrete slab; ν_f = Poisson's ratio of foundation; and h = thickness of slab. For computing deflections, they suggested that only 42% of the value obtained from equation 8 be used. Based on equation 8, the corresponding k for $E_c = 4 \times 10^6$ psi (28 GN/m²), $E_f = 1 \times 10^4$ psi (69 MN/m²), $\nu_f = 0.4$ and h = 8 in. (203 mm) is 150 pci (41 MN/m³).

It was found in this study that equation 8 is applicable only to loads in the interior but not to loads near to the edge or joint. Figure 5 shows a comparison between liquid and solid foundations for the slab shown in Figure 4 using the fine mesh. When a k value of 250 pci (68 MN/m³) is used, the critical stress decreases and checks more closely with the solid foundation, but the differences in stresses at other locations increase. There is not a single k value which can yield stresses at all points comparable to those obtained from the solid

foundation. A comparison of deflections between liquid and solid foundations indicates that the solid foundation has a much larger deflection basin and distributes the load over a larger area.

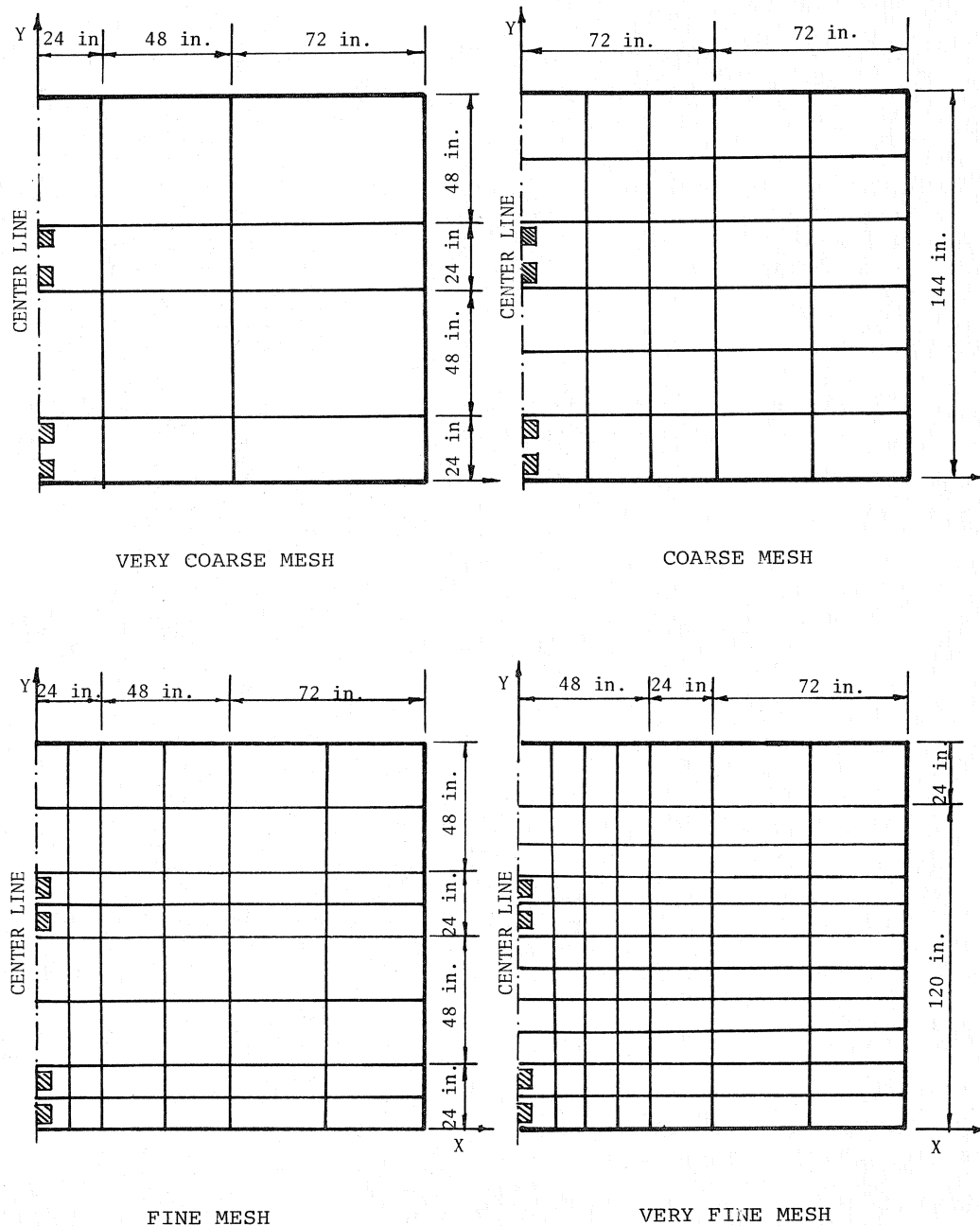
Figure 6 shows a comparison between liquid and solid foundations when the single axle load is applied at the joint. The loading position and the division of the two slabs into finite elements are shown in Figure 7. The joint has a dowel diameter of 1 in. (25.4 mm), a dowel spacing of 12 in. (30.5 mm), and a modulus of dowel support of 1×10^6 psi (270 GN/m³). Figure 6 shows that at some locations, both solutions check quite well; but at other locations, a large discrepancy exists. However, the general trend of variations is similar between the two solutions.

A comparison between Figures 5 and 6 clearly indicates that the most critical loading position is not at the joint but at the pavement edge far from the joints.

Effect of Warping on Loading Stresses

It is well known that temperature warping changes the contact condition between slab and foundation and thus affects the stresses due to loading. During the day when the temperature on the top of slab is higher than that at the bottom, the slab

Figure 4. Four different grid sizes with loading near edge.
(1 in. = 25.4 mm)



warps down and causes the interior of slab to rise and not in contact with the foundation. At night when the temperature on top is lower than that at bottom, the slab warps up and causes the edge of slab to curl and out of contact with the foundation. It is interesting to see how temperature warping affects the stresses due to loading. The slab with the fine mesh shown in Figure 4 is selected for analysis. The temperature differential between the top and bottom of slab is assumed to be 10°F (5.6°C) for upward warping and 20°F (11.1°C) for downward warping. These are reasonable values for a 8 in.

slab as measured in the several road tests.

Figure 8 shows the effect of temperature warping on loading stresses when the slab is placed on a solid foundation. The solid lines are based on full contact and the dashed lines and small circles are based on partial contact. In the latter case, the analysis is divided into two steps. First, the gaps and precompressions due to the weight of slab and temperature warping are determined, which are then used as initial conditions to compute the stresses due to applied loads. It can be seen from the figure that the maximum stress is not affected by

Table 4. Effect of grid sizes on stresses and deflections.

x Coord. (in.)	y Coord. (in.)	Stress or Deflection	Grid Size			
			Very Coarse	Coarse	Fine	Very Fine
0	0	σ_x (psi)	200.6	204.2	227.6	229.8
		w (0.001 in.)	15.5	15.7	16.8	16.8
0	24	σ_x (psi)	128.6	142.3	150.3	154.6
		σ_y (psi)	9.7	34.5	21.8	29.1
		w (0.001 in.)	14.7	14.6	15.3	15.3
		σ_x (psi)	114.6	127.5	132.0	136.2
0	72	σ_y (psi)	36.2	63.0	52.7	60.7
		w (0.001 in.)	13.5	13.4	13.7	13.6
		σ_x (psi)	110.6	118.9	123.1	127.0
0	96	σ_y (psi)	53.1	69.5	60.1	68.5
		w (0.001 in.)	12.1	12.0	12.2	12.2
		σ_x (psi)	-2.2	7.5	16.3	19.3
24	0	w (0.001 in.)	13.2	13.3	14.2	14.2
		σ_x (psi)	-34.4	-29.6	-32.9	-33.1
72	0	w (0.001 in.)	6.8	6.8	7.1	7.1

Note: (1) 1 in. = 25.4 mm and 1 psi = 6.9 kN/m².

(2) Positive stresses indicate tension at bottom and negative stresses indicate tension at top.

(3) σ_x = longitudinal stress; σ_y = transverse stress; and w = vertical deflection.

warping. The upward warping has very little effect on the stresses due to loading, although a slight increase in the negative stresses along the transverse direction is noted. The downward warping results in a significant stress increase at the interior of slab but not along the edge because, when the slab warps down, the edge is still in contact but the interior is not. It was found that if there are initial gaps at the edge, the maximum stress and the negative stresses in the transverse direction will increase as the slab warps up but the interior stresses will decrease as the slab warps down. However, these changes are not very significant.

Conclusions

A finite element computer package was developed for the analysis and design of concrete pavements. The package can be applied to multiple jointed slabs on liquid foundations, solid foundations or layer foundations. The method for developing the package was presented and its capabilities and limitations were described. The package has been fully tested and a user's manual is available.

The package can be applied to any number of slabs arranged in an arbitrary pattern, as long as the storage capacity of the computer is not exceeded. To reduce the storage requirement, an iteration method was developed to solve the large

number of simultaneous equations. The package can analyze the contact condition between slab and foundation due to initial gaps, temperature warping and loading, either separately or jointly. Both shear and moment transfer across the joints and the looseness of dowel support can also be considered.

Examples are given to illustrate the application of the package to the analysis of concrete pavements. Based on the results of analysis, the following conclusions can be drawn:

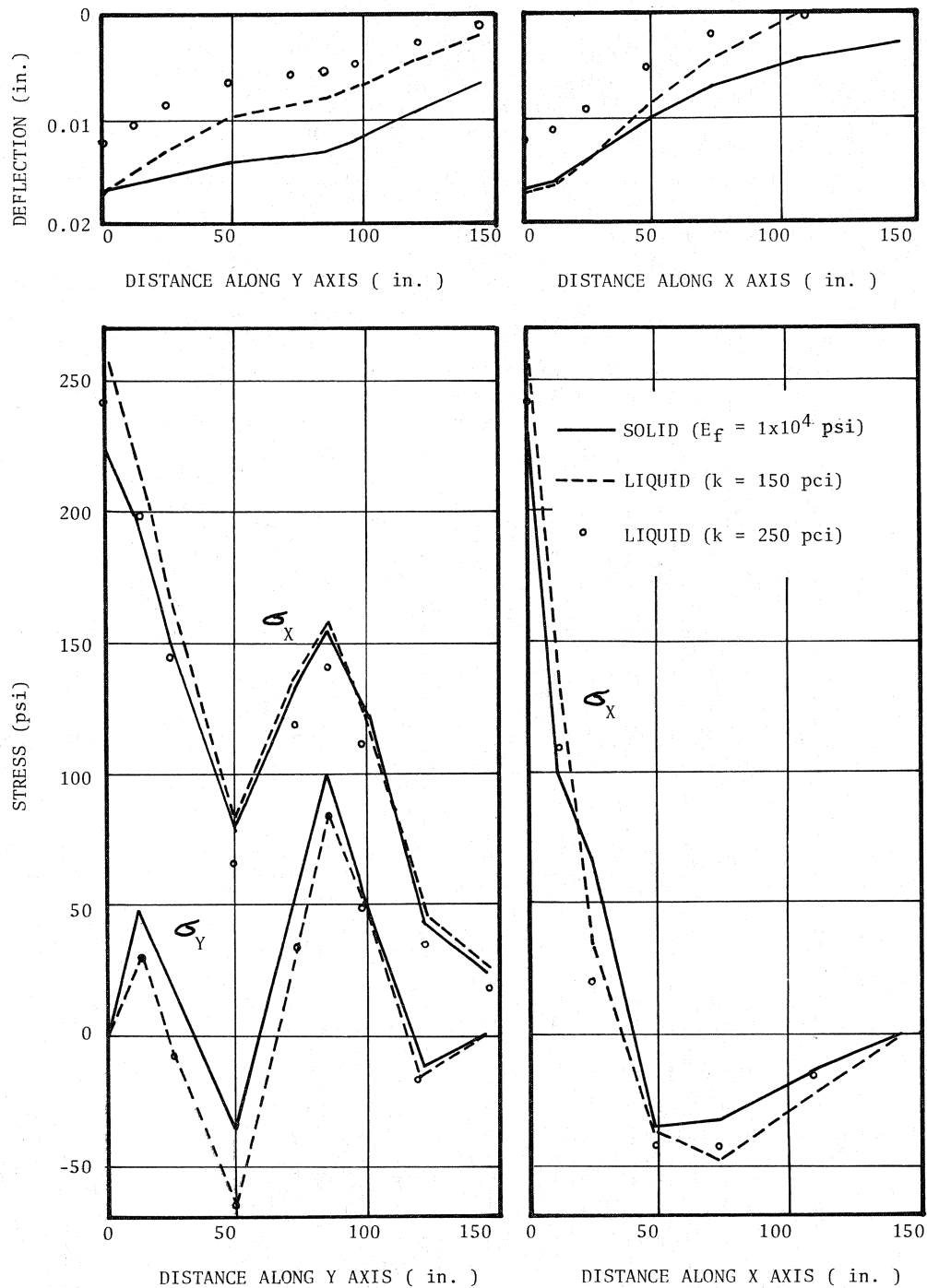
1. When the concrete slab is much stiffer than the foundation and the load is applied in the interior of the slab, the finite element solutions based on plate theory, as obtained from the package, check closely with the solutions from Burmister's layer theory, indicating that the plate theory and the finite element method result in reasonable solutions.

2. The use of an equivalent layer based on the transformed section concept yields vertical deflections and stresses in the bottom layer comparable to those determined by Burmister's layer theory. However, the stresses in the top layer do not check with the layer theory.

3. When two layers of slab are unbonded, the bottom layer can be considered either as a second layer of the slab or as a top layer in the layer foundation. Both solutions yield comparable results.

4. A comparison of results on four different grid sizes indicates that even a very coarse mesh can produce reasonable solutions.

Figure 5. Comparison between liquid and solid foundations with loading near edge.
 (1 in. = 25.4 mm, 1 psi = 6.9 kN/m², 1 pci = 272 kN/m³)



5. For interior loading, it is possible to select a modulus of subgrade reaction, or k value, for liquid foundations such that the distribution of stresses in the slab will bear a close resemblance to that obtained by solid foundations. However, this is not true for loading at the edge or near to the joint. In spite of this fact, the general pattern of stress distribution based on liquid foundations is very similar to that based on solid foundations.

6. The most critical loading position for a

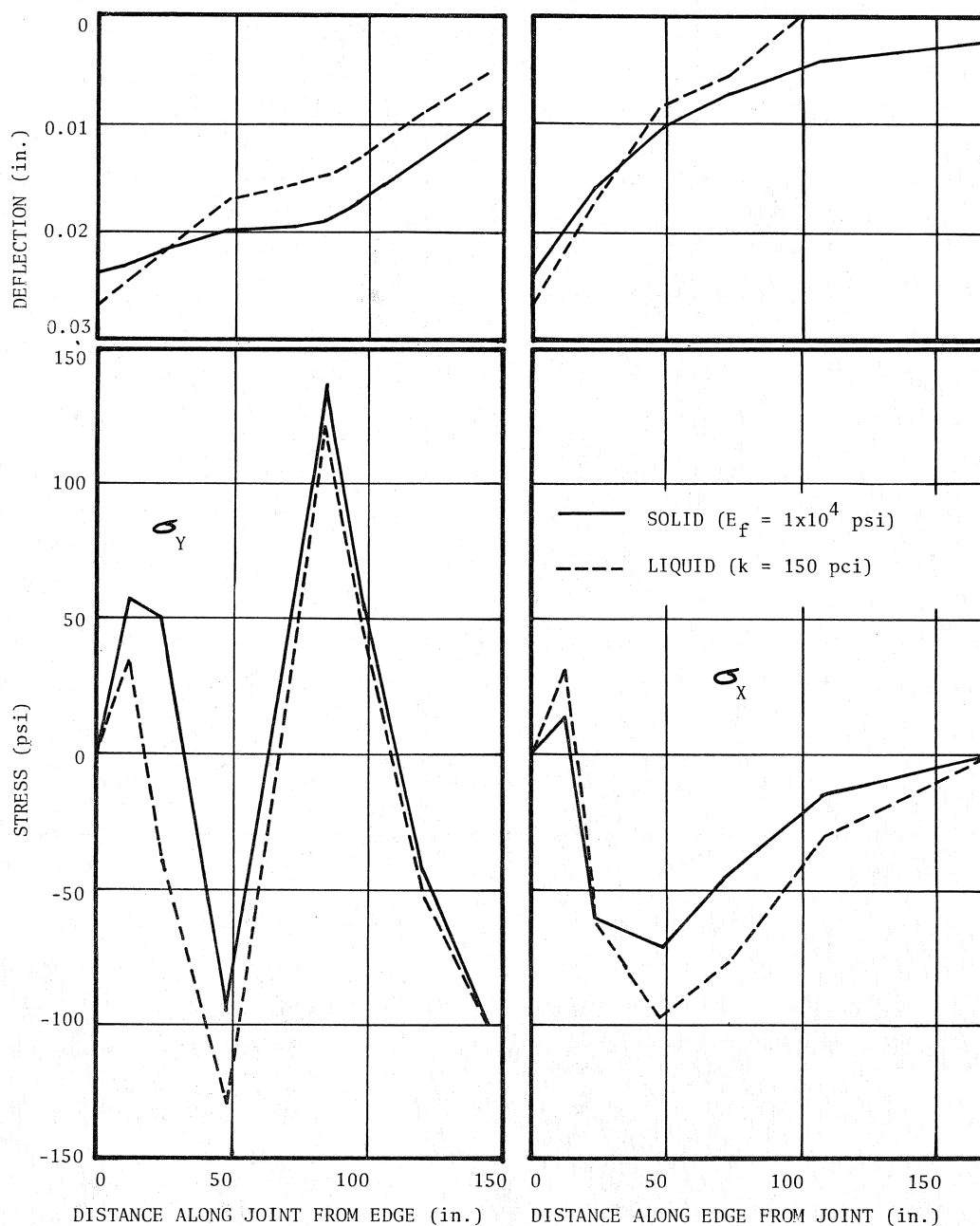
conventional highway pavement with standard dowel joints is not at the joint but at the pavement edge far from the joints.

7. If a design is based on the edge loading, it appears that temperature warping has very little effect on the critical stress due to loading. Because the most critical stress under edge loading is in the longitudinal direction, even some initial gaps near the edge will not cause an appreciable increase in the critical stress.

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Figure 6. Comparison between liquid and solid foundations with loading at joint.
 (1 in. = 25.4 mm, 1 psi = 6.9 kN/m², 1 pci = 272 kN/m³)



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Figure 7. Two slabs with loading at joint.
(1 in. = 25.4 mm)

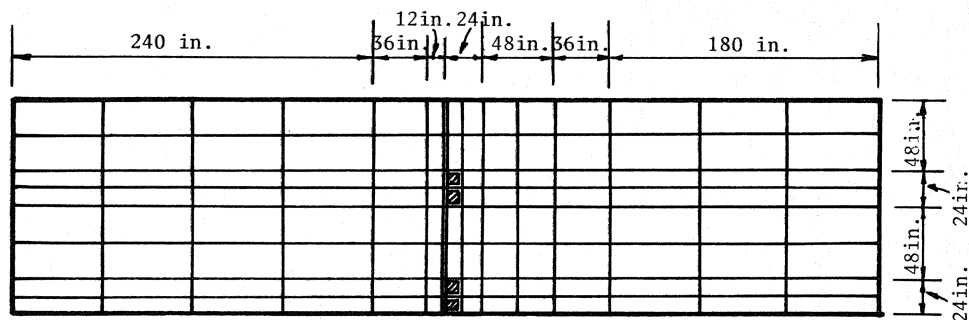


Figure 8. Effect of temperature warping on loading stresses.
(1 in. = 25.4 mm, 1 psi = 6.9 kN/m²)

